

# Optical Waveguide Theory (H)



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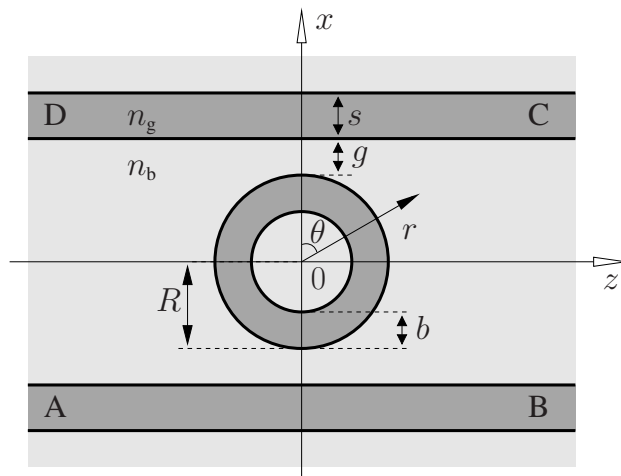
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## Course overview

### Optical waveguide theory

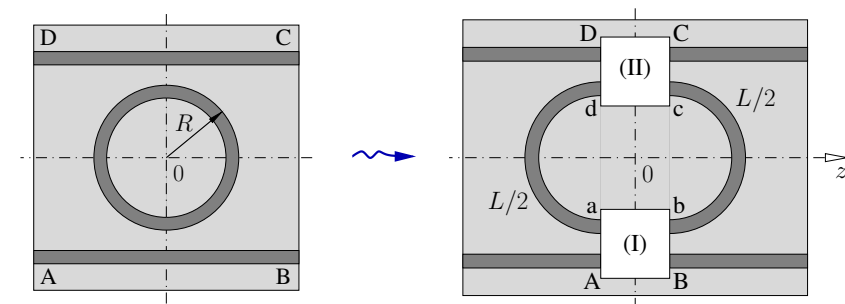
- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
  - Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
  - Oblique semi-guided waves: 2-D integrated optics.
  - Summary, concluding remarks.

### Circular traveling wave resonators



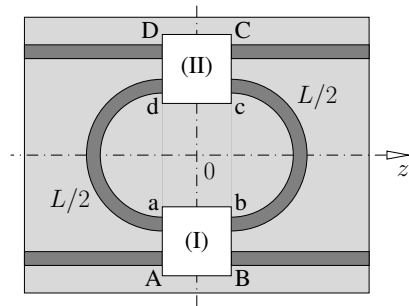
Integrated optical micro-ring or micro-disk resonators.

### Ringresonator: Abstract model



- Ringresonator  $\approx$  2 couplers + 2 cavity segments
- CW description:  $\mathbf{E}, \mathbf{H} \sim e^{i\omega t}$ ,  $\omega = kc$ ,  $k = 2\pi/\lambda$ .

## Couplers: Scattering matrices



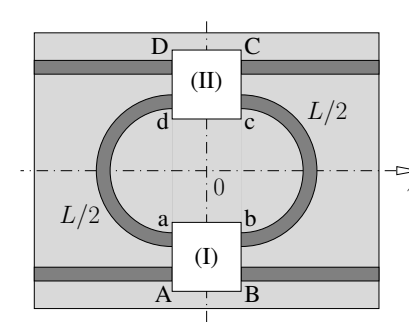
- Uniform polarization, single mode waveguides.
- Linear, nonmagnetic (attenuating) elements.
- Backreflections are negligible.
- Interaction restricted to the couplers  $\leftrightarrow$  "port" definition.

$\hookrightarrow$  Symmetric coupler scattering matrices:

$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \chi & \tau \\ \rho & \chi & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$A_{\pm}, B_{\pm}, a_{\pm}, b_{\pm}$ : Amplitudes of waves traveling in  $\pm z$ -direction.

## Coupler symmetries



Symmetry  $z \rightarrow -z$ :

$$A_+ \rightarrow b_+ \stackrel{!}{=} B_- \rightarrow a_-$$

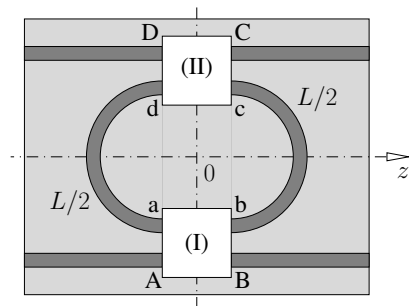
$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \kappa & \tau \\ \rho & \kappa & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$$\hookrightarrow \begin{pmatrix} A_- \\ a_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} B_- \\ b_- \end{pmatrix}, \quad \begin{pmatrix} B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \end{pmatrix}.$$

Symmetry  $x \rightarrow -x$ , (I) = (II):

$$\hookrightarrow \begin{pmatrix} D_- \\ d_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} C_- \\ c_- \end{pmatrix}, \quad \begin{pmatrix} C_+ \\ c_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} D_+ \\ d_+ \end{pmatrix}.$$

## Cavity segments



Field evolution  $\sim e^{-i\gamma s}$  along the cavity core, propagation distance  $s$ .

$$\gamma = \beta - i\alpha,$$

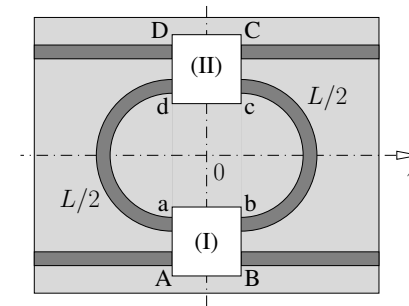
$\beta$ : phase propagation constant,  
 $\alpha$ : attenuation constant.

( $\leftrightarrow$  bend modes, to come.)

$\hookrightarrow$  Relations of amplitudes at the ends of the cavity segments:

$$\begin{aligned} c_- &= b_+ e^{-i\beta L/2} e^{-\alpha L/2}, & a_+ &= d_- e^{-i\beta L/2} e^{-\alpha L/2}, \\ b_- &= c_+ e^{-i\beta L/2} e^{-\alpha L/2}, & d_+ &= a_- e^{-i\beta L/2} e^{-\alpha L/2}. \end{aligned}$$

## Output amplitudes



Coupler scattering matrices  
+ Cavity field evolution  
+ External input amplitudes

$$A_+ = \sqrt{P_{in}},$$

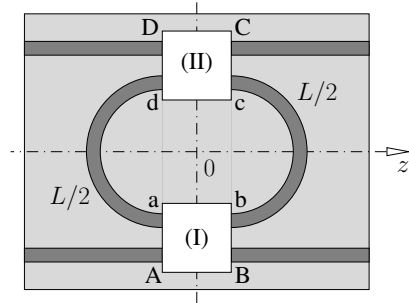
$$B_- = C_- = D_+ = 0$$

External output amplitudes:

$$A_- = 0, \quad C_+ = 0, \quad D_- = \frac{\kappa^2 p}{1 - \tau^2 p^2} A_+, \quad B_+ = \left( \rho + \frac{\kappa^2 \tau p^2}{1 - \tau^2 p^2} \right) A_+,$$

$$p = e^{-i\beta L/2} e^{-\alpha L/2}.$$

## Power transfer



Power drop:  $P_D = |D_-|^2$ ,  
 Transmission:  $P_T = |B_+|^2$ .

