Modal Analysis of Dielectric Rib-Waveguides by a Wave Matching Method

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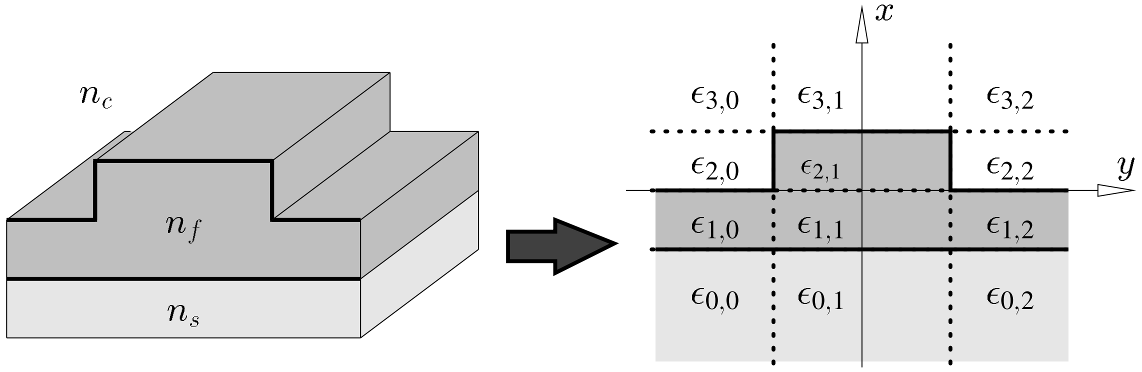
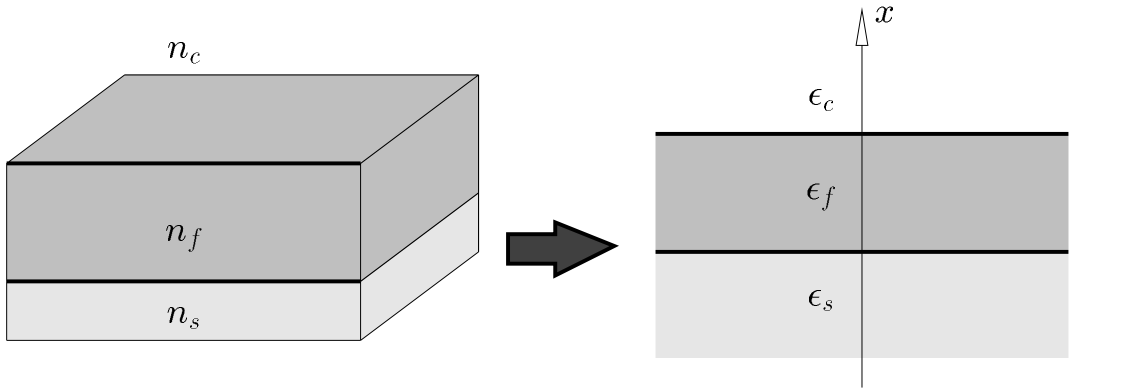
Frequently a waveguide cross section can be divided into rectangles with constant refractive index:

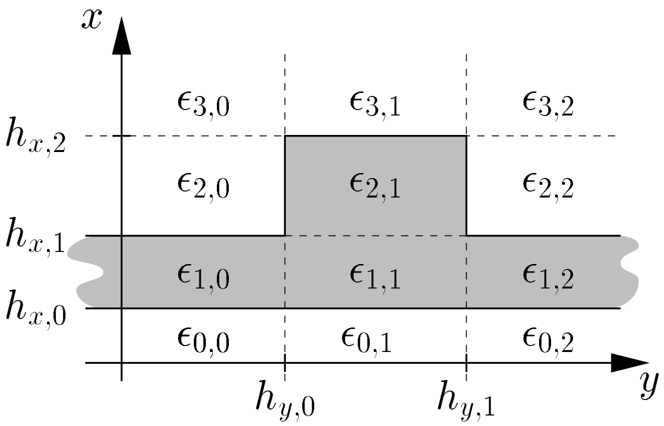


Slab waveguide analysis:



- Ansatz for the modal field, separately on each slab: sum of physically reasonable fundamental solutions of the wave equation.
- Continuity requirements for the electromagnetic fields connect the unknown coefficients within a system of linear equations.
- This system must be solved for a nonvanishing field.

WMM: Attempt to establish an analogous procedure for two transverse dimensions.



- Guided modes:
  Square integrable fields, all components are of the form
  $$F(x,y,z,t) = \Phi(x,y)\,\mathrm{e}^{\mathrm{i}(\omega t - \beta z)}, \quad \omega = kc = 2\pi c/\lambda\,.$$

- Wave equation, valid inside regions with constant permittivity:
  $$\partial_x^2\Phi + \partial_y^2\Phi = (\beta^2 - k^2\epsilon)\Phi.$$

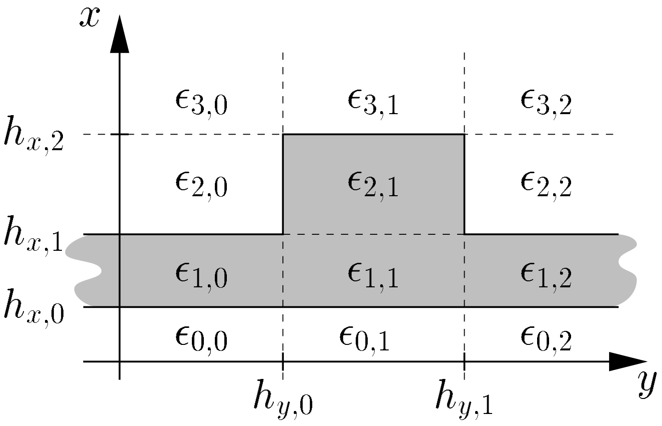
- Semivectorial calculations:

polarization	set to zero	Φ represents
QTE	E_x	E_y
QTM	H_x	H_y

- Vectorial calculations:
  (Φ_1, Φ_2) represents (E_x, E_y) or (H_x, H_y) or (E_z, H_z).

