Optical Waveguide Theory (F)

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Course overview

Optical waveguide theory

A Photonics / integrated optics; theory, motto; phenomena, introductory examples.

B Brush up on mathematical tools.

C Maxwell equations, different formulations, interfaces, energy and power flow.

D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.

E Normal modes of dielectric optical waveguides, mode interference.

F Examples for dielectric optical waveguides.

G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.

H Bent optical waveguides; whispering gallery resonances; circular microresonators.

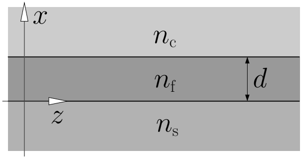
I Coupled mode theory, perturbation theory.

- Hybrid analytical / numerical coupled mode theory.

J A touch of photonic crystals; a touch of plasmonics.

- Oblique semi-guided waves: 2-D integrated optics.
- Summary, concluding remarks.

2-D waveguide configurations



$\epsilon \in \mathbb{R},\ \mu = 1,\ \sim \exp(\mathrm{i}\omega t)$ (FD)

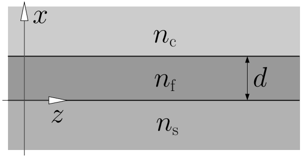
- 2-D waveguide, 1-D cross section.
- Permittivity $\epsilon = n^2$, refractive index $n(x)$. (1-D waveguide)

- $\partial_y \epsilon = 0$ ⟷ $\partial_y \boldsymbol{E} = 0,\ \partial_y \boldsymbol{H} = 0$, 2-D TE / TM setting.
- $\partial_z \epsilon = 0$ ⟷ Modal solutions that vary harmonically with z:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x)\ \mathrm{e}^{-\mathrm{i}\beta z},$$

mode profile $\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$,
propagation constant β,
effective index $n_{\rm eff} = \beta/k$.

2-D waveguide configurations



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mode profile $\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$,
propagation constant β,
effective index $n_{\text{eff}} = \beta/k$.

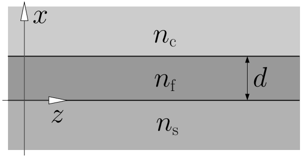
(TE): principal component $\bar{E}_y$, $\quad \partial_x^2 \bar{E}_y + (k^2\epsilon - \beta^2)\bar{E}_y = 0,$

$$\bar{E}_x = 0, \quad \bar{E}_z = 0, \quad \bar{H}_x = \frac{-\beta}{\omega\mu_0}\bar{E}_y, \quad \bar{H}_y = 0, \quad \bar{H}_z = \frac{\mathrm{i}}{\omega\mu_0}\partial_x \bar{E}_y,$$

$\bar{E}_y$ & $\partial_x \bar{E}_y$ continuous at dielectric interfaces.

2-D waveguide configurations

$\epsilon \in \mathbb{R},\ \mu = 1,\ \sim \exp(\mathrm{i}\omega t)$ (FD)



- 2-D waveguide, 1-D cross section.
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(1-D waveguide)

- $\partial_y \epsilon = 0 \;\leftrightsquigarrow\;$ $\partial_y \boldsymbol{E} = 0$, $\partial_y \boldsymbol{H} = 0$, 2-D TE / TM setting.
- $\partial_z \epsilon = 0 \;\leftrightsquigarrow\;$ Modal solutions that vary harmonically with z:

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x)\ \mathrm{e}^{-\mathrm{i}\beta z},$$

mode profile $\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$,
propagation constant β,
effective index $n_{\text{eff}} = \beta / k$.

(TM): principal component $\bar{H}_y$, $\quad \epsilon \partial_x \frac{1}{\epsilon} \partial_x \bar{H}_y + (k^2\epsilon - \beta^2)\bar{H}_y = 0,$

$$\bar{E}_x = \frac{\beta}{\omega\epsilon_0\epsilon}\bar{H}_y, \quad \bar{E}_y = 0, \quad \bar{E}_z = \frac{-\mathrm{i}}{\omega\epsilon_0\epsilon}\partial_x \bar{H}_y, \quad \bar{H}_x = 0, \quad \bar{H}_z = 0,$$

$\bar{H}_y$ & $\epsilon^{-1}\partial_x \bar{H}_y$ continuous at dielectric interfaces.

Guided 2-D TE / TM modes, orthogonality properties

(→ *Exercise.*)

- A set (index m) of guided modes of a 2-D waveguide (ϵ),
  $\psi_m^{\mathrm{p}} = (\bar{\boldsymbol{E}}_m, \bar{\boldsymbol{H}}_m)$, p=TE,TM & β_m, $\beta_m \neq \beta_l$, if $l \neq m$.

- $$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2) := \frac{1}{4} \int \left(E_{1x}^* H_{2y} - E_{1y}^* H_{2x} + H_{1y}^* E_{2x} - H_{1x}^* E_{2y} \right) \mathrm{d}x \,.$$

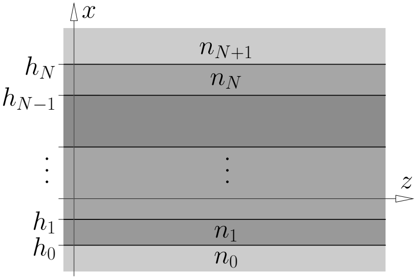
- Power P_m per lateral (y) unit length carried by mode $\psi_m^{\mathrm{p}}, \beta_m$:

$$P_m := \int S_z \,\mathrm{d}x = (\psi_m^{\mathrm{p}}; \psi_m^{\mathrm{p}}) = \begin{cases} \dfrac{\beta_m}{2\omega\mu_0} \displaystyle\int |E_{m,y}|^2 \,\mathrm{d}x, & \text{if } \mathrm{p} = \mathrm{TE}, \\ \dfrac{\beta_m}{2\omega\epsilon_0} \displaystyle\int \frac{1}{\epsilon} \,|H_{m,y}|^2 \,\mathrm{d}x, & \text{if } \mathrm{p} = \mathrm{TM}. \end{cases}$$

$$(\psi_l^{\mathrm{TE}}; \psi_m^{\mathrm{TM}}) = 0 \,, \qquad (\psi_l^{\mathrm{TE}}; \psi_m^{\mathrm{TE}}) = \frac{\beta_m}{2\omega\mu_0} \int E_{l,y}^* \, E_{m,y} \,\mathrm{d}x = \delta_{lm} P_m \,,$$

$$(\psi_l^{\mathrm{TM}}; \psi_m^{\mathrm{TE}}) = 0 \,, \qquad (\psi_l^{\mathrm{TM}}; \psi_m^{\mathrm{TM}}) = \frac{\beta_m}{2\omega\epsilon_0} \int \frac{1}{\epsilon} \, H_{l,y}^* \, H_{m,y} \,\mathrm{d}x = \delta_{lm} P_m \,.$$

