Optical Waveguide Theory (D)

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Paderborn University — Summer Semester 2024

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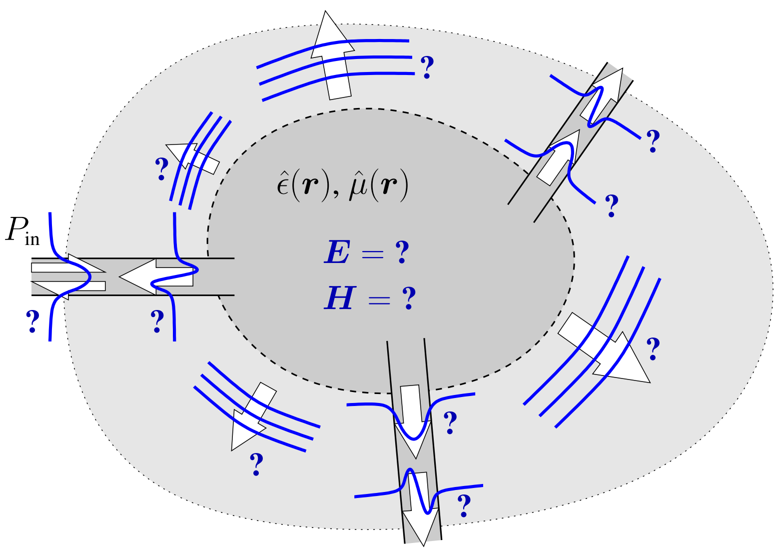
Course overview

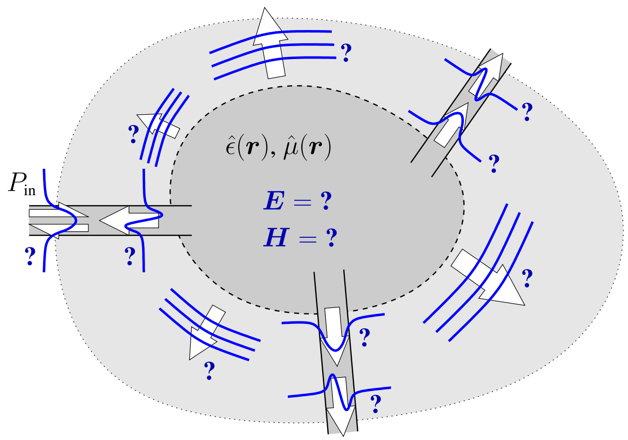
Optical waveguide theory

A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
B Brush up on mathematical tools.
C Maxwell equations, different formulations, interfaces, energy and power flow.
D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
E Normal modes of dielectric optical waveguides, mode interference.
F Examples for dielectric optical waveguides.
G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
H Bent optical waveguides; whispering gallery resonances; circular microresonators.
I Coupled mode theory, perturbation theory.
J A touch of photonic crystals; a touch of plasmonics.
- Hybrid analytical / numerical coupled mode theory.
- Oblique semi-guided waves: 2-D integrated optics.

Guided wave scattering problems, schematically



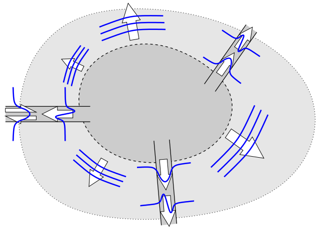
Guided wave scattering problems, schematically



Given $\hat\epsilon(\boldsymbol{r}), \hat\mu(\boldsymbol{r})$ & external excitation (incoming guided mode),
determine $\boldsymbol{E}, \boldsymbol{H}$ within the computational domain
& determine the optical power carried by outgoing waves.