- Continuity requirements:

polarization	quantities continuous on horizontal boundaries	quantities continuous on vertical boundaries
QTE	E_y, $\partial_x E_y$	ϵE_y, $\partial_y E_y$
QTM	H_y, $\epsilon^{-1}\partial_x H_y$	H_y, $\partial_y H_y$
VEC	ϵE_x, E_y, E_z, H_x, H_y, H_z	E_x, ϵE_y, E_z, H_x, H_y, H_z



Ansatz for the modal field, separately on each rectangle:

$$\Phi(x, y) = \sum_j a_j \phi_j(x, y).$$

ϕ: factorizing solutions of the wave equation,

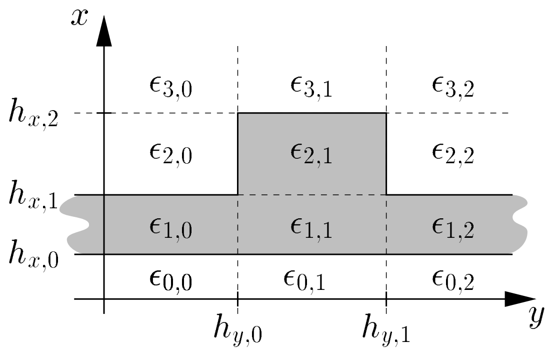
$$\phi(x, y) = c \cdot F(p(x - x_0)) \cdot G(q(y - y_0)).$$

c:	normalization constants,
x_0, y_0:	local coordinate offsets,
F, G	$\in \{\sin, \cos, \exp\}$,
p, q:	transverse wave vector components with $\pm p^2 \pm q^2 = \beta^2 - k^2\epsilon\,.$

For each subset of trial functions ($\leftrightarrow$ each choice of F and G):
Select a number of parameters p_j and q_j, such that the corresponding trial functions

- are representative for that subset,
- are numerically linearly independent.

On unbounded rectangles:
Consider only outwards exponentially decaying trial functions.



Least squares expression for the field mismatch:

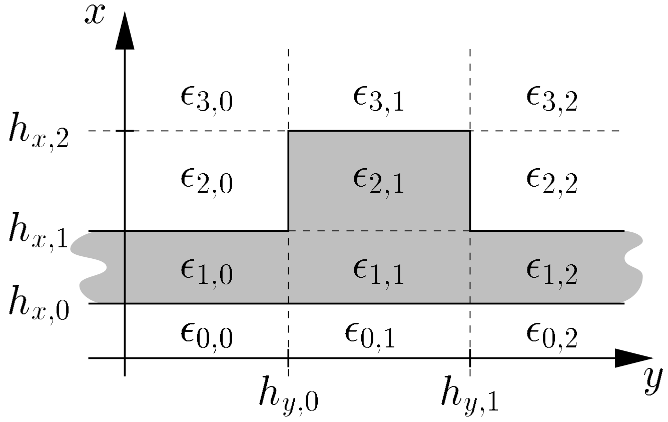
$$D_\beta(\vec{a}) = \sum_{\substack{\text{quantities}\\ \text{required}\\ \text{to be}\\ \text{continuous}}} \; \sum_{\substack{\text{horizontal}\\ \text{/ vertical}\\ \text{boundary}\\ \text{sections}}} \; \int_{\substack{\text{boundary}\\ \text{line}}} \begin{matrix}\text{suitable}\\ \text{weighting}\\ \text{factors}\end{matrix} \cdot \left| \begin{matrix}\text{difference}\\ \text{in field /}\\ \text{derivative}\end{matrix} \right|^2 \mathrm{d}s$$

Side condition: normalized fields

$$N_\beta(\vec{a}) = \sum_{\text{rectangles}} \; \iint_{\substack{\text{rectangle}\\ \text{domain}}} \begin{matrix}\text{suitable}\\ \text{weighting}\\ \text{factors}\end{matrix} \cdot \left| \quad \text{field} \quad \right|^2 \mathrm{d}x\, \mathrm{d}y$$

polarization	$\mid$ field $\mid^2$
QTE	$\lvert E_y \rvert^2$
QTM	$\lvert H_y \rvert^2$
VEC	$\lvert E_x \rvert^2 + \lvert E_y \rvert^2 + \lvert E_z \rvert^2$ or $\lvert H_x \rvert^2 + \lvert H_y \rvert^2 + \lvert H_z \rvert^2$ or (positive) part of S_z or ?

Guided mode: prescribed by $(\beta, \vec{a})$ with $D_\beta(\vec{a}) = 0$ and $N_\beta(\vec{a}) = 1$.



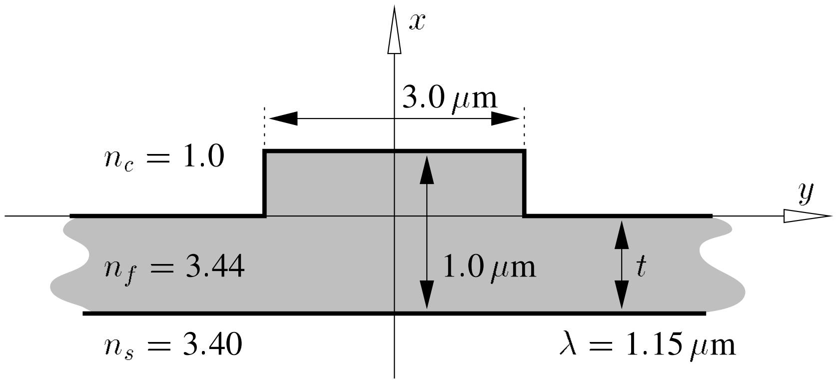
D_β and N_β are quadratic forms:

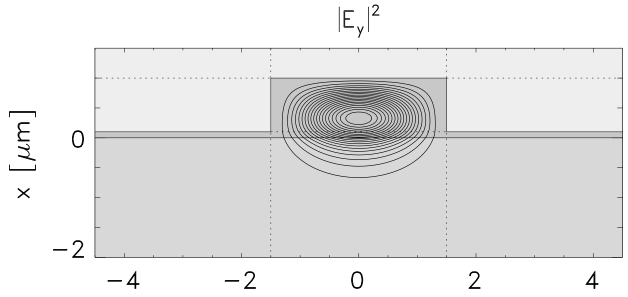
$$D_\beta(\vec{a}) = \vec{a}^\top \mathsf{D}_\beta \, \vec{a}, \qquad N_\beta(\vec{a}) = \vec{a}^\top \mathsf{N}_\beta \, \vec{a}.$$

- For fixed trial value β find the set of coefficients $\vec{a}$ with minimum error $\mu_\beta \Leftrightarrow$ solve the eigenvalue equation

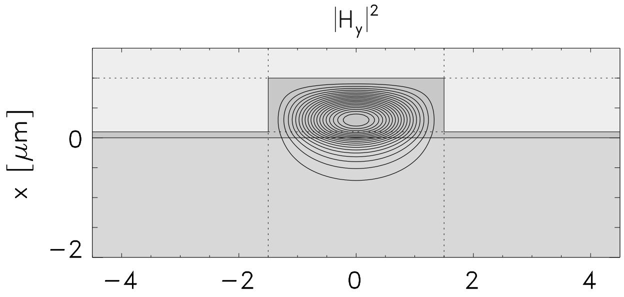
$$\mathsf{D}_\beta \, \vec{a} = \mu_\beta \, \mathsf{N}_\beta \vec{a} \, .$$

- Minimize μ_β with respect to β
  $\rightsquigarrow$ estimation for a valid propagation constant.