Dielectric multilayer slab waveguide



$\epsilon \in \mathbb{R},\ \mu = 1,\ \sim \exp(\mathrm{i}\omega t)$ (2-D, FD)

- N interior layers, piecewise constant $\epsilon = n^2$:

$$n(x) = \begin{cases} n_{N+1} & \text{if } h_N < x, \\ n_l & \text{if } h_{l-1} < x < h_l, \\ n_0 & \text{if } x < h_0. \end{cases}$$

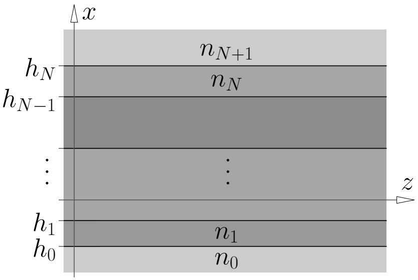
- Principal component $\phi(x)$ (TE: $\phi = \bar{E}_y$, TM: $\phi = \bar{H}_y$).

- $\partial_x^2 \phi + (k^2 n_l^2 - \beta^2)\phi = 0, \quad x \in$ layer $l, \quad l = 0, \ldots, N+1$

(Half-infinte substrate ($l = 0$) and cover ($l = N+1$) layers.)

- ϕ & $\eta\partial_x\phi$ continuous at $x = h_l$, (TE: $\eta = 1$, TM: $\eta = n^{-2}$).

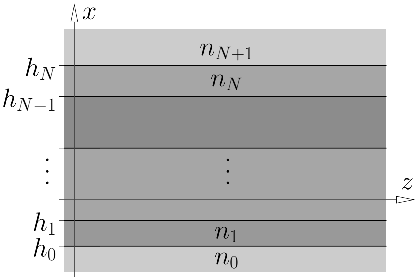
Dielectric multilayer slab waveguide



- Interior layer l,
  $h_{l-1} < x < h_l$,
  local refractive index n_l,
- $\partial_x^2 \phi = (\beta^2 - k^2\, n_l^2)\phi$.
- Consider a trial value $\beta^2 \in \mathbb{R}$.

- $\beta^2 < k^2\, n_l^2 \rightsquigarrow \quad \partial_x^2 \phi = -\kappa_l^2\, \phi, \quad \kappa_l := \sqrt{k^2\, n_l^2 - \beta^2},$
  $$\phi(x) = A_l \sin(\kappa_l\, x) + B_l \cos(\kappa_l\, x).$$
- $\beta^2 > k^2\, n_l^2 \rightsquigarrow \quad \partial_x^2 \phi = \kappa_l^2\, \phi, \quad \kappa_l := \sqrt{\beta^2 - k^2\, n_l^2},$
  $$\phi(x) = A_l\, \mathrm{e}^{\kappa_l\, x} + B_l\, \mathrm{e}^{-\kappa_l\, x}.$$
- Unknowns $A_l, B_l \in \mathbb{C}$. (Local coordinate offsets required to cope with the exponentials.)

Dielectric multilayer slab waveguide, guided modes



- Substrate region,
  $x < h_0$,
  local refractive index n_0,
- $\partial_x^2 \phi = (\beta^2 - k^2\, n_0^2)\phi$.
- Consider a trial value $\beta^2 \in \mathbb{R}$.

- $\beta^2 < k^2\, n_0^2 \rightsquigarrow \quad \partial_x^2 \phi = -\kappa_0^2\, \phi, \quad \kappa_0 := \sqrt{k^2\, n_0^2 - \beta^2},$

$$\cancel{\phi(x) = A_0 \sin(\kappa_0\, x) + B_0 \cos(-\kappa_0\, x)}.$$

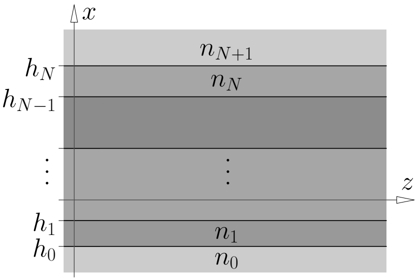
- $\beta^2 > k^2\, n_0^2 \rightsquigarrow \quad \partial_x^2 \phi = \kappa_0^2\, \phi, \quad \kappa_0 := \sqrt{\beta^2 - k^2\, n_0^2},$

$$\phi(x) = A_0\, \mathrm{e}^{\kappa_0\, x} + \cancel{B_0\, \mathrm{e}^{-\kappa_0\, x}}.$$

- Unknown $A_0 \in \mathbb{C}$.

Guided modes: $n_{\mathrm{eff}} = \beta / k > n_0$.

Dielectric multilayer slab waveguide, guided modes



- Cover region,
  $h_N < x$,
  local refractive index n_{N+1},
- $\partial_x^2 \phi = (\beta^2 - k^2\, n_{N+1}^2)\phi$.
- Consider a trial value $\beta^2 \in \mathbb{R}$.

- $\beta^2 < k^2\, n_{N+1}^2 \quad \rightsquigarrow \quad \partial_x^2 \phi = -\kappa_{N+1}^2\, \phi, \quad \kappa_{N+1} := \sqrt{k^2\, n_{N+1}^2 - \beta^2}$,

  $$\cancel{\phi(x) = A_{N+1} \sin(\kappa_{N+1}\, x) + B_{N+1} \cos(-\kappa_{N+1}\, x)}.$$

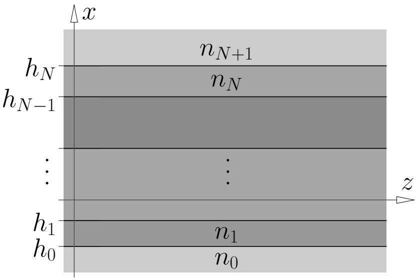
- $\beta^2 > k^2\, n_{N+1}^2 \quad \rightsquigarrow \quad \partial_x^2 \phi = \kappa_{N+1}^2\, \phi, \quad \kappa_{N+1} := \sqrt{\beta^2 - k^2\, n_{N+1}^2}$,

  $$\phi(x) = \cancel{A_{N+1}\, \mathrm{e}^{\kappa_{N+1}\, x}} + B_{N+1}\, \mathrm{e}^{-\kappa_{N+1}\, x}.$$

- Unknown $B_{N+1} \in \mathbb{C}$.

Guided modes: $n_{\text{eff}} = \beta/k > n_{N+1}$.