$$P_D = P_{in} \frac{|\kappa|^4 e^{-\alpha L}}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$P_T = P_{in} \frac{|\rho|^2 (1 + |\tau|^2 d^2 e^{-2\alpha L} - 2|\tau| d e^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$\tau =: |\tau| e^{i\varphi}, \quad d e^{i\psi} := \tau - \kappa^2/\rho, \quad L \neq 2\pi R.$$

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## Resonances

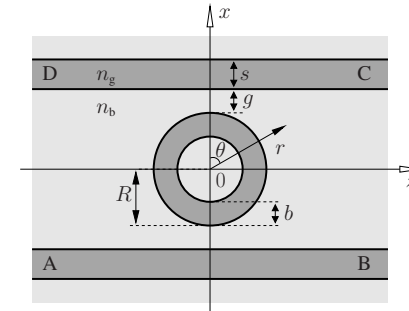
$$P_D = P_{in} \frac{|\kappa|^4 e^{-\alpha L}}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)} (\lambda)$$

$$P_T = P_{in} \frac{|\rho|^2 (1 + |\tau|^2 d^2 e^{-2\alpha L} - 2|\tau| d e^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)} (\lambda)$$

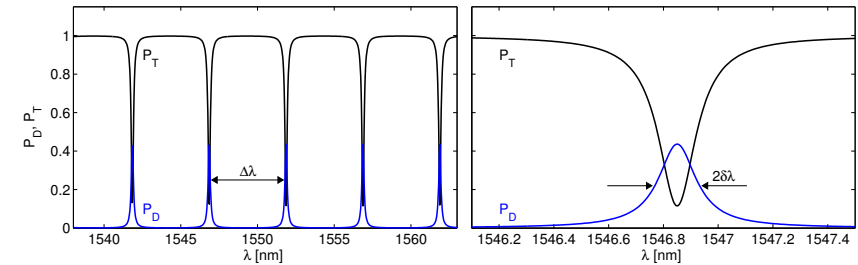
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## Spectral response



$R = 50 \mu\text{m}$ ,  $b = s = 1.0 \mu\text{m}$ ,  $g = 0.9 \mu\text{m}$ ,  
 $n_b = 1.45$ ,  $n_g = 1.60$ ; 2-D, TE.  
 $\Delta\lambda = 5.0 \text{ nm}$ ,  $2\delta\lambda = 0.17 \text{ nm}$ ,  
 $F = 30$ ,  $Q = 9400$ ,  $P_{D,\text{res}} = 0.44$ .



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## Resonances

$$P_D = P_{in} \frac{|\kappa|^4 e^{-\alpha L}}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$P_T = P_{in} \frac{|\rho|^2 (1 + |\tau|^2 d^2 e^{-2\alpha L} - 2|\tau| d e^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L - 2\varphi)}$$

- Resonances:  
 $\approx$  Singularities in the denominators of  $P_D$ ,  $P_T$ , origin:  $\beta(\lambda)$ .
- Correction for finite coupler length  $l$ :  
 $\beta L - 2\varphi = \beta L_{\text{cav}} - \phi$ ,  $\phi = 2\beta l + 2\varphi$ ,  $L_{\text{cav}} = 2\pi R$ ,  $\partial_\lambda \phi \approx 0$ .
- Resonance condition:  $\cos(\beta L_{\text{cav}} - \phi) = 1$ , or

$$\beta = \frac{2m\pi + \phi}{L_{\text{cav}}} =: \beta_m \quad \text{integer } m; \quad P_D|_{\beta=\beta_m} = P_{in} \frac{|\kappa|^4 e^{-\alpha L}}{(1 - |\tau|^2 e^{-\alpha L})^2}.$$

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## Free spectral range

- Resonance next to  $\beta_m$  :

$$\beta_{m-1} = \frac{2(m-1)\pi + \phi}{L_{\text{cav}}} = \beta_m - \frac{2\pi}{L_{\text{cav}}} \approx \beta_m + \left. \frac{\partial \beta}{\partial \lambda} \right|_m \Delta \lambda$$

- $\partial_\lambda \beta = ?$

$q_j$  : waveguide parameters with dimension length,

$$\beta(a\lambda, aq_j) = \beta(\lambda, q_j)/a, \quad \partial_a |_{a=1}$$

$$\hookrightarrow \frac{\partial \beta}{\partial \lambda} = -\frac{1}{\lambda} \left( \beta + \sum_j q_j \frac{\partial \beta}{\partial q_j} \right) \approx -\frac{\beta}{\lambda}.$$

$$\text{FSR:} \quad \Delta \lambda = -\frac{2\pi}{L_{\text{cav}}} \left( \left. \frac{\partial \beta}{\partial \lambda} \right|_m \right)^{-1} \approx \frac{\lambda^2}{n_{\text{eff}} L_{\text{cav}} |}_m, \quad n_{\text{eff}} = \beta/k.$$

(Free spectral range, the spectral distance (here: wavelength) between the drop peaks / the transmission dips).

## Finesse & Q-factor

$$\text{Finesse:} \quad F = \frac{\Delta \lambda}{2\delta \lambda} = \pi \frac{|\tau| e^{-\alpha L/2}}{1 - |\tau|^2 e^{-\alpha L}}.$$

$$\text{Q-factor:} \quad Q = \frac{\lambda}{2\delta \lambda} = \pi \frac{n_{\text{eff}} L_{\text{cav}}}{\lambda} \frac{|\tau| e^{-\alpha L/2}}{1 - |\tau|^2 e^{-\alpha L}} = \frac{n_{\text{eff}} L_{\text{cav}}}{\lambda} F.$$

$$\text{or} \quad Q = k R n_{\text{eff}} F \quad \text{for} \quad L_{\text{cav}} = 2\pi R.$$

## Spectral width of the resonances

$$P_D = P_{\text{in}} \frac{|\kappa|^4 e^{-\alpha L}}{1 + |\tau|^4 e^{-2\alpha L} - 2|\tau|^2 e^{-\alpha L} \cos(\beta L_{\text{cav}} - \phi)},$$

$$P_D|_{\beta_m} = P_{D,\text{res}}.$$

$$P_D|_{\beta_m + \delta \beta} = P_{D,\text{res}}/2. \quad \delta \beta = ?$$

- Expansion of cos-terms

$$\hookrightarrow \delta \beta = \pm \frac{1}{L_{\text{cav}}} \left( \frac{1}{|\tau|} e^{\alpha L/2} - |\tau| e^{-\alpha L/2} \right) \approx -\frac{\beta_m}{\lambda} \delta \lambda$$