Scattering problems, time domain

(TD)

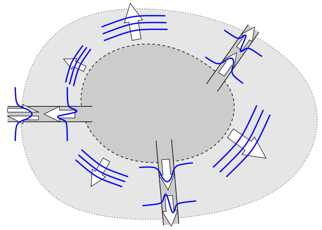


$\boldsymbol{E}(\boldsymbol{r},t),\ \boldsymbol{H}(\boldsymbol{r},t),$
$\boldsymbol{\nabla}\times\boldsymbol{E} = -\mu_0\hat{\mu}\dot{\boldsymbol{H}},$
$\boldsymbol{\nabla}\times\boldsymbol{H} = \epsilon_0\hat{\epsilon}\dot{\boldsymbol{E}}.$

- $\begin{pmatrix}\text{3-D}\\ \text{2-D}\\ \text{1-D}\end{pmatrix}$ computational domain $\times$ time interval.
- Initial & boundary conditions $\longleftrightarrow$ incident waves.
- "Local" time-explicit iterative schemes possible (e.g. FDTD).
- Time evolution available; direct modeling of pulse propagation.
- Dispersion (. . . ?).
- Guided wave excitation (. . . ?).
- Fourier transform $\longrightarrow$ spectral information.

Scattering problems, frequency domain

(FD)

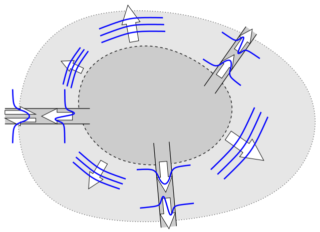


$\boldsymbol{E}(\boldsymbol{r}),\ \boldsymbol{H}(\boldsymbol{r}),\ \sim\exp(\mathrm{i}\omega t),$
$\boldsymbol{\nabla}\times\boldsymbol{E} = -\mathrm{i}\omega\mu_0\hat{\mu}\boldsymbol{H},$
$\boldsymbol{\nabla}\times\boldsymbol{H} = \mathrm{i}\omega\epsilon_0\hat{\epsilon}\boldsymbol{E}.$

- $\begin{pmatrix}\text{3-D}\\ \text{2-D}\\ \text{1-D}\end{pmatrix}$ computational domain.
- "$\mathsf{M}(\overrightarrow{\text{field}}) = (\overrightarrow{\text{excitation}})$";
  matrix needs to be determined, stored; system needs to be solved.
- Spectral information directly available.
- Dispersion — straightforward.
- Guided wave excitation — straightforward.
- Fourier transform $\longrightarrow$ time evolution / pulse propagation.

Open problems

(TD & FD)



"Open" spatial computational domain
$\rightsquigarrow$ boundary conditions need to

- permit outgoing radiated fields
  & outgoing (reflected) guided modes to exit the domain,
- launch the incoming external excitation.

$\leftrightsquigarrow$ simulate a nonexisting boundary, an unlimited domain.

Keywords:
- transparent-influx boundary conditions,
- absorbing boundary conditions,
- perfectly matched layers (PMLs).

2-D problems

$\hat{\epsilon} = \epsilon\hat{1},\ \hat{\mu} = \mu\hat{1},\ \sim\exp(\mathrm{i}\omega t)$ (FD)

Assume $\partial_y\epsilon = 0,\ \partial_y\mu = 0;$ consider solutions $\partial_y\boldsymbol{E} = 0,\ \partial_y\boldsymbol{H} = 0:$

$$\begin{pmatrix}-\partial_z E_y\\ \partial_z E_x - \partial_x E_z\\ \partial_x E_y\end{pmatrix} = -\mathrm{i}\omega\mu_0\mu\begin{pmatrix}H_x\\ H_y\\ H_z\end{pmatrix},\qquad \begin{pmatrix}-\partial_z H_y\\ \partial_z H_x - \partial_x H_z\\ \partial_x H_y\end{pmatrix} = \mathrm{i}\omega\epsilon_0\epsilon\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}.$$

$\hookrightarrow$ Two decoupled sets of equations:

- $\{E_y, H_x, H_z\}:$ transverse electric (TE) fields, $\boldsymbol{E}\perp$ x-z-plane.
- $\{H_y, E_x, E_z\}:$ transverse magnetic (TM) fields, $\boldsymbol{H}\perp$ x-z-plane.

(Different conventions on the use of TE, TM.)

(Applies also to the TD.)