Dielectric multilayer slab waveguide



Trial value $\beta^2 \in \mathbb{R}$,

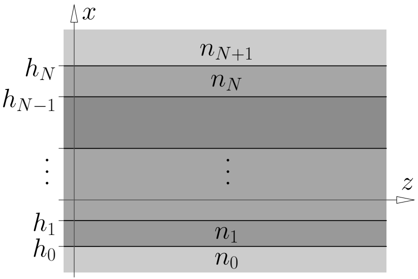
$\beta/k > n_0, n_{N+1}$,

$\rightsquigarrow \quad \kappa_l, \quad l = 0, \ldots, N+1.$

$$\phi(x) = \begin{cases} B_{N+1}\, \mathrm{e}^{-\kappa_{N+1} x}, & \text{for } h_N < x, \\ \left\{ \begin{array}{ll} A_l \sin(\kappa_l x) + B_l \cos(\kappa_l x), & \text{if } \beta^2 < k^2 n_l^2, \\ A_l\, \mathrm{e}^{\kappa_l x} + B_l\, \mathrm{e}^{-\kappa_l x}, & \text{if } \beta^2 > k^2 n_l^2, \end{array} \right\} & \text{for } h_{l-1} < x < h_l, \\ A_0\, \mathrm{e}^{\kappa_0 x}, & \text{for } x < h_0. \end{cases}$$

- $2N+2$ unknowns $A_0, A_1, B_1, \ldots, A_N, B_N, B_{N+1}$.
- Continuity of $\phi, \eta\partial_x\phi$ at $N+1$ interfaces $\rightsquigarrow$ $2N+2$ equations.

Dielectric multilayer slab waveguide



Trial value $\beta^2 \in \mathbb{R}$,
$\beta/k > n_0, n_{N+1}$.

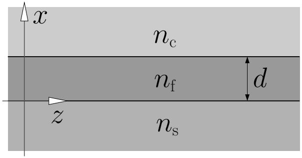
- $2N+2$ unknowns $A_0, A_1, B_1, \ldots, A_N, B_N, B_{N+1}$.
- Continuity of $\phi, \eta\partial_x\phi$ at $N+1$ interfaces $\rightsquigarrow$ $2N+2$ equations.
- Arrange as linear system of equations $\mathsf{M}(\beta^2)\,(A_0, \ldots, B_{N+1})^{\mathsf{T}} = 0$.
- Identify propagation constants where $\mathsf{M}(\beta^2)$ becomes singular.

(Equations relate to the series of interfaces $\leftrightarrow$ A transfer-matrix technique can be applied.)

- Choose e.g. $A_0 = 1$, fill $A_1, \ldots, B_{N+1}$, normalize. $(\cdot, \cdot\, ; \cdot, \cdot)$

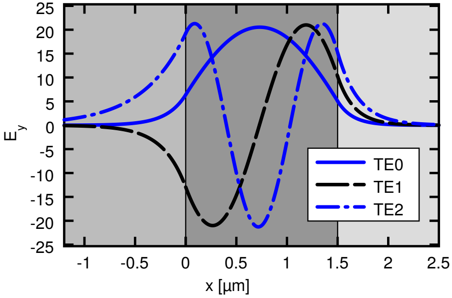
Guided modes $\{\beta_m, (\bar{\boldsymbol{E}}_m, \bar{\boldsymbol{H}}_m)\}$.

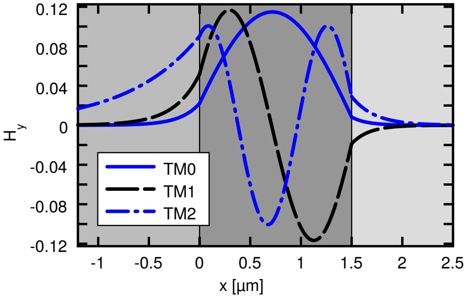
A nonsymmetric 3-layer slab waveguide



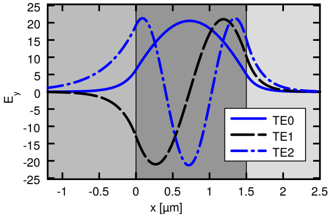
$n_s = 1.45,\ n_f = 1.99,\ n_c = 1.0,$
$d = 1.5\ \mu\text{m},\ \lambda = 1.55\ \mu\text{m}.$

$\text{TE}_0\!:\ n_{\text{eff}} = 1.944,\quad \text{TM}_0\!:\ n_{\text{eff}} = 1.933,$
$\text{TE}_1\!:\ n_{\text{eff}} = 1.804,\quad \text{TM}_1\!:\ n_{\text{eff}} = 1.759,$
$\text{TE}_2\!:\ n_{\text{eff}} = 1.562,\quad \text{TM}_2\!:\ n_{\text{eff}} = 1.490.$





Dielectric multilayer slab waveguide, nodal properties



(Fixed polarization, TE / TM.)

$$\partial_x(\partial_x \phi) = -(k^2\, n^2 - \beta^2)\phi.$$

$k^2 n^2 - \beta^2$ determines the rate of change of the slope of ϕ.

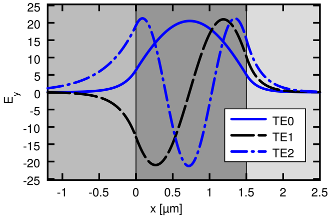
Imagine a numerical ODE algorithm of "shooting-type".

- Guided modes with a growing number of nodes (x with $\phi(x) = 0$) with decreasing effective indices

  $\longleftrightarrow$ mode indices = number of nodes in ϕ.

"Quantum numbers".

Dielectric multilayer slab waveguide, nodal properties



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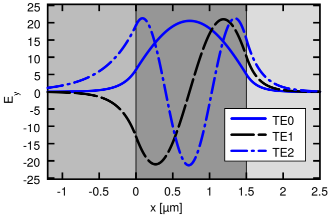
- A fundamental mode with zero nodes and highest effective index.

Dielectric multilayer slab waveguide, nodal properties



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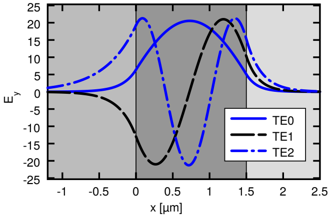
- Guided modes with a growing number of nodes (x with $\phi(x) = 0$) with decreasing effective indices
  $\leftrightsquigarrow$ mode indices = number of nodes in ϕ.

“Quantum numbers”.

- A fundamental mode with zero nodes and highest effective index.
- Modes of the same polarization are non-degenerate.