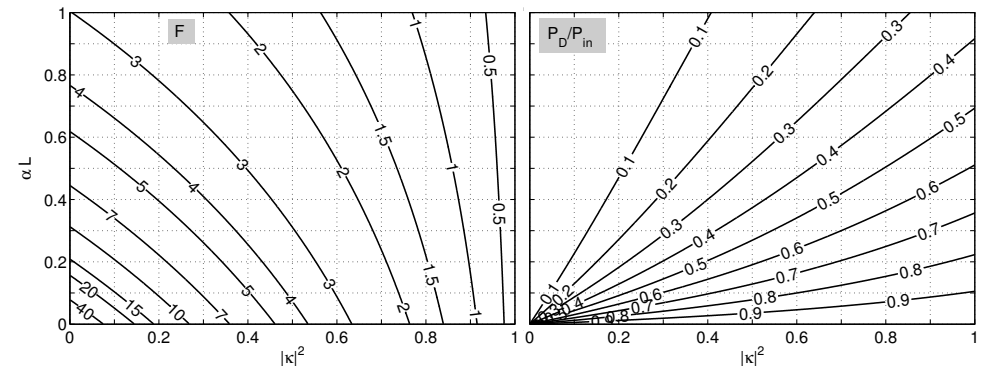
$$\text{FWHM:} \quad 2\delta \lambda = \frac{\lambda^2}{\pi L_{\text{cav}} n_{\text{eff}} |}_m \left( \frac{1}{|\tau|} e^{\alpha L/2} - |\tau| e^{-\alpha L/2} \right).$$

(Full width at half maximum of the spectral drop peaks / the transmission dips (wavelength).)

## Performance versus coupling strength & losses

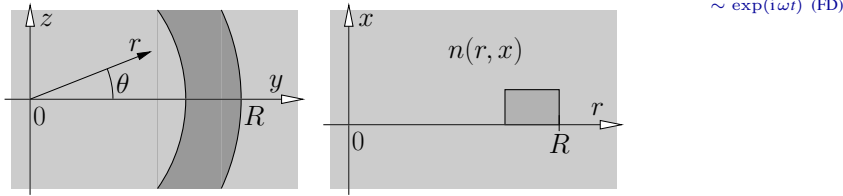
Assumption: Lossless coupler elements,  $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$ .

$$F = \pi \frac{(\sqrt{1 - |\kappa|^2}) e^{-\alpha L/2}}{1 - (1 - |\kappa|^2) e^{-\alpha L}}, \quad P_D|_{\text{res}} = P_{\text{in}} \frac{|\kappa|^4 e^{-\alpha L}}{(1 - (1 - |\kappa|^2) e^{-\alpha L})^2}.$$



$\alpha, \kappa = ?$

## Modes of bent waveguides



- Constant curvature  $\leftrightarrow$  cylindrical coordinates  $r, \theta, x$ .
- Bend radius  $R$ ,  $\partial_\theta \epsilon = 0$ ,  $\partial_\theta n = 0$

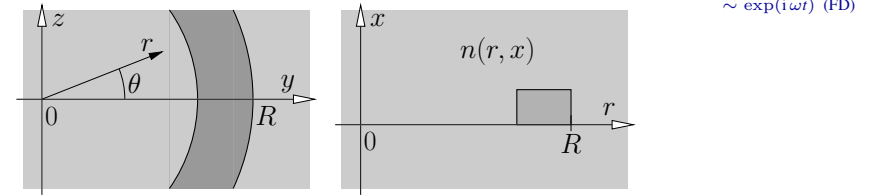
$$\hookrightarrow \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (r, \theta, x) = \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{H}} \end{pmatrix} (r, x) e^{-i\gamma R\theta}, \quad \text{bend modes,}$$

$\bar{\mathbf{E}}, \bar{\mathbf{H}}$ : bend mode profile, components  $\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x$ ,  
 $\gamma = \beta - i\alpha \in \mathbb{C}$ : propagation constant,  
 $\beta \in \mathbb{R}$ : phase constant,  
 $\alpha \in \mathbb{R}$ : attenuation constant.

(Exponent  $i\gamma R\theta$ : a convention, "propagation distance"  $R\theta$ .)

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## Modes of bent waveguides



- Piecewise constant  $n(r, x)$ ,  $\psi \in \{\bar{E}_r, \bar{E}_\theta, \bar{E}_x, \bar{H}_r, \bar{H}_\theta, \bar{H}_x\}$ ,

$$\hookrightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \left( k^2 n^2 - \frac{\gamma^2 R^2}{r^2} \right) \psi = 0, \quad \text{where } \partial n = 0,$$

& continuity conditions at interfaces (cylindrical coordinates),

& boundary conditions:

regularity at  $r = 0$ , outgoing waves at  $x = \pm\infty$ ,  $r = \infty$ .

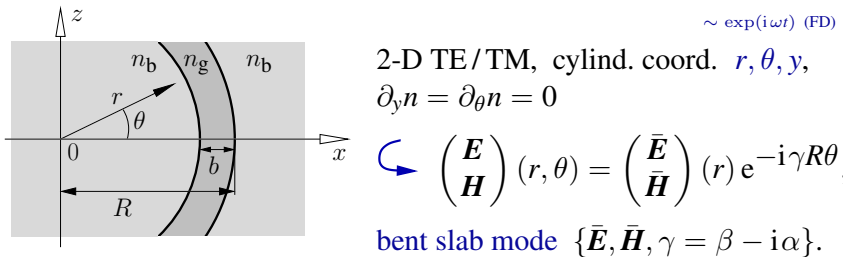
(or: normalizability versus  $x$ .)

Vectorial 3-D bend mode eigenvalue problem.

(Practical setting: computational domain  $r_1 < r < r_0$ ,  $x_b < x < x_t$ , PML boundary conditions /  $\psi = 0$  at  $r = r_1$ .)

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## Modes of bent slab waveguides



2-D TE/TM, cylind. coord.  $r, \theta, y$ ,  
 $\partial_y n = \partial_\theta n = 0$

$$\hookrightarrow \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} (r, \theta) = \begin{pmatrix} \bar{\mathbf{E}} \\ \bar{\mathbf{H}} \end{pmatrix} (r) e^{-i\gamma R\theta},$$

bent slab mode  $\{\bar{\mathbf{E}}, \bar{\mathbf{H}}, \gamma = \beta - i\alpha\}$ .

