HCMT interaction of whispering gallery modes in circuits of circular integrated optical micro-resonators

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Integrated Optical MicroSystems
MESA$^+$ Institute for Nanotechnology
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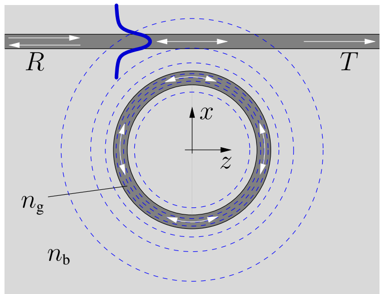
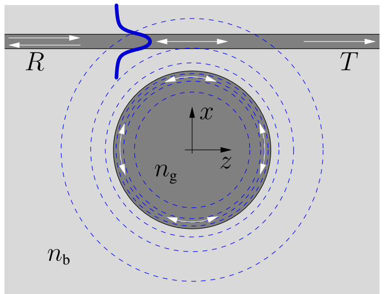
Excitation of whispering gallery resonances





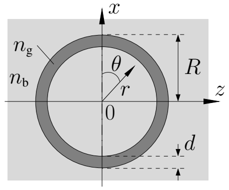
2-D, $n_g > n_b$, $\left\{ \omega_j^c, \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_j^c (x,z) \right\}$, $P_{in}(\omega)$ given: $T(\omega)$, $R(\omega)$ = ?

Outline

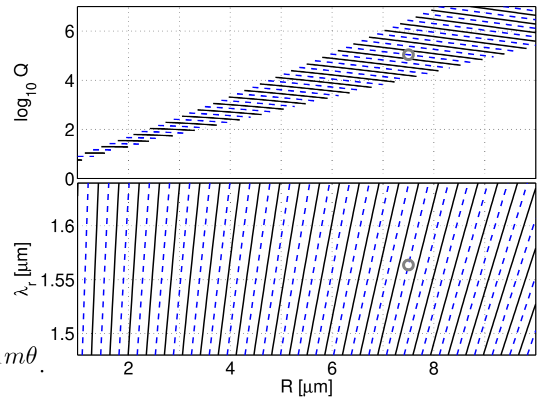
Localized resonances & guided wave excitation

- Whispering gallery modes
- Hybrid analytical / numerical coupled mode theory
- Benchmarks, micro-ring and -disk
- Supermode analysis, perturbations
- CROW
- Three-ring molecule

Micro-ring, resonances



TE, $d = 0.75\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



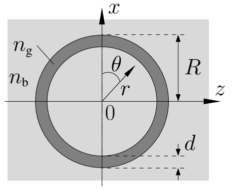
WGM(n, m):

$\omega^c_{n,m} \in \mathbb{C}$, $m \in \mathbb{Z}$,

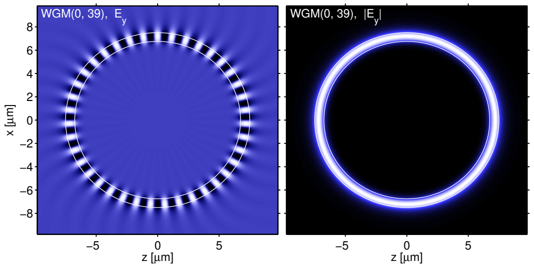
$$\psi^c_{n,m}(r,\theta) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}^c_{n,m} (r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

$Q = \mathrm{Re}\,\omega^c/(2\mathrm{Im}\,\omega^c)$, $\lambda_r = 2\pi c/\mathrm{Re}\,\omega^c$, outgoing radiation, FWHM: $\Delta\lambda = \lambda_r/Q$.

Micro-ring, resonances

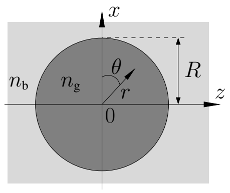


TE, $R = 7.5\,\mu$m, $d = 0.75\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.

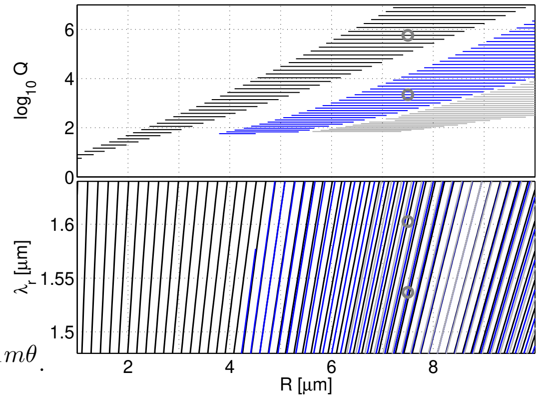


WGM(0, 39): $\lambda_r = 1.5637\,\mu$m, $Q = 1.1 \cdot 10^5$, $\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

Micro-disk, resonances