Dielectric multilayer slab waveguide, nodal properties



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$$\partial_x(\partial_x\phi) = -(k^2\,n^2 - \beta^2)\phi.$$

$k^2n^2 - \beta^2$ determines the rate of change of the slope of ϕ.

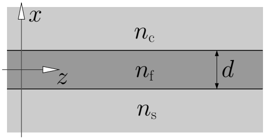
Imagine a numerical ODE algorithm of "shooting-type".

- A sign change of $\partial_x\phi$ is required to form a guided mode
  $\rightsquigarrow$ There must be some region (layer) with $k^2n^2 - \beta^2 > 0$.

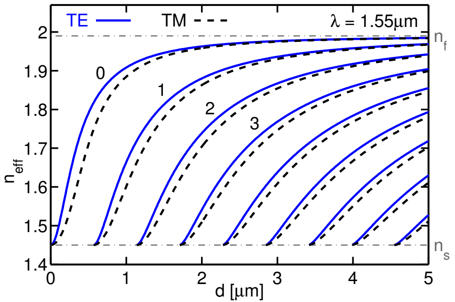
Interval for effective indices n_{eff} of guided modes:

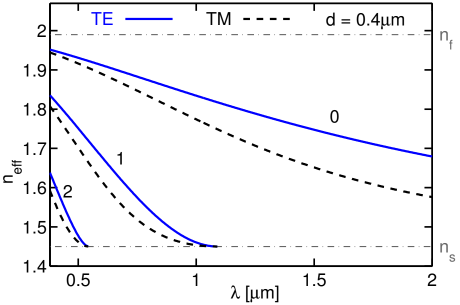
$$\max\{n_0, n_{N+1}\} < n_{\text{eff}} < \max_l\{n_l\}.$$

3-layer slab waveguide, dispersion curves



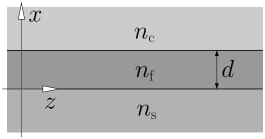
Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$.





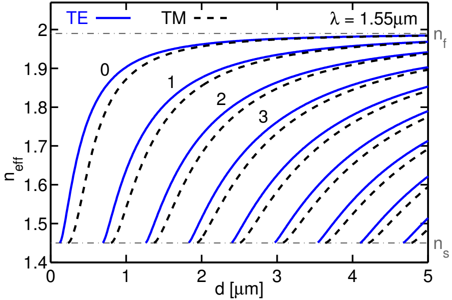
(Caution: $\partial_\lambda \epsilon = 0$ assumed !)

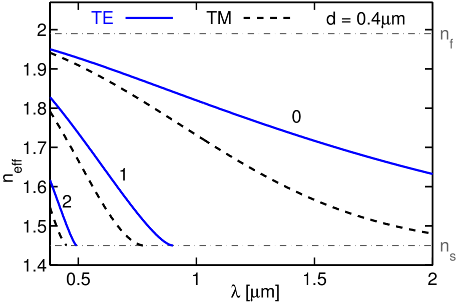
3-layer slab waveguide, dispersion curves



Nonsymmetric waveguide,
moderate refractive index contrast,

$n_s = 1.45,\ n_f = 1.99,\ n_c = 1.0.$





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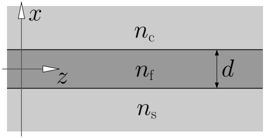
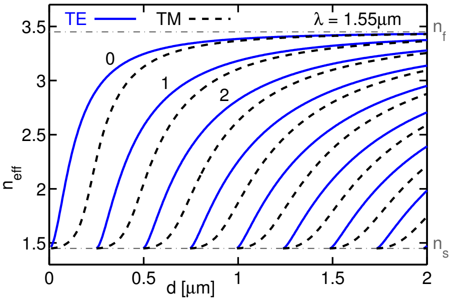
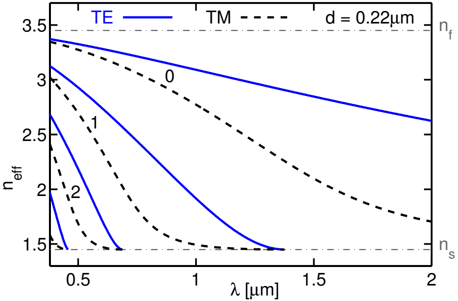
3-layer slab waveguide, dispersion curves



Symmetric waveguide,
high refractive index contrast,

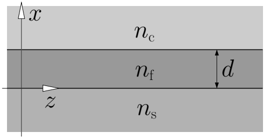
$$n_{\mathrm{s}} = 1.45,\ n_{\mathrm{f}} = 3.45,\ n_{\mathrm{c}} = 1.45.$$





(Caution: $\partial_\lambda \epsilon = 0$ assumed !)

3-layer slab waveguide, dispersion curves



Nonsymmetric waveguide,
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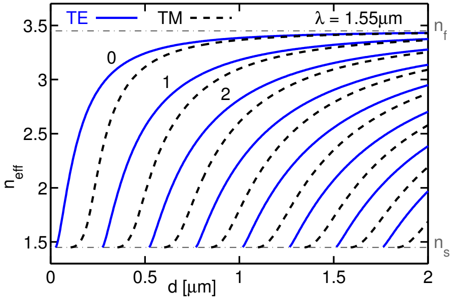
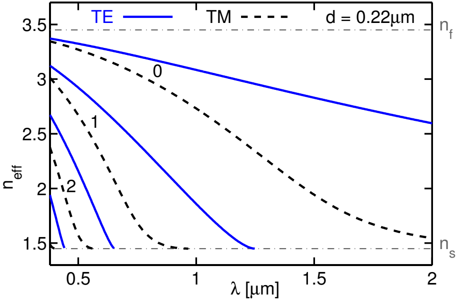
$n_s = 1.45, \; n_f = 3.45, \; n_c = 1.0.$





(Caution: $\partial_\lambda \epsilon = 0$ assumed !)

3-layer slab waveguide, dispersion curves

Remarks / observations:

- At large core thicknesses, or short wavelengths, for all modes: n_{eff} approaches the level n_{f} of bulk waves in the core material.
- Modes of higher order at the same n_{eff} supported by waveguides with thickness increased by specific distances.

  Guided mode, layer l with $\kappa_l^2 = (k^2 n^2 - \beta^2) > 0$, field $\phi(x) \sim \cos(\kappa_l x + \chi)$ for $x \in$ layer l;
  increase layer thickness by $\Delta x = \pi / \kappa_l$, such that $\kappa_l (x + \Delta x) = \kappa_l x + \pi$
  $\longrightarrow$ the thicker waveguide supports a mode of order $+1$ with the same propagation constant.

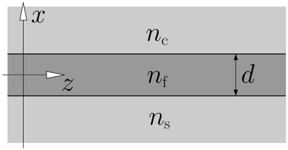
- Cutoff thicknesses at fixed wavelength.