- Piecewise constant  $n(r)$ ,  $\phi = \bar{E}_y$  (TE),  $\phi = \bar{H}_y$  (TM)

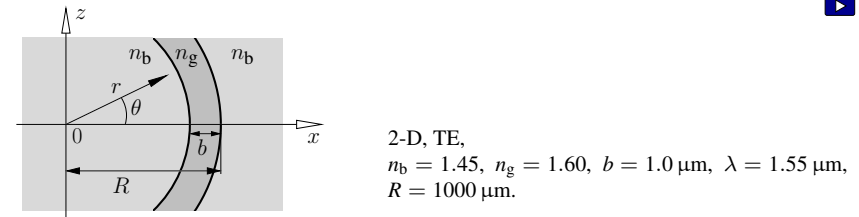
$$\hookrightarrow \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \left( k^2 n^2 - \frac{\gamma^2 R^2}{r^2} \right) \phi = 0,$$

(Bessel differential equation with (complex) order  $\gamma R$ .)

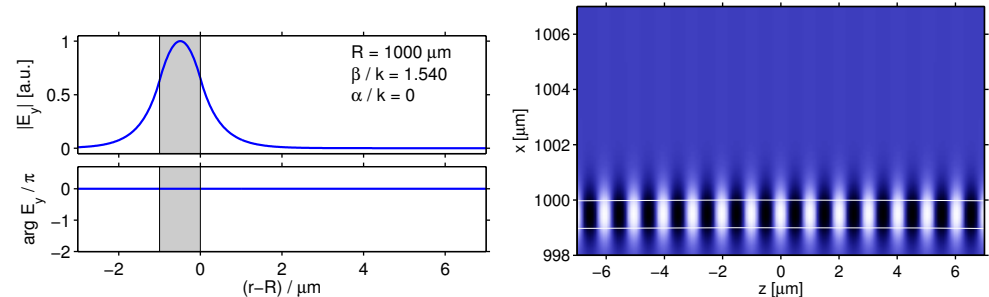
- Nonzero solutions,
- bounded at the origin,  $\sim J_{\gamma R}(nkr)$  for  $r < R - b$ ,
- outgoing exterior fields,  $\sim H_{\gamma R}^{(2)}(nkr)$  for  $r > R$ , ( $\sim \exp(i\omega t)$ ),
- continuity at interfaces:  $\phi$ ,  $\partial_r \phi$  (TE),  $\phi$ ,  $(\partial_r \phi)/n^2$  (TM).

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## Bend modes, 2-D examples

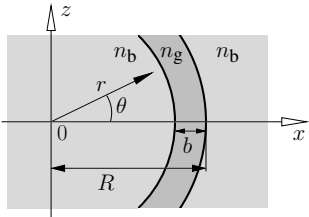


2-D, TE,  
 $n_b = 1.45$ ,  $n_g = 1.60$ ,  $b = 1.0 \mu\text{m}$ ,  $\lambda = 1.55 \mu\text{m}$ ,  
 $R = 1000 \mu\text{m}$ .

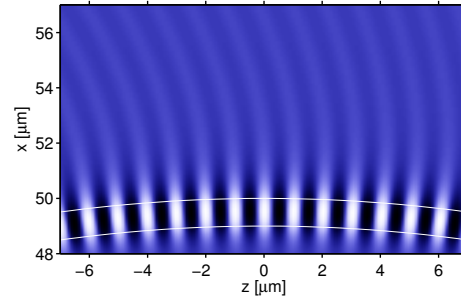
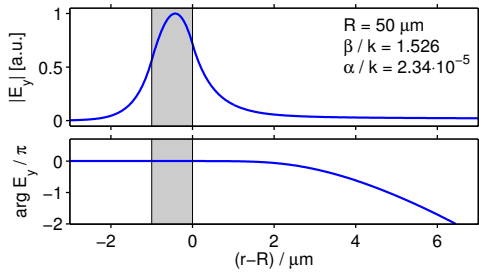


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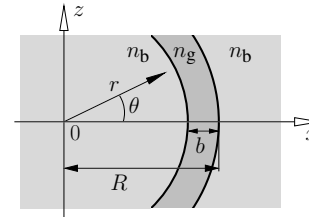
## Bend modes, 2-D examples



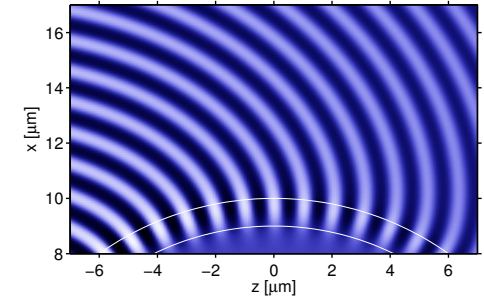
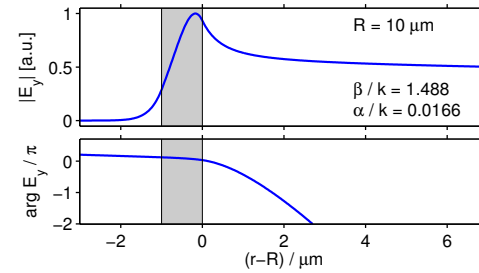
2-D, TE,  
 $n_b = 1.45$ ,  $n_g = 1.60$ ,  $b = 1.0 \mu\text{m}$ ,  $\lambda = 1.55 \mu\text{m}$ ,  
 $R = 50 \mu\text{m}$ .



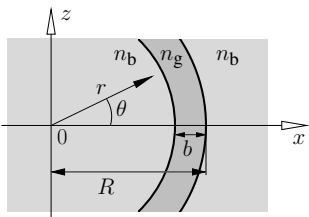
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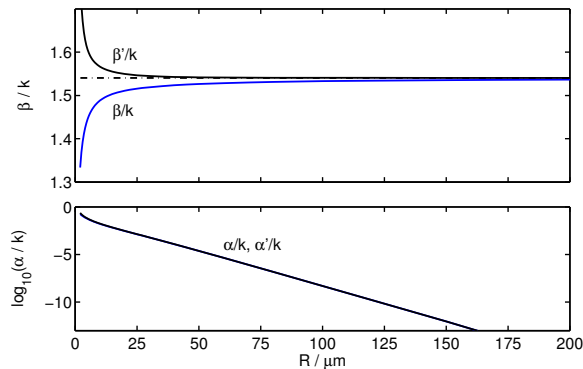
2-D, TE,  
 $n_b = 1.45$ ,  $n_g = 1.60$ ,  $b = 1.0 \mu\text{m}$ ,  $\lambda = 1.55 \mu\text{m}$ ,  
 $R = 10 \mu\text{m}$ .



## Propagation constant vs. bend radius



2-D, TE,  
 $n_b = 1.45$ ,  $n_g = 1.60$ ,  $b = 1.0 \mu\text{m}$ ,  $\lambda = 1.55 \mu\text{m}$ ,  
 $R \in [2, 200] \mu\text{m}$ .