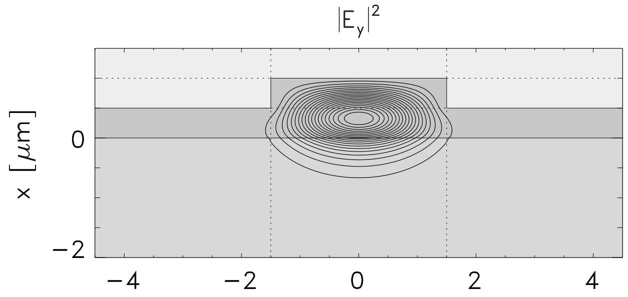




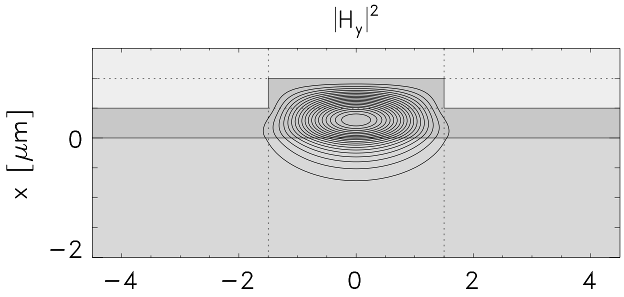
t = **0.1 μm, QTE**



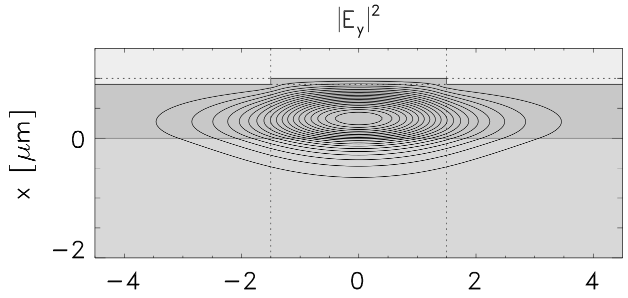
t = **0.1 μm, QTM**



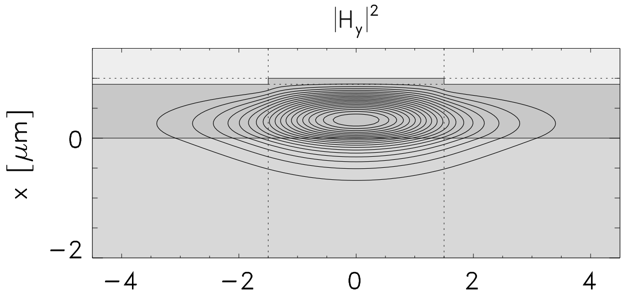
t = **0.5 μm, QTE**



t = **0.5 μm, QTM**



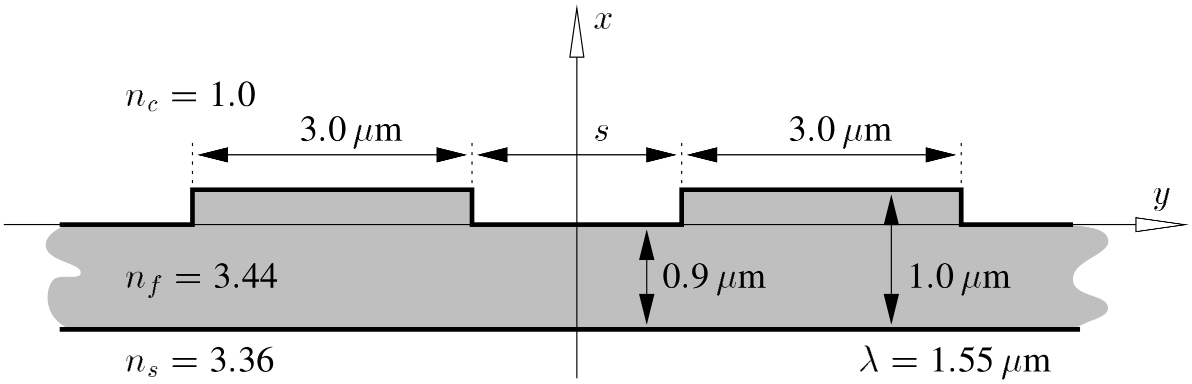
t = **0.9 μm, QTE**

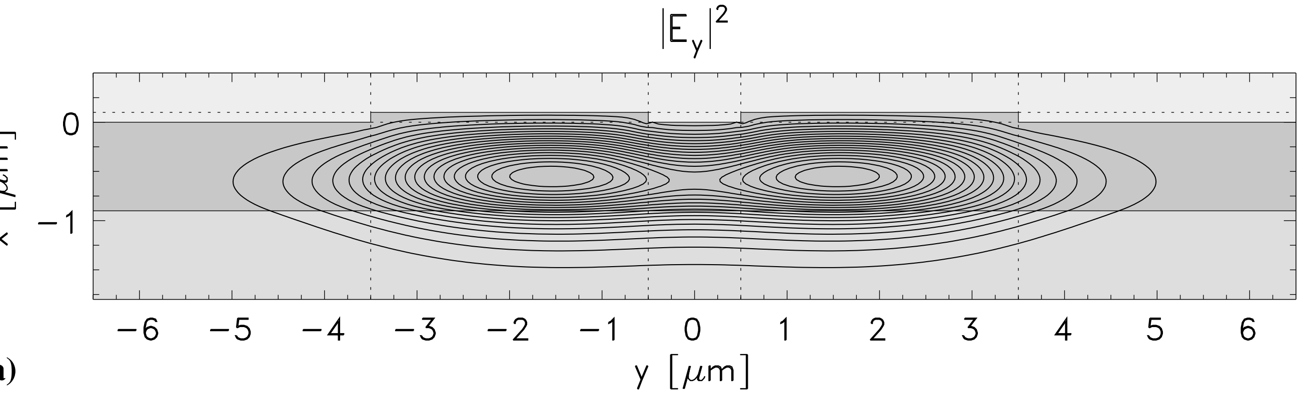


t = **0.9 μm, QTM**

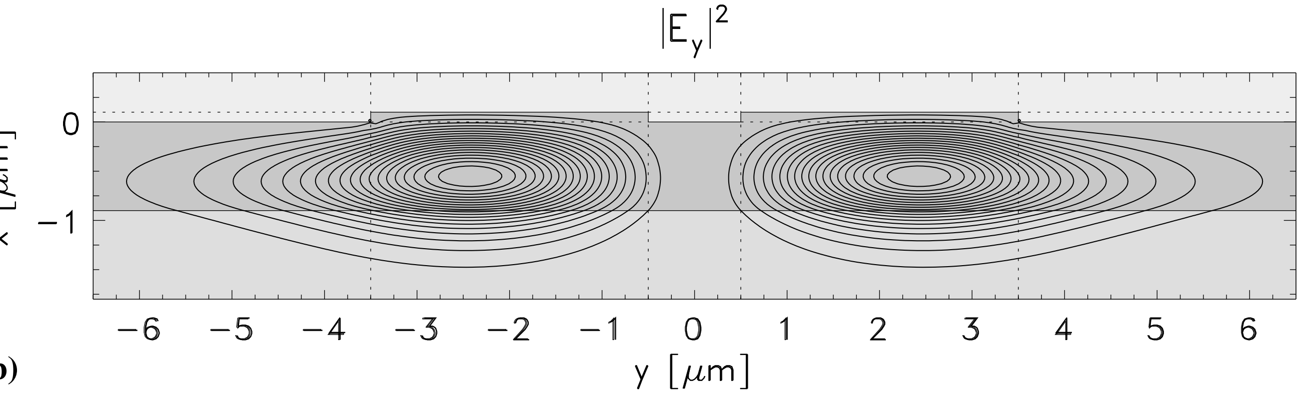
		t/μm	0.1	0.3	0.5	0.7	0.9