  Nonsymmetric 3-layer waveguide $n_{\mathrm{s}} \neq n_{\mathrm{c}}$: There exist cutoff thicknesses for all modes.
  Symmetric 3-layer waveguide $n_{\mathrm{s}} = n_{\mathrm{c}}$: Cutoff thicknesses exist for all modes of order ≥ 1,
  no cutoff thickness for the fundamental TE/TM modes.

- λ is the "length-defining" quantity; wavelength scaling, factor a :
  $n_{\mathrm{eff}}(\lambda, d) = n_{\mathrm{eff}}(a\lambda, ad), \quad \beta(\lambda, d) = a^{-1}\,\beta(a\lambda, ad)$.
- Cutoff wavelengths for waveguides with fixed thickness.

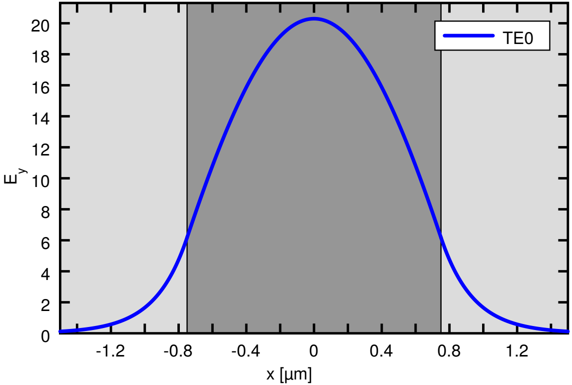
  For all modes; exception: no cutoff wavelength for the fundamental TE/TM modes in a symmetric 3-layer waveguide.

3-layer slab waveguide, mode confinement

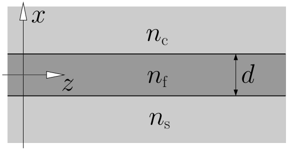


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55\,\mu$m,

$d = 1.50\,\mu$m, TE_0: $n_{\mathrm{eff}} = 1.946$.

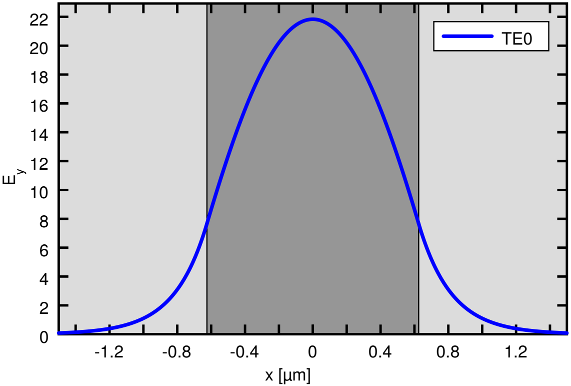


3-layer slab waveguide, mode confinement

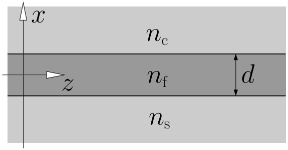


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55$ µm,

$d = 1.25$ µm, TE$_0$: $n_{eff} = 1.932$.

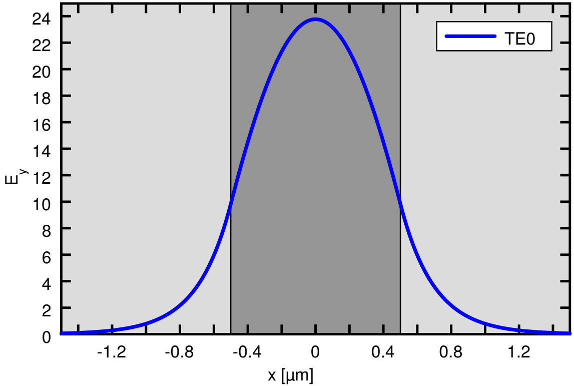


3-layer slab waveguide, mode confinement

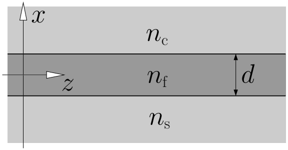


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55\ \mu$m,

$d = 1.00\ \mu$m, TE$_0$: $n_{\text{eff}} = 1.908$.

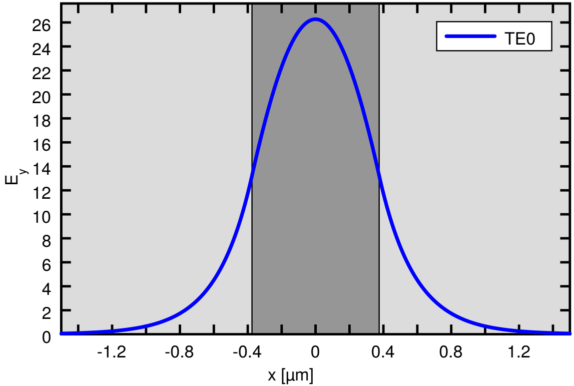


3-layer slab waveguide, mode confinement

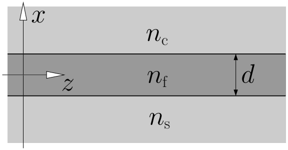


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55$ µm,

$d = 0.75$ µm, TE$_0$: $n_{\text{eff}} = 1.868$.

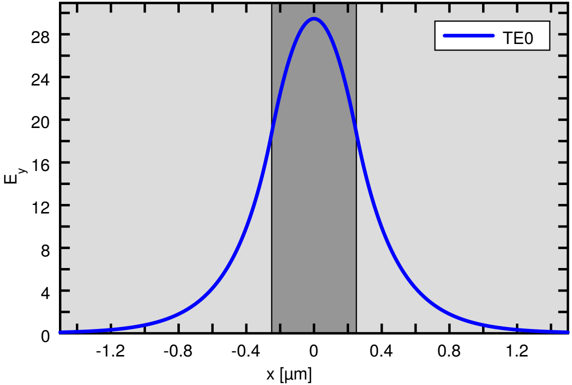


3-layer slab waveguide, mode confinement

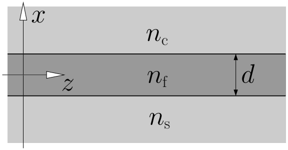


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55$ µm,

$d = 0.50$ µm, TE$_0$: $n_{eff} = 1.791$.