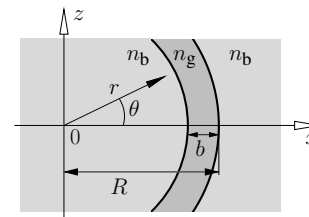
Alternative definition:  
 $R' = R - b/2$ .

Identical physical fields

$$\gamma'R' = \gamma R,$$

$$\gamma' = \gamma \frac{R}{R - b/2}.$$

## Power & orthogonality



2-D TE/TM bend modes:

- Power flow:  $S_r \neq 0$ ,  $S_r, S_\theta \sim e^{-2\alpha R\theta}$ ,  $S_\theta \sim |\phi|^2/r$

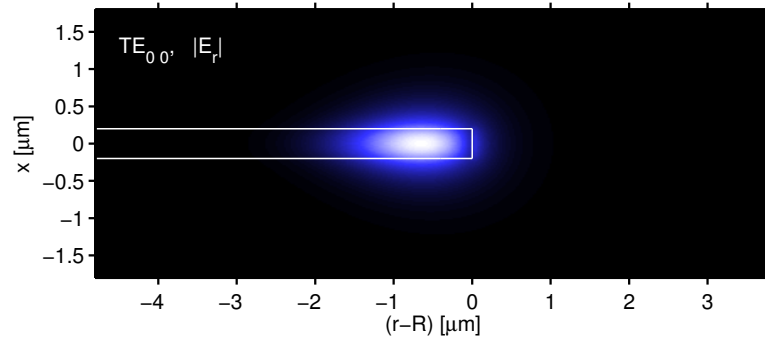
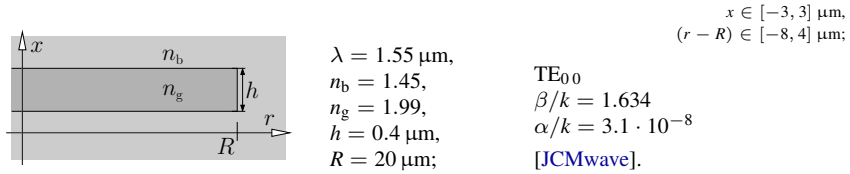
$$\int_0^\infty S_\theta(r) dr < \infty \quad \text{power normalization.}$$

- Orthogonality of nondegenerate bend modes, product

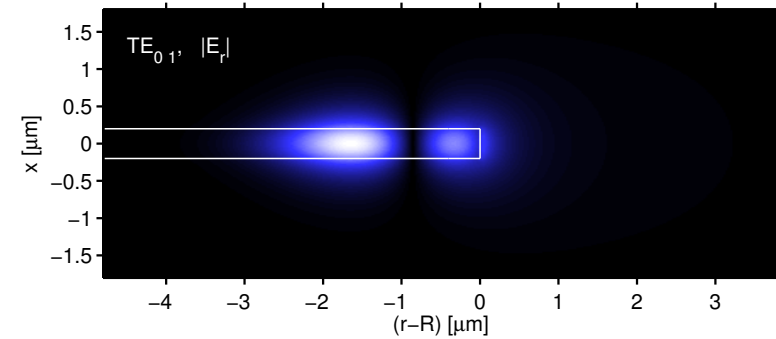
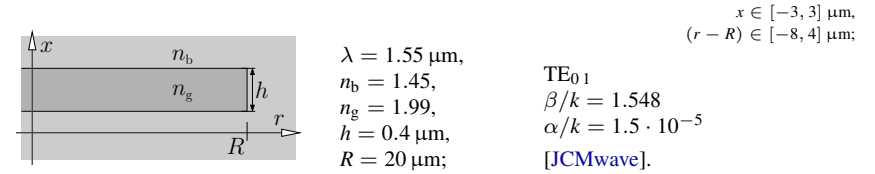
$$[\mathbf{E}_1, \mathbf{H}_1; \mathbf{E}_2, \mathbf{H}_2] = \int_0^\infty (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1) \cdot \mathbf{e}_\theta dr.$$

(Here  $[\cdot, \cdot; \cdot, \cdot]$  is complex valued.)  
 (Expressions  $\sim \phi^2/r \rightarrow$  convergence of the integrals.)

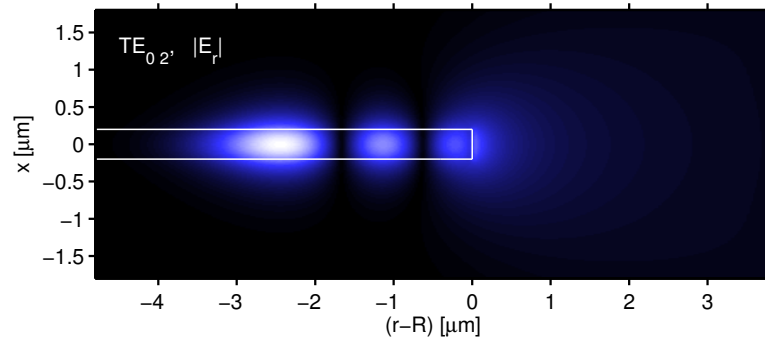
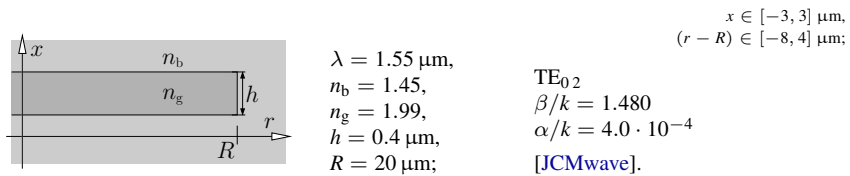
### Bend modes supported by an angular disc segment



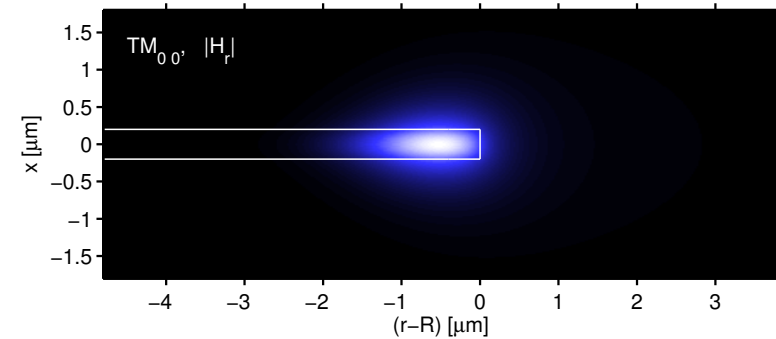
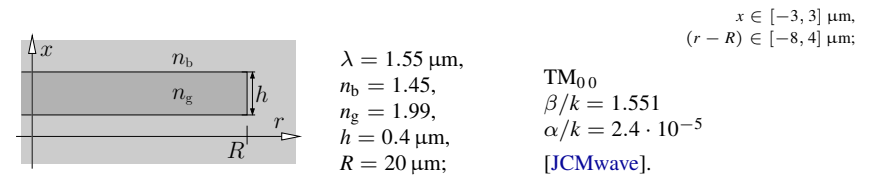
### Bend modes supported by an angular disc segment