2-D TE waves

$k^2 = \omega^2/c^2 = \omega^2\epsilon_0\mu_0$ (FD)

- Principal component E_y,

$$H_x = \frac{-\mathrm{i}}{\omega\mu_0\mu}\,\partial_z E_y, \quad H_z = \frac{\mathrm{i}}{\omega\mu_0\mu}\,\partial_x E_y, \quad \mathrm{i}\omega\epsilon_0\epsilon E_y = \partial_z H_x - \partial_x H_z$$

$$\hookrightarrow \quad \partial_x \frac{1}{\mu}\partial_x E_y + \partial_z \frac{1}{\mu}\partial_z E_y + k^2\epsilon E_y = 0. \qquad (*)$$

- Continuity of E_y, $\frac{1}{\mu}\partial_n E_y$ required at interfaces with normal $\boldsymbol{n}$.

- If $\mu = 1$: $\epsilon(x,z)$ (!)

$$\hookrightarrow \quad \partial_x^2 E_y + \partial_z^2 E_y + k^2\epsilon E_y = 0, \qquad (**)$$

scalar 2-D (TE) Helmholtz equation (E_y, $\partial_n E_y$ continuous).

(Reflection / transmission problems: s-polarized waves satisfy $(*)$, $(**)$.)

2-D TM waves

$k^2 = \omega^2/c^2 = \omega^2\epsilon_0\mu_0$ (FD)

- Principal component H_y,

$$E_x = \frac{\mathrm{i}}{\omega\epsilon_0\epsilon}\,\partial_z H_y, \quad E_z = \frac{-\mathrm{i}}{\omega\epsilon_0\epsilon}\,\partial_x H_y, \quad -\mathrm{i}\omega\mu_0\mu H_y = \partial_z E_x - \partial_x E_z$$

$$\hookrightarrow \quad \partial_x \frac{1}{\epsilon}\partial_x H_y + \partial_z \frac{1}{\epsilon}\partial_z H_y + k^2\mu H_y = 0. \qquad (*)$$

- Continuity of H_y, $\frac{1}{\epsilon}\partial_n H_y$ required at interfaces with normal $\boldsymbol{n}$.

- If $\mu = 1$: $\epsilon(x,z)$ (!)

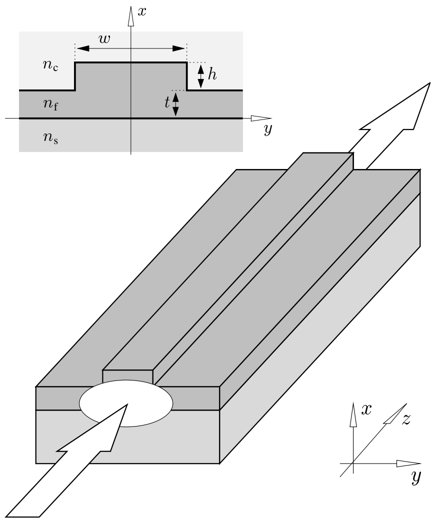
$$\hookrightarrow \quad \partial_x \frac{1}{\epsilon}\partial_x H_y + \partial_z \frac{1}{\epsilon}\partial_z H_y + k^2 H_y = 0, \qquad (**)$$

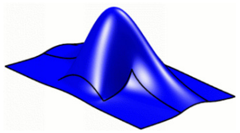
scalar 2-D (TM) Helmholtz equation (H_y, $\frac{1}{\epsilon}\partial_n H_y$ continuous).

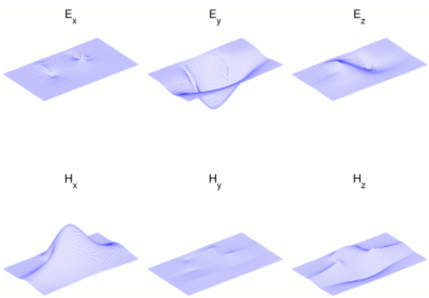
(Reflection / transmission problems: p-polarized waves satisfy $(*)$, $(**)$.)

Rib waveguide

... variant of an integrated optical waveguide with 2-D confinement







Waveguides: Mode problems

$$\nabla \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0\mu\boldsymbol{H}, \quad \nabla \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}.$$

$\sim \exp(\mathrm{i}\omega t)$ (FD)

- Waveguide: a system that is homogeneous along its axis z,
  $\partial_z\epsilon = 0$, $\partial_z\mu = 0$, $\partial_z n = 0$.

- Look for solutions (modes) that vary harmonically with z:
  $\boldsymbol{E}(x,y,z) = \bar{\boldsymbol{E}}(x,y)\,\mathrm{e}^{-\mathrm{i}\beta z}$, $\boldsymbol{H}(x,y,z) = \bar{\boldsymbol{H}}(x,y)\,\mathrm{e}^{-\mathrm{i}\beta z}$,

$\hookrightarrow$ mode profile $\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$, propagation constant β. (drop $^-$)

$$\begin{pmatrix} \partial_y E_z + \mathrm{i}\beta E_y \\ -\mathrm{i}\beta E_x - \partial_x E_z \\ \partial_x E_y - \partial_y E_x \end{pmatrix} = -\mathrm{i}\omega\mu_0\mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} \partial_y H_z + \mathrm{i}\beta H_y \\ -\mathrm{i}\beta H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix} = \mathrm{i}\omega\epsilon_0\epsilon \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix},$$

$\hookrightarrow$ vectorial mode equations, variants. (...)

Waveguides: Mode equations

- Where $\epsilon\,\cancel{(\boldsymbol{r})}$, $\mu\,\cancel{(\boldsymbol{r})}$: $\sim \exp(\mathrm{i}\omega t)$ (FD)

$$\Delta\tilde{\boldsymbol{E}} + k^2\epsilon\mu\tilde{\boldsymbol{E}} = 0, \quad \Delta\tilde{\boldsymbol{H}} + k^2\epsilon\mu\tilde{\boldsymbol{H}} = 0$$

$$\partial_x^2\boldsymbol{E} + \partial_y^2\boldsymbol{E} + (k^2\epsilon\mu - \beta^2)\boldsymbol{E} = 0,$$
$$\partial_x^2\boldsymbol{H} + \partial_y^2\boldsymbol{H} + (k^2\epsilon\mu - \beta^2)\boldsymbol{H} = 0,$$

scalar mode equation, valid for all components of $\boldsymbol{E}$, $\boldsymbol{H}$,
to be supplemented by suitable boundary and interface conditions.

Eigenvalue problem with eigenvalue β, eigenfunction $\boldsymbol{E}$, $\boldsymbol{H}$,
"$\mathsf{M}(\beta)\,(\overrightarrow{\text{profile}}) = 0$".

- Guided modes: discrete $\beta \in \mathbb{R}$, $\iint S_z\,\mathrm{d}x\mathrm{d}z < \infty$. $(\epsilon, \mu \in \mathbb{R})$

Waveguides: Mode equations

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"$\mathsf{M}(\beta)\,(\overrightarrow{\text{profile}}) = 0$".

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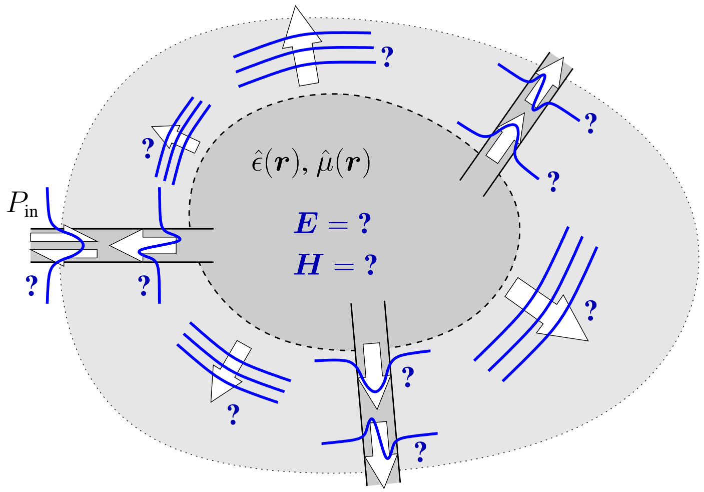
(Radiation modes: continuum of $\beta^2 \in \mathbb{R}$, oscillating external fields.)
(Leaky modes: discrete $\beta \in \mathbb{C}$, outgoing wave boundary conditions.)
$(\ldots)$

Guided wave scattering problems

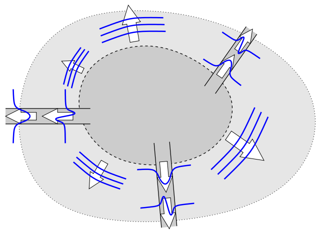
(FD)



Given external excitation $\sim \exp(\mathrm{i}\omega t)$, $\omega \in \mathbb{R}$.

Resonance problems

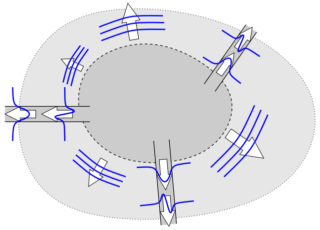
(FD ...)



$\boldsymbol{E}(\boldsymbol{r})$, $\boldsymbol{H}(\boldsymbol{r})$, $\sim \exp(\mathrm{i}\omega t)$, $\omega = ?$
$\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0\hat{\mu}\boldsymbol{H}$,
$\boldsymbol{\nabla} \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\hat{\epsilon}\boldsymbol{E}$,
& outgoing wave boundary conditions.

Resonance problems

(FD . . .)



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$\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{i}\omega\mu_0\hat{\mu}\boldsymbol{H},$
$\boldsymbol{\nabla} \times \boldsymbol{H} = \mathrm{i}\omega\epsilon_0\hat{\epsilon}\boldsymbol{E},$
& outgoing wave boundary conditions.

- Look for nonzero solutions with $\omega \in \mathbb{C}$ that oscillate and decay (slowly . . .) in time.
- "$\mathsf{M}(\omega)\,(\overrightarrow{\text{field}}) = 0$", eigenvalue problem.
- Solutions: discrete eigenfrequencies ω, resonant mode profiles.

Keyword: "Quasi-Normal-Modes", QNMs.

Beam propagation method

- Starting point: $\Delta\psi + k^2\epsilon\,\psi = 0,$ $\sim \exp(\mathrm{i}\omega t)$ (FD)
  "small" changes in $\epsilon = n^2$ along a propagation coordinate z.
- Ansatz: $\psi(x,y,z) = \psi_0(x,y,z)\ \mathrm{e}^{-\mathrm{i}kn_\mathrm{r}z},$
  reference effective index n_r,
  assume that ψ_0 varies "slowly" along z $\longleftrightarrow$ neglect $\partial_z^2\psi_0$.
  $$-\mathrm{i}2kn_\mathrm{r}\partial_z\psi_0 + (\partial_x^2 + \partial_y^2)\psi_0 + k^2(\epsilon - n_\mathrm{r}^2)\psi_0 = 0,$$
  PDE of first order in z, solved as an initial value problem.

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  $$-\mathrm{i}2kn_\mathrm{r}\partial_z\psi_0 + (\partial_x^2 + \partial_y^2)\psi_0 + k^2(\epsilon - n_\mathrm{r}^2)\psi_0 = 0,$$
  PDE of first order in z, solved as an initial value problem.
- Restriction to unidirectional propagation, reflections are neglected.
- Paraxial propagation, errors for waves with effective indices $\neq n_\mathrm{r}$.

(Many variants (vectorial, wide-angle, bi-directional, ...) have been proposed.)
(Other ways of motivating the approximation exist.)
(Term "BPM" in use also for other types of methods.)

- Keywords: Paraxial approximation,
  Slowly-varying-envelope approximation (SVEA),
  Beam propagation method (BPM).