$\dfrac{(\beta/k)^2 - n_s^2}{n_f^2 - n_s^2}$	QTE	MTRM	0.3019	0.3110	0.3270	0.3512	0.3883
		WMM	0.3024	0.3112	0.3268	0.3508	0.3881
	QTM	MTRM	0.2674	0.2751	0.2890	0.3107	0.3455
		WMM	0.2676	0.2759	0.2892	0.3102	0.3451

MTRM: Modal Transverse Resonance Method by A. S. Sudbø;
data from C. Vassallo, *Opt. Quant. Electr.* **29**, 1997





(a)

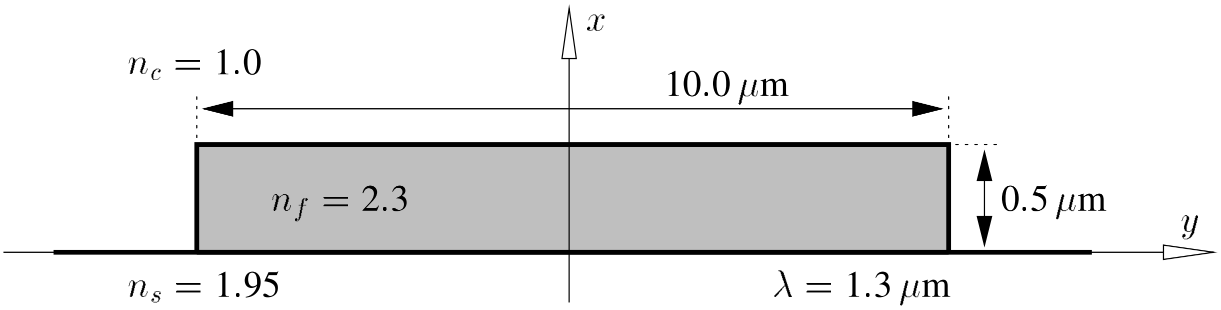


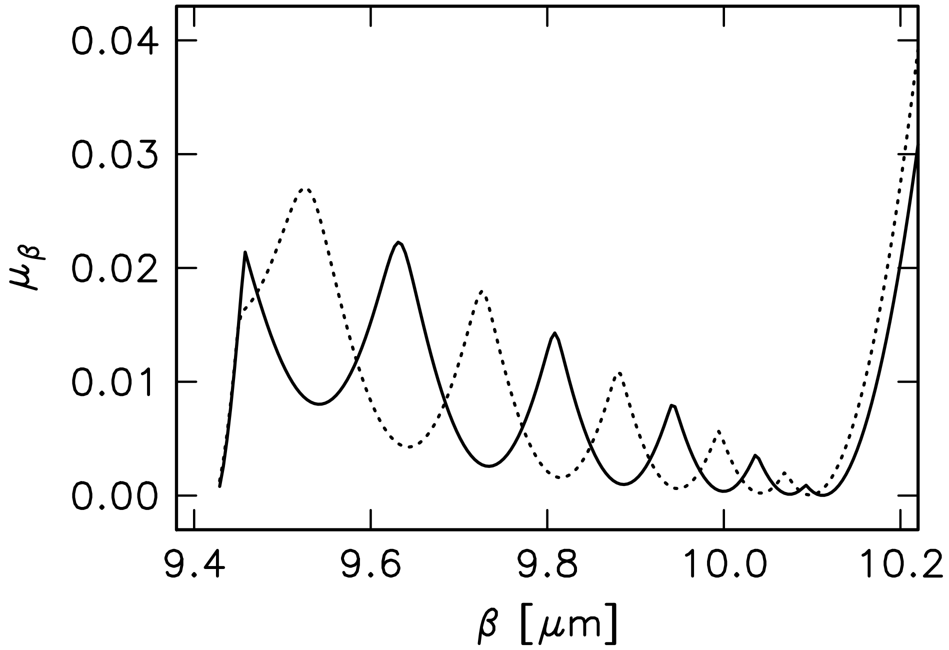
(b)