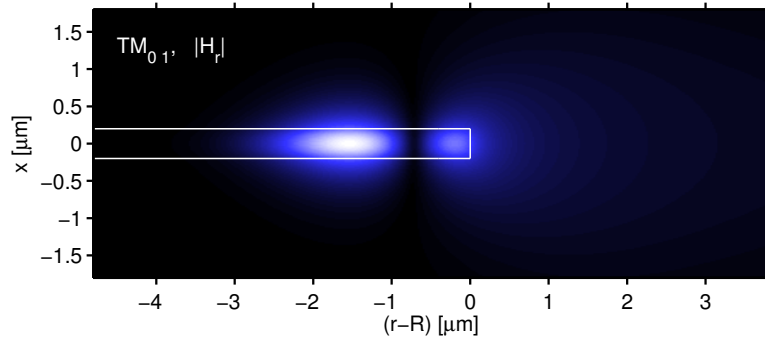
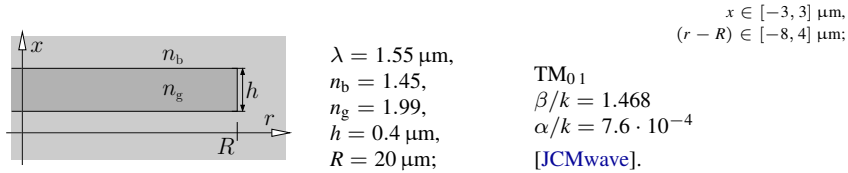
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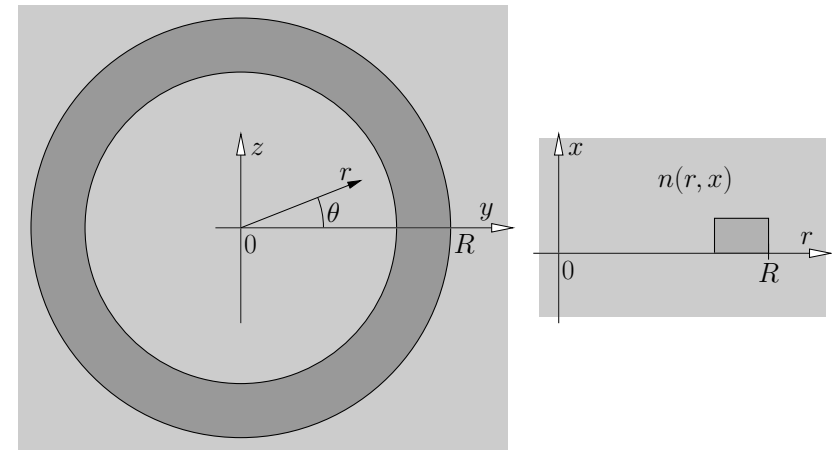
### Bend modes supported by an angular disc segment



## Bend modes supported by an angular disc segment



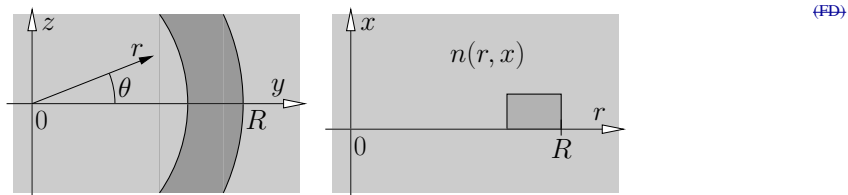
## Circular microcavity



Bend modes  $\longleftrightarrow$  Whispering gallery resonances.

(Terms not always clearly distinguished.)

## Whispering gallery resonances



- Full cavity,  $\theta \in [0, 2\pi]$ :  
Look for resonances in the form of **whispering gallery modes**

$$\left( \begin{matrix} \mathbf{E} \\ \mathbf{H} \end{matrix} \right) (r, \theta, x, t) = \left( \begin{matrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{H}} \end{matrix} \right) (r, x) e^{i\omega_c t - im\theta}, \quad \text{+c.c.}$$

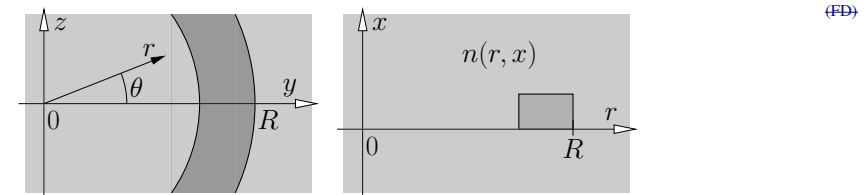
$\tilde{\mathbf{E}}, \tilde{\mathbf{H}}$ : **WGM profile**, components  $\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x$ ,

$m \in \mathbb{Z}$ : **angular order**,

$\omega_c = \omega'_c + i\omega''_c \in \mathbb{C}$ : **eigenfrequency**,  $\omega'_c, \omega''_c \in \mathbb{R}$ .

Q-factor  $Q = \omega'_c / (2\omega''_c)$ , resonance wavelength  $\lambda_r = 2\pi c / \omega'_c$ , outgoing radiation, FWHM:  $2\delta\lambda = \lambda_r / Q$ .

## Whispering gallery resonances



- Piecewise constant  $n(r, x)$ ,  $\psi \in \{\tilde{E}_r, \tilde{E}_\theta, \tilde{E}_x, \tilde{H}_r, \tilde{H}_\theta, \tilde{H}_x\}$ ,  
(Dispersion ?)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \left( \frac{\omega_c^2}{c^2} n^2 - \frac{m^2}{r^2} \right) \psi = 0, \quad \text{where } \partial n = 0,$$

& continuity conditions at interfaces (cylindrical coordinates),

& boundary conditions:

regularity at  $r = 0$ , outgoing waves at  $x = \pm\infty$ ,  $r = \infty$ .

(or: normalizability versus  $x$ .)

Vectorial eigenproblem for whispering gallery resonances.