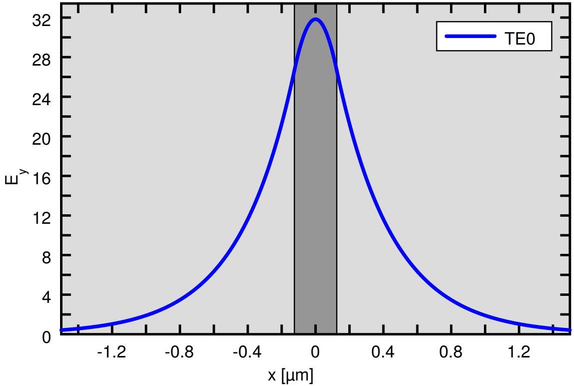


3-layer slab waveguide, mode confinement

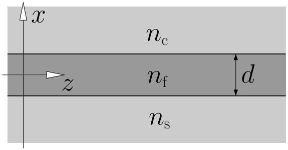


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55$ µm,

$d = 0.25$ µm, $\quad TE_0$: $n_{eff} = 1.630$.

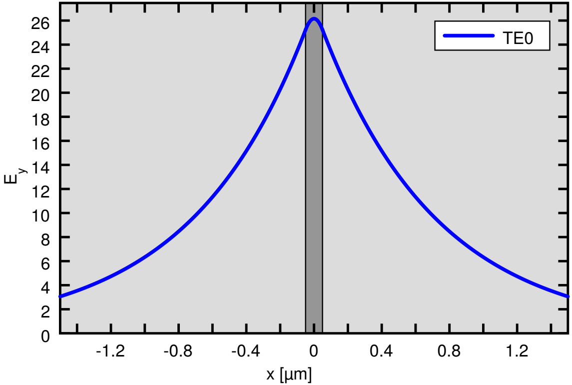


3-layer slab waveguide, mode confinement

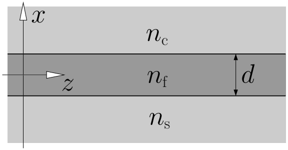


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45$, $n_f = 1.99$, $n_c = 1.45$, $\lambda = 1.55$ µm,

$d = 0.10$ µm, TE_0: $n_{\text{eff}} = 1.494$.

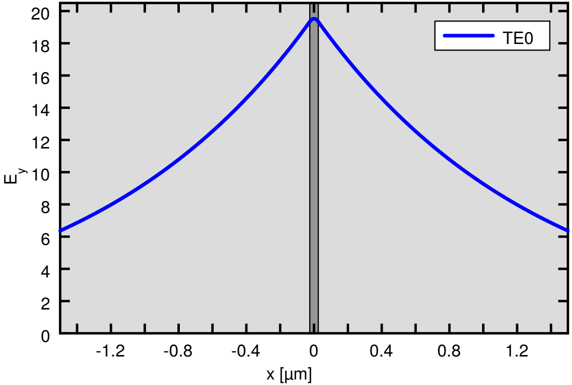


3-layer slab waveguide, mode confinement

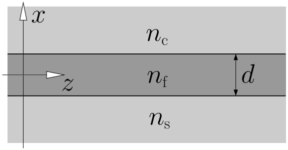


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45,\ n_f = 1.99,\ n_c = 1.45,\ \lambda = 1.55\ \mu m$,

$d = 0.05\ \mu m,\quad TE_0{:}\ \ n_{eff} = 1.462.$

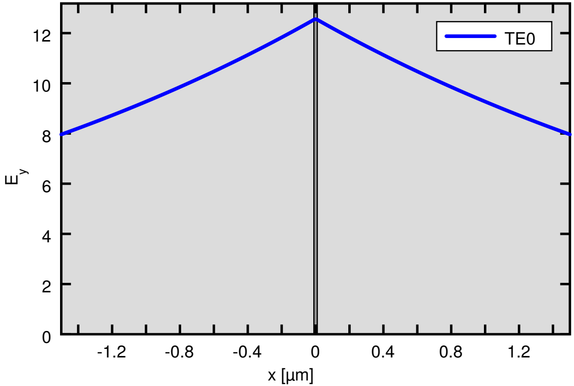


3-layer slab waveguide, mode confinement

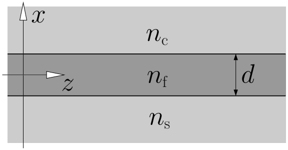


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45,\ n_f = 1.99,\ n_c = 1.45,\ \lambda = 1.55\ \mu m,$

$d = 0.02\ \mu m, \quad TE_0{:}\ \ n_{eff} = 1.452.$

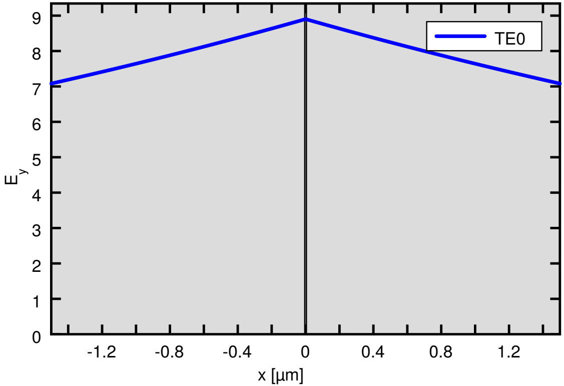


3-layer slab waveguide, mode confinement

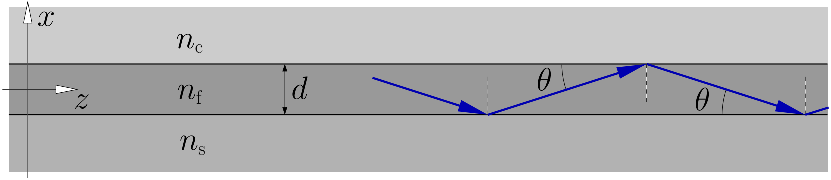


Symmetric waveguide,
moderate refractive index contrast,
$n_s = 1.45,\ n_f = 1.99,\ n_c = 1.45,\ \lambda = 1.55\ \mu\text{m},$

$d = 0.01\ \mu\text{m}, \quad \text{TE}_0\text{:}\ \ n_{\text{eff}} = 1.450.$



3-layer slab waveguide, ray model



Field in the core:

$$\sim a_u \, e^{-i(\kappa x + \beta z)} + a_d \, e^{-i(-\kappa x + \beta z)}, \qquad k^2 n_f^2 = \beta^2 + \kappa^2$$

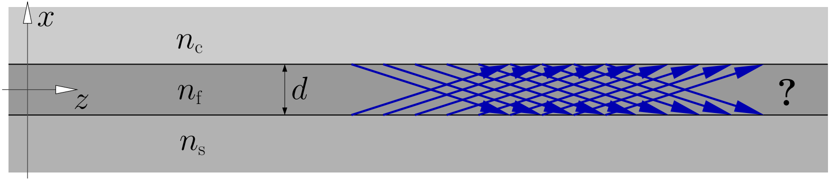
$\leftrightsquigarrow$ propagation angle θ with $\beta = k n_f \cos\theta, \quad \kappa = k n_f \sin\theta.$

Guided mode formation:

- Repleated total internal reflection of waves in the core at upper and lower interfaces
- Calculate optical phase gain, including phase jumps for reflection at interfaces (polarization dependent).
- Phase gain of 2π for one "round trip", "transverse resonance condition" $\longleftrightarrow$ constructive interference of waves.