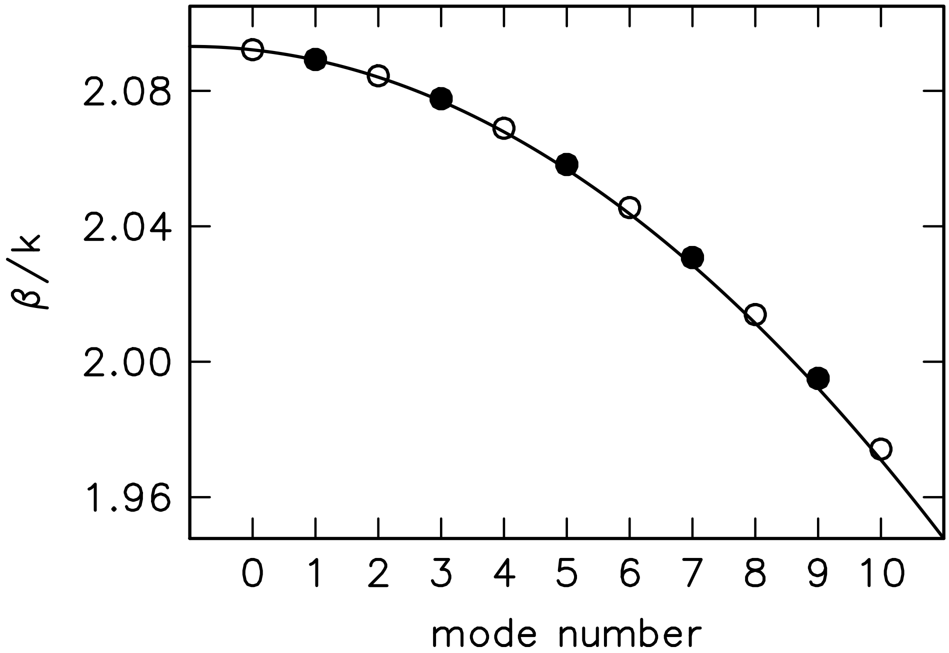
$$L_c = \frac{\pi}{\beta_s - \beta_a}$$

s/μm	1.0	3.0	
L_c/mm	0.45	1.47	FDM
	0.46	1.49	WMM

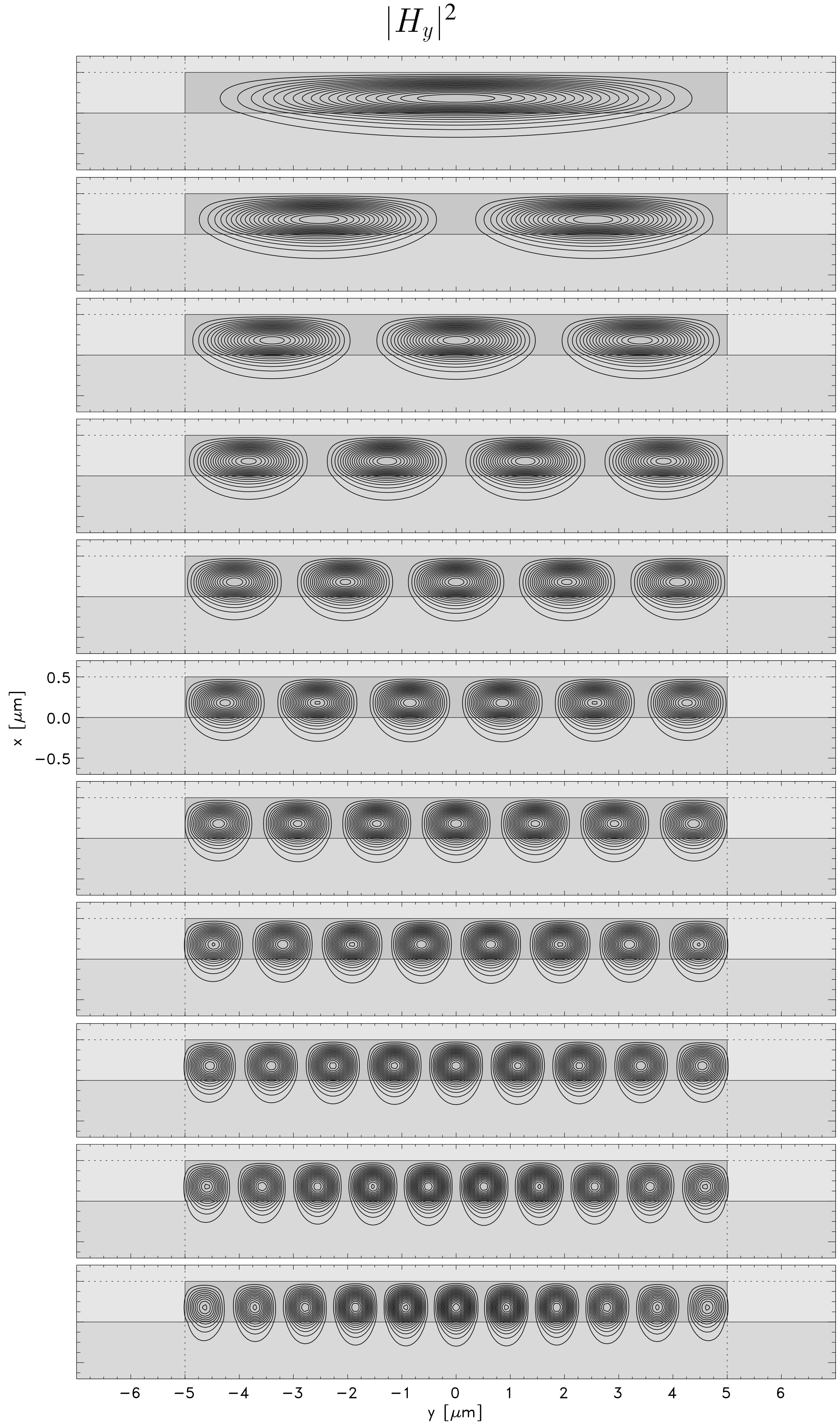
FDM: H. Noro and T. Nakayama, *J. Lightw. Techn.* **14**, 1996