(Practical setting: computational domain  $r_1 < r < r_0$ ,  $x_b < x < x_t$ , PML boundary conditions /  $\psi = 0$  at  $r = r_1$ )

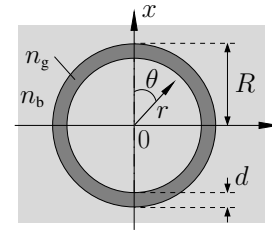


## 2-D whispering gallery resonances

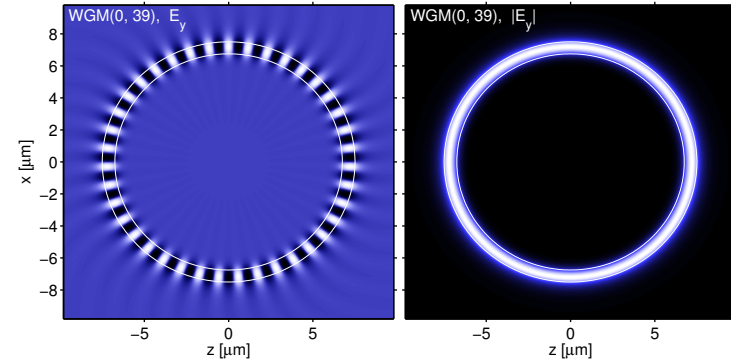
... as discussed for the 2-D TE/TM bend modes.

(WGMs: Bessel differential equation of integer order.)  
 (Notation: WGM( $\rho, m$ ) — mode of radial order  $\rho$  and angular order  $m$ .)

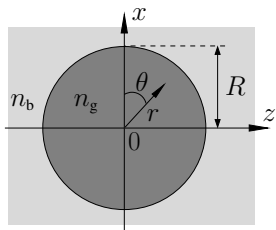
## 2-D whispering gallery resonances



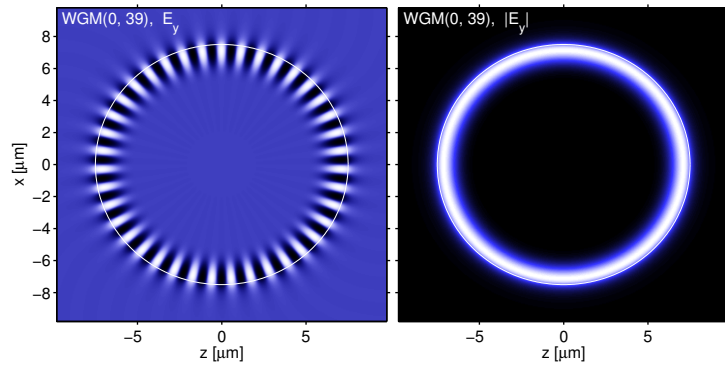
TE,  $R = 7.5 \mu\text{m}$ ,  $d = 0.75 \mu\text{m}$ ,  $n_g = 1.5$ ,  $n_b = 1.0$ .  
 WGM(0,39):  
 $\lambda_r = 1.5637 \mu\text{m}$ ,  $Q = 1.1 \cdot 10^5$ ,  $2\delta\lambda = 1.4 \cdot 10^{-5} \mu\text{m}$ .



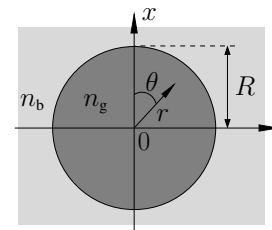
## 2-D whispering gallery resonances



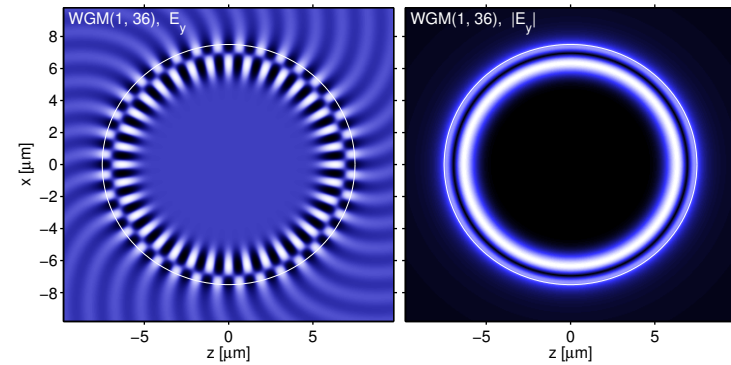
TE,  $R = 7.5 \mu\text{m}$ ,  $n_g = 1.5$ ,  $n_b = 1.0$ .  
 WGM(0,39):  
 $\lambda_r = 1.6025 \mu\text{m}$ ,  $Q = 5.7 \cdot 10^5$ ,  $2\delta\lambda = 2.8 \cdot 10^{-6} \mu\text{m}$ .



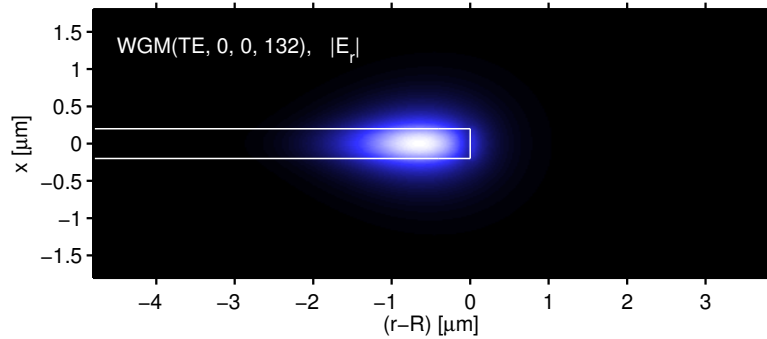
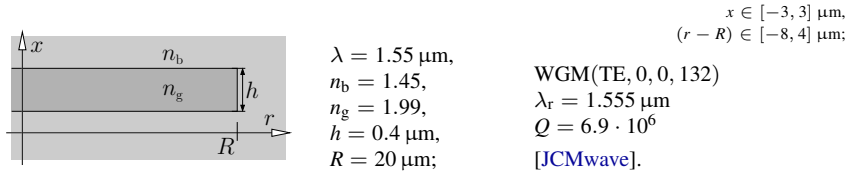
## 2-D whispering gallery resonances



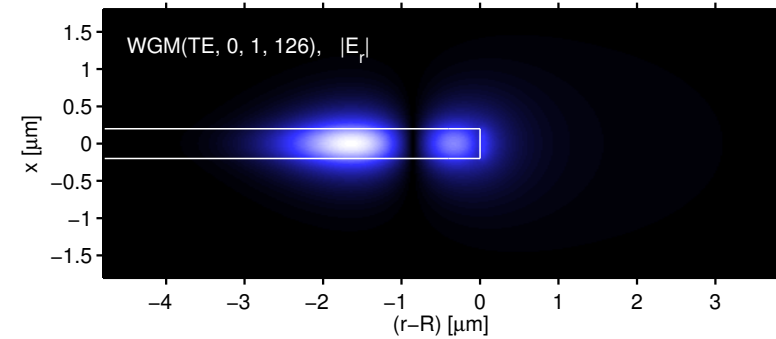
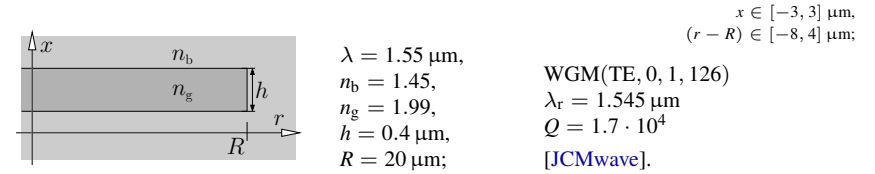
TE,  $R = 7.5 \mu\text{m}$ ,  $n_g = 1.5$ ,  $n_b = 1.0$ .  
 WGM(1,36):  
 $\lambda_r = 1.5367 \mu\text{m}$ ,  $Q = 2.2 \cdot 10^3$ ,  $2\delta\lambda = 7.0 \cdot 10^{-4} \mu\text{m}$ .



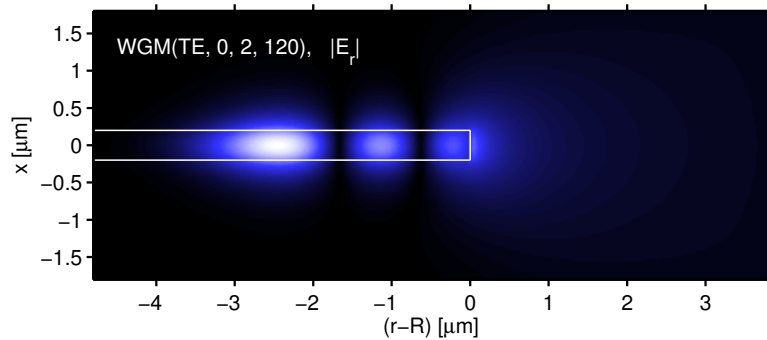
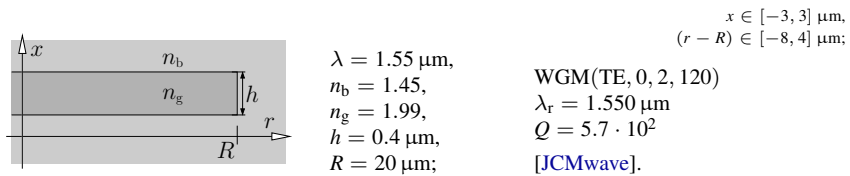
### WGMs supported by a circular slab disc



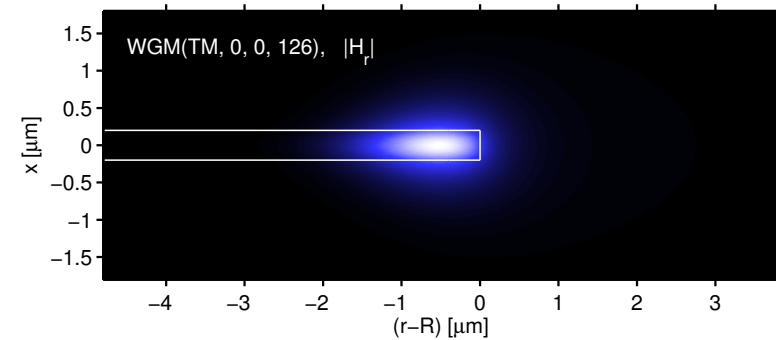
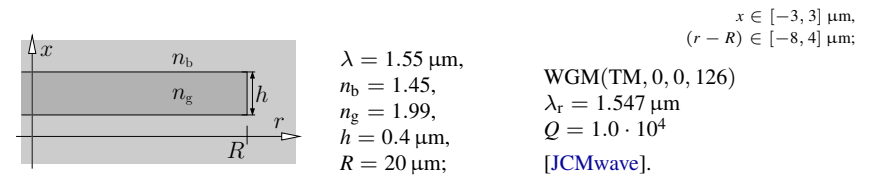
### WGMs supported by a circular slab disc



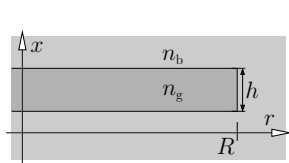
### WGMs supported by a circular slab disc



### WGMs supported by a circular slab disc



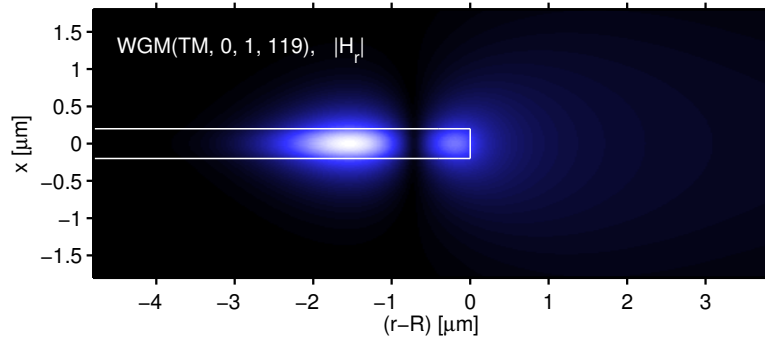
## WGMs supported by a circular slab disc



$\lambda = 1.55 \mu\text{m}$ ,  
 $n_b = 1.45$ ,  
 $n_g = 1.99$ ,  
 $h = 0.4 \mu\text{m}$ ,  
 $R = 20 \mu\text{m}$ ;

WGM(TM, 0, 1, 119)  
 $\lambda_r = 1.550 \mu\text{m}$   
 $Q = 3.0 \cdot 10^2$   
 [JCMwave].

$x \in [-3, 3] \mu\text{m}$ ,  
 $(r - R) \in [-8, 4] \mu\text{m}$ ;



## Bend modes versus whispering gallery resonances

(Field supported by a full circular cavity.)  
 (Incompatible models, in principle.)

[BWG]  $\omega \in \mathbb{R}$  given,  $\gamma = \beta - i\alpha \in \mathbb{C}$  eigenvalue,

$$\Phi(r, \theta, t) = \phi(r) e^{i\omega t - i\beta R\theta} e^{-\alpha R\theta}.$$

[WGM]  $\omega_c = \omega_c + i\omega_c'' \in \mathbb{C}$  eigenvalue,  $m \in \mathbb{Z}$  given,

$$\Psi(r, \theta, t) = \psi(r) e^{i\omega_c' t - im\theta} e^{-\omega_c'' t}.$$

Look at a resonant low-loss configuration:

- Translate  $\omega \approx \omega_c'$ ,  $m \approx \beta R$ .
- Equate the power loss during one time period  $T = 2\pi/\omega \approx 2\pi/\omega_c'$   
 $\rightsquigarrow \beta/\alpha \approx \omega_c'/\omega_c'' = 2Q.$

## Upcoming

Next lectures:

- Coupled mode theory, perturbation theory.
- Hybrid analytical / numerical coupled mode theory.
- A touch of photonic crystals; a touch of plasmonics.