(A frequently encountered intuitive model ... of very limited applicability.)

3-layer slab waveguide, ray model



Field in the core:

$$\sim a_u\, e^{-i(\kappa x + \beta z)} + a_d\, e^{-i(-\kappa x + \beta z)}, \qquad k^2 n_f^2 = \beta^2 + \kappa^2$$

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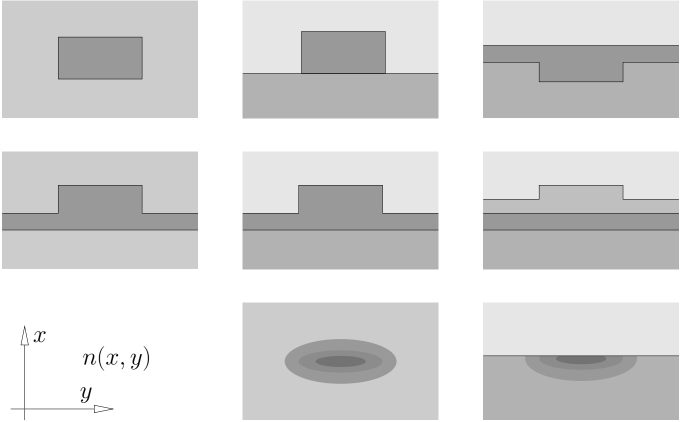
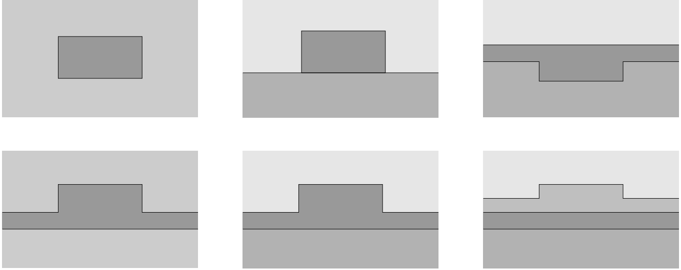
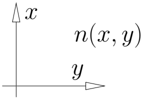
3-D waveguides



Cross sections (2-D) of typical integrated-optical waveguides.

3-D rectangular waveguides

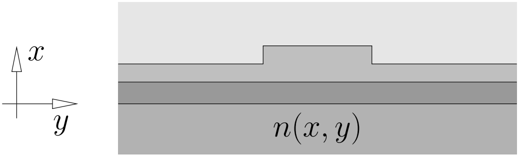




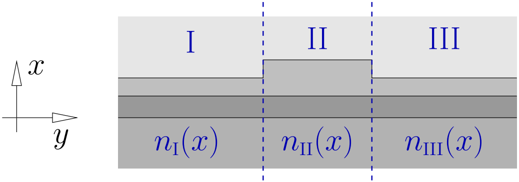
No analytical solutions :

- numerical mode solvers.
- approximations.

Effective index method



Effective index method



Outline: (!)

- Divide into slices $\rho = \mathrm{I, II, III}$: $n(x,y) = n_\rho(x)$, if $y \in$ slice ρ.
- Compute polarized modes $X_\rho(x)$, β_ρ, $X_\rho'' + (k^2 n_\rho^2 - \beta_\rho^2)X_\rho = 0$, $N_\rho = \beta_\rho / k$.
- Consider a scalar mode equation for the principal component Ψ of the 3-D waveguide

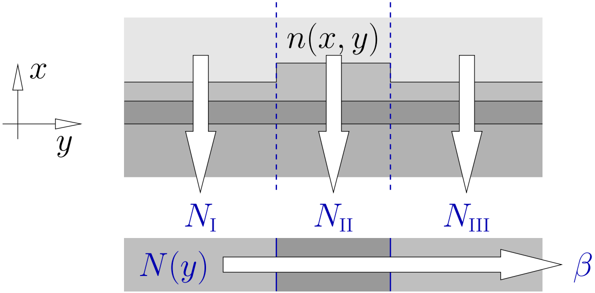
$$\partial_x^2 \Psi + \partial_y^2 \Psi + (k^2 n^2 - \beta^2)\Psi = 0, \qquad \Psi = E_y \text{ (TE)}, \ \Psi = H_y \text{ (TM)}.$$

- Ansatz: $\Psi(x,y) = X_\rho(x)\, Y(y)$, if $y \in$ slice ρ; require continuity of Y and Y'.
- Effective index profile: $N(y) := N_\rho$, if $y \in$ slice ρ.

$$\hookrightarrow \quad Y'' + (k^2 N^2 - \beta^2) Y = 0,$$

a 1-D mode equation for Y, β with the effective index profile N in place of the refractive indices.

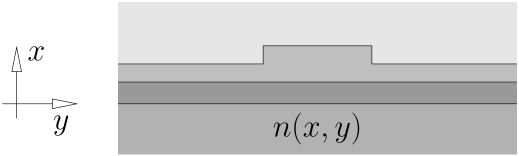
Effective index method, schematically



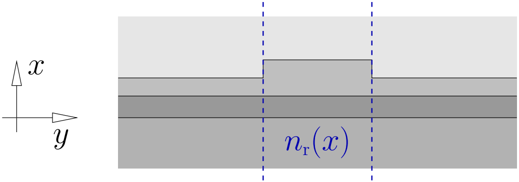
Remarks / issues:

- A popular, quite intuitive method.
- Frequently an (often informal) basis for discussion of waveguide properties.
- $\leftrightarrow$ Relevance of the slab waveguide model.
- Manifold variants / ways of improvements exist.
- What if a slice does not support a guided slab mode?
- What about higher order modes?
- How to evaluate modal fields? What about other than principal components?
- . . .

Variational effective index method



Variational effective index method



Outline: (!)