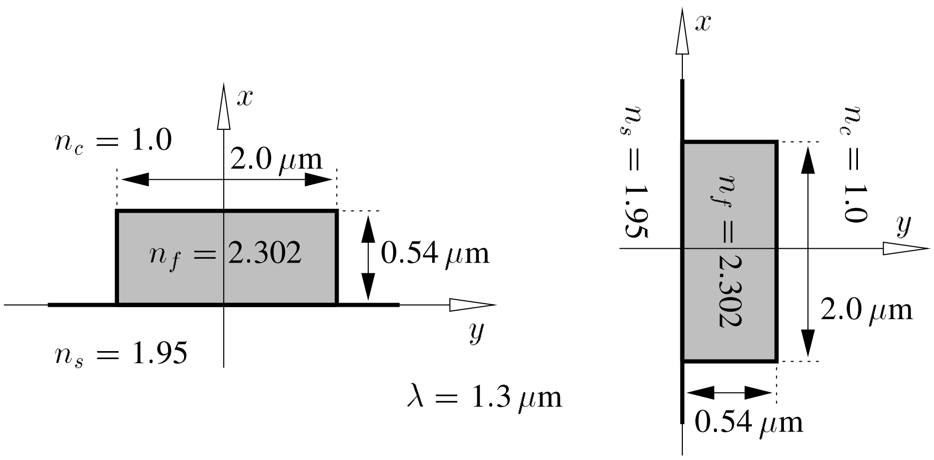




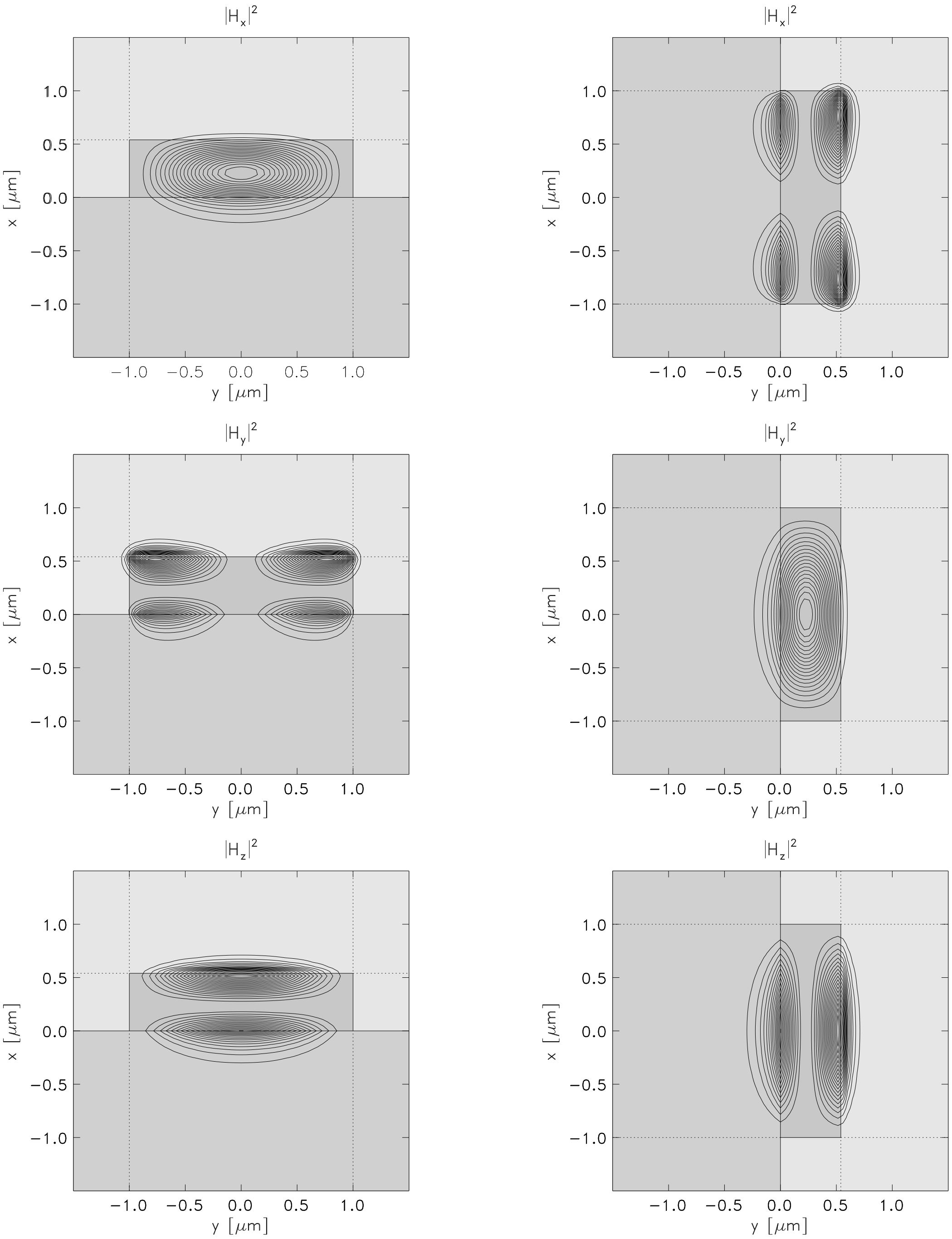


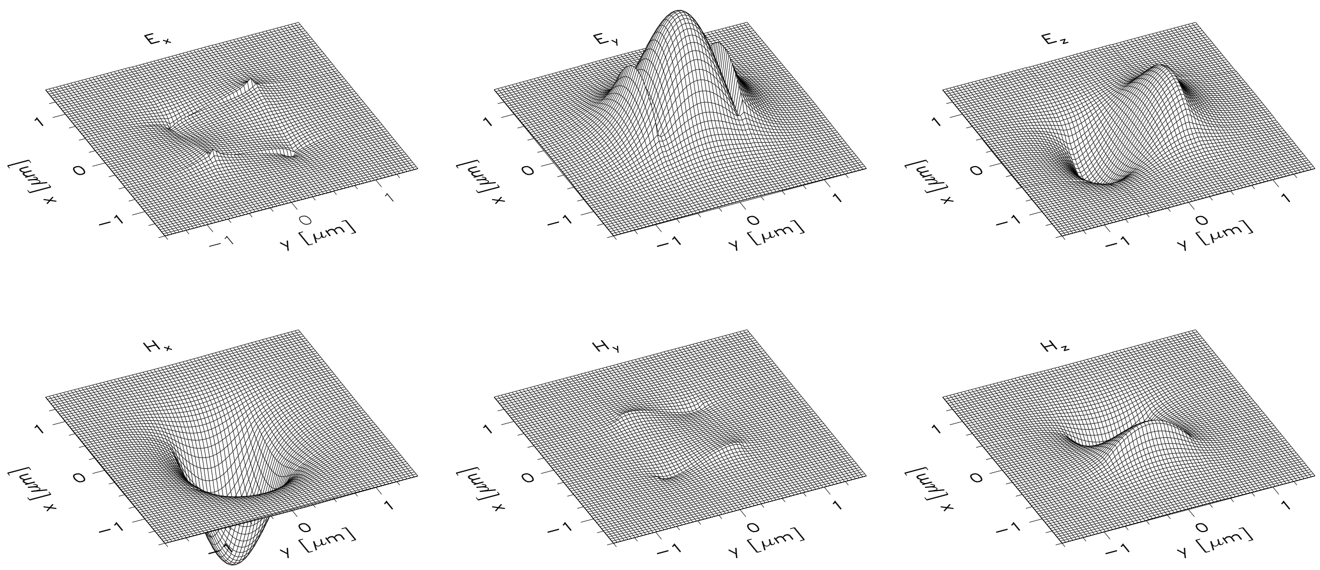
$\bullet, \circ$: WMM ——— : $\beta_v/k = n_p - \dfrac{\lambda^2(\nu+1)^2}{8 n_p w^2}$

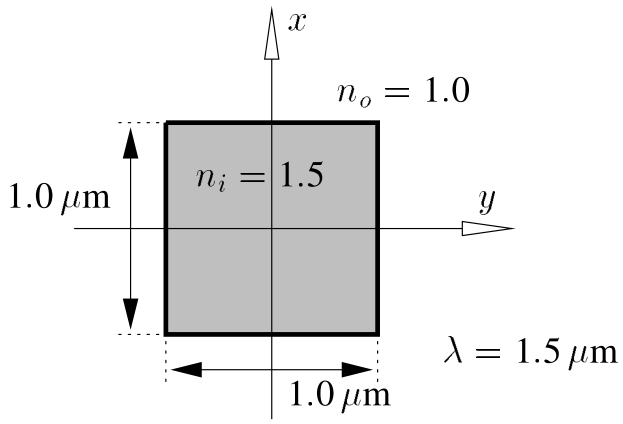

$|H_y|^2$
0.5
0.0
−0.5
x [μm]
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
y [μm]

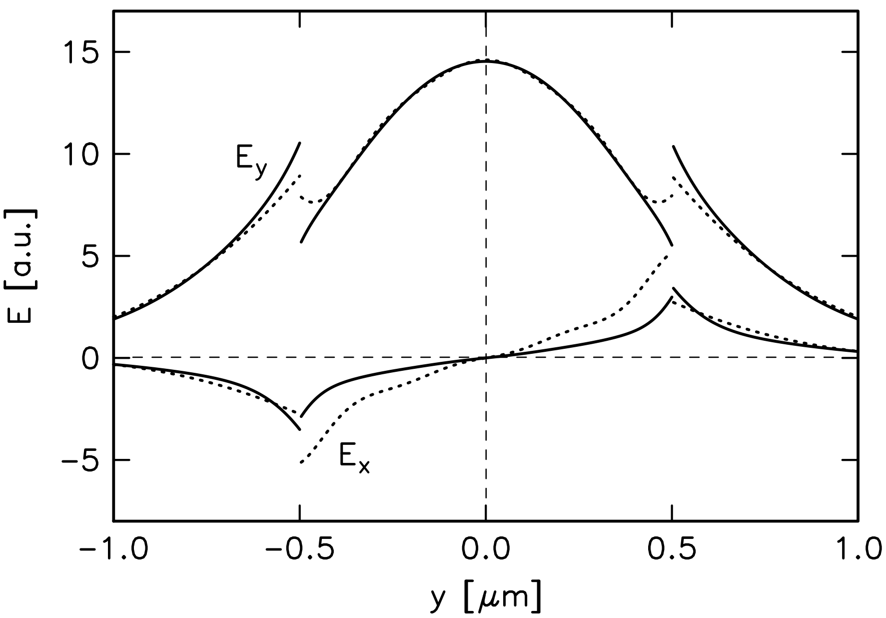


$$\beta = 10.3432\,\mu\mathrm{m}^{-1}$$



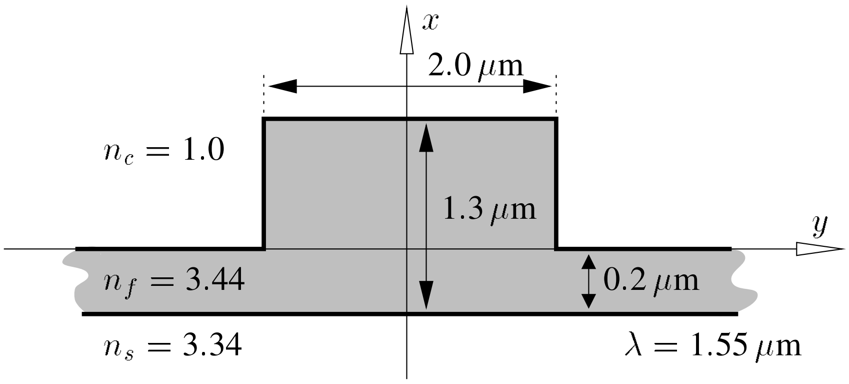




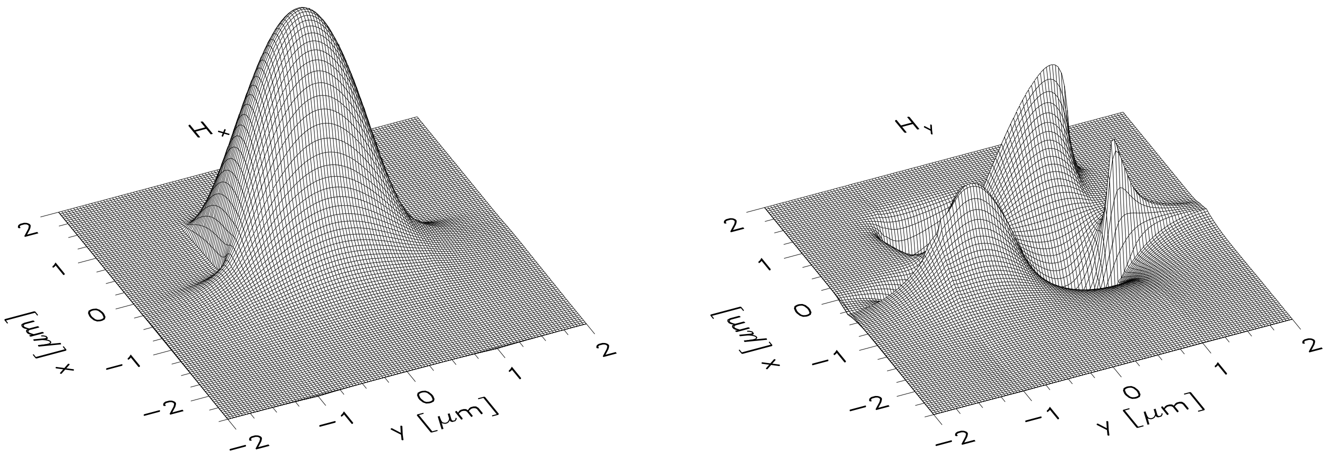


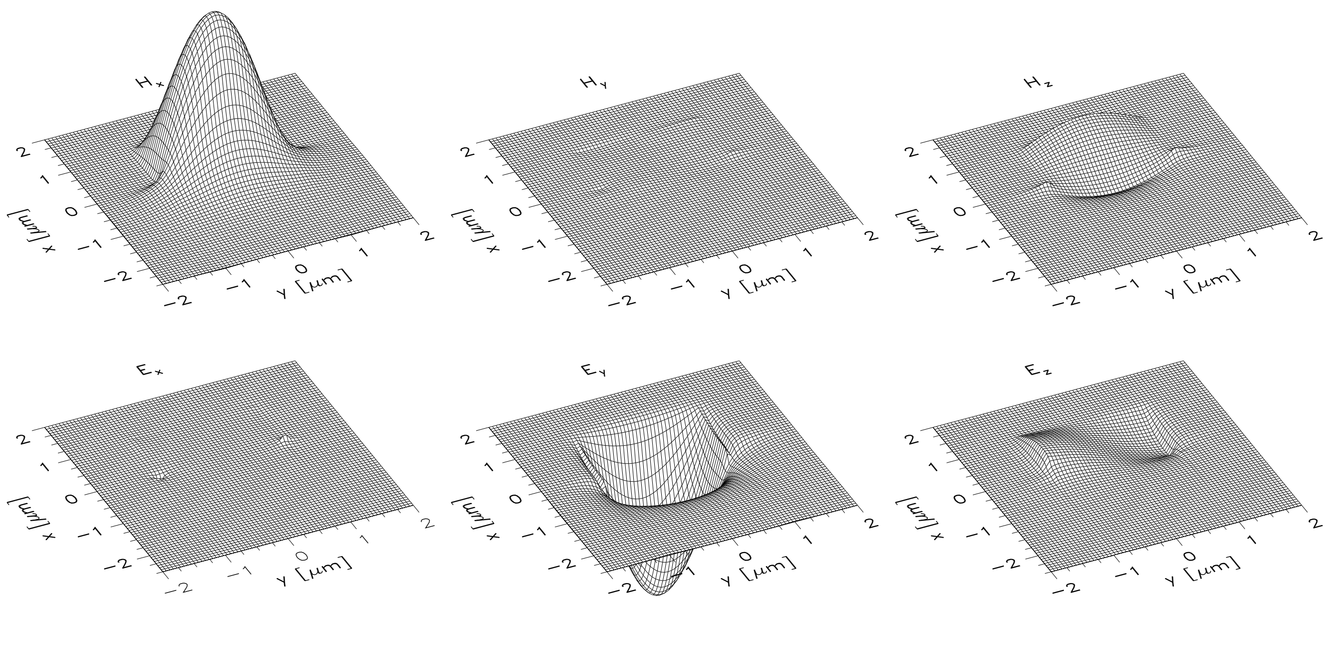
$\cdots\cdots$: $E_{x,y}(0.5\,\mu\text{m} + \delta, y)$ —— : $E_{x,y}(0.5\,\mu\text{m} - \delta, y)$

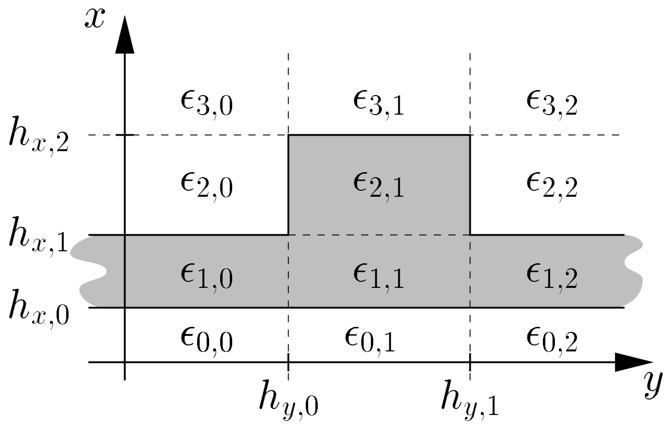
(c.f. A. S. Sudbø, *J. Lightw. Techn.* **10**, 1992)



$\beta/k =$ 3.38869, vectorial FDM,
P. Lüsse et. al., *J. Lightw. Techn.* **12**, 1994
3.38866, semivectorial WMM, QTE-polarization
3.38854, vectorial WMM







WMM:

- a method for calculating guided modes of waveguides with piecewise constant and rectangular permittivity profile,
- the simple ansatz of factorizing functions can approximate modal fields well,
- Maxwell's equations are satisfied exactly inside homogeneous regions,
- the field mismatch on the boundaries allows to estimate propagation constants via the minimization of a least squares expression,
- can be implemented for semivectorial and vectorial calculations,
- vectorial simulations treat both transverse directions exactly in the same way,
- reasonable results, good agreement with standard examples from the literature,
- moderate in memory and computational time consumption, well qualified for an implementation on modern personal computers,
- yields continuous field representations, not restricted to a computational window; forms an ideal basis for a more extensive tool for integrated optics design.