- Identify a reference slice, refractive index profile $n_{\mathrm{r}}(x)$.
- Compute polarized guided slab modes $(\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}})_{\mathrm{r}}$, β_{r} for the reference slice.
- For each each reference slab mode : $\ \dots$
- Choose an ansatz: (VEIM)

$$\begin{pmatrix} E_x, E_y, E_z \\ H_x, H_y, H_z \end{pmatrix}(x,y,z) = \begin{pmatrix} 0, & \bar{E}_{\mathrm{r},y}(x)Y^{E_y}(y), & \bar{E}_{\mathrm{r},y}(x)Y^{E_z}(y) \\ \bar{H}_{\mathrm{r},x}(x)Y^{H_x}(y), & \bar{H}_{\mathrm{r},z}(x)Y^{H_y}(y), & \bar{H}_{\mathrm{r},z}(x)Y^{H_z}(y) \end{pmatrix} \qquad \text{(TE)}$$

$$\begin{pmatrix} E_x, E_y, E_z \\ H_x, H_y, H_z \end{pmatrix}(x,y,z) = \begin{pmatrix} \bar{E}_{\mathrm{r},x}(x)Y^{E_x}(y), & \bar{E}_{\mathrm{r},z}(x)Y^{E_y}(y), & \bar{E}_{\mathrm{r},z}(x)Y^{E_z}(y) \\ 0, & \bar{H}_{\mathrm{r},y}(x)Y^{H_y}(y), & \bar{H}_{\mathrm{r},y}(x)Y^{H_z}(y) \end{pmatrix} \qquad \text{(TM)}$$

$$Y^{\cdot}(y) = \,?$$

A functional for guided modes of 3-D dielectric waveguides

(→ *Exercise.*)

- $$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,y,z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix}(x,y)\ \mathrm{e}^{-\mathrm{i}\beta z}, \qquad \beta \in \mathbb{R},$$
  $$\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}} \to 0 \ \text{ for } \ x, y \to \pm\infty.$$

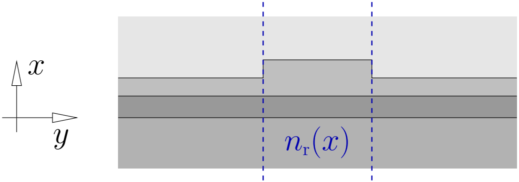
- $$(\mathsf{C} + \mathrm{i}\beta\mathsf{R})\bar{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\bar{\boldsymbol{H}}, \quad (\mathsf{C} + \mathrm{i}\beta\mathsf{R})\bar{\boldsymbol{H}} = \mathrm{i}\omega\epsilon_0\epsilon\bar{\boldsymbol{E}},$$
  $$\mathsf{R} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathsf{C} = \begin{pmatrix} 0 & 0 & \partial_y \\ 0 & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}.$$

- $$\mathcal{B}(\boldsymbol{E}, \boldsymbol{H}) := \frac{\omega\epsilon_0\langle \boldsymbol{E}, \epsilon\boldsymbol{E}\rangle + \omega\mu_0\langle \boldsymbol{H}, \boldsymbol{H}\rangle + \mathrm{i}\langle \boldsymbol{E}, \mathsf{C}\boldsymbol{H}\rangle - \mathrm{i}\langle \boldsymbol{H}, \mathsf{C}\boldsymbol{E}\rangle}{\langle \boldsymbol{E}, \mathsf{R}\boldsymbol{H}\rangle - \langle \boldsymbol{H}, \mathsf{R}\boldsymbol{E}\rangle},$$
  $$\langle \boldsymbol{F}, \boldsymbol{G}\rangle = \iint \boldsymbol{F}^* \cdot \boldsymbol{G}\,\mathrm{d}x\,\mathrm{d}y.$$

$$\mathcal{B}(\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}) = \beta, \qquad \frac{\mathrm{d}}{\mathrm{d}s}\mathcal{B}(\bar{\boldsymbol{E}} + s\,\delta\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}} + s\,\delta\bar{\boldsymbol{H}})\Big|_{s=0} = 0$$

at valid mode fields $\bar{\boldsymbol{E}}, \bar{\boldsymbol{H}}$, for arbitrary $\delta\bar{\boldsymbol{E}}$, $\delta\bar{\boldsymbol{H}}$.

Variational effective index method



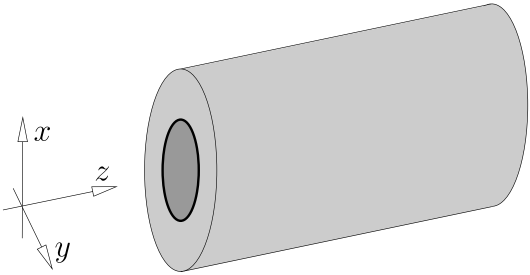
Outline, continued: (!)

- Restrict $\mathcal{B}$ to the VEIM ansatz, require stationarity with respect to the $\{Y^{\cdot}\}$.

↪ 1-D mode ("-like") equations for principal unknowns Y^{H_x} (TE) and Y^{E_x} (TM)

with effective quantities in place of refractive indices, all other $Y^{\cdot}$ can be computed.

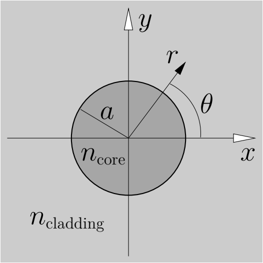
Optical fibers



[Optical Communication A-D]

Circular step index optical fibers

(FD)



Circular symmetry

$\leftrightsquigarrow$ cylindrical coordinates r, θ, z.

$$\epsilon = n^2, \quad n(r) = \begin{cases} n_{\text{core}}, & r \leq a, \\ n_{\text{cladding}}, & r > a. \end{cases}$$

Circular and axial symmetry:

$$\hookrightarrow \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} (r, \theta, z) = \begin{pmatrix} \bar{\boldsymbol{E}} \\ \bar{\boldsymbol{H}} \end{pmatrix} (r) \, \mathrm{e}^{-\mathrm{i} l \theta - \mathrm{i} \beta z}, \qquad l \in \mathbb{Z}, \beta \in \mathbb{R}.$$

$(E_r, E_\theta, E_z, H_r, H_\theta, H_z)$

Where $\partial \epsilon = 0$: $\quad \Delta \psi + k^2 n^2 \psi = 0, \quad \psi \in \{E_r, \ldots H_z\}$.

$$\hookrightarrow \quad \partial_r^2 \phi + \frac{1}{r} \partial_r \phi + (k^2 n^2 - \beta^2 - \frac{l^2}{r^2}) \phi = 0, \quad \phi \in \{\bar{E}_r, \ldots \bar{H}_z\}$$

(An ODE of Bessel type.)

& vectorial interface conditions at $r = a$. (Alternatively: Scalar theory, LP modes.)

(. . .)

Next lectures:

- Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- Bent optical waveguides; whispering gallery resonances; circular microresonators.
- Coupled mode theory, perturbation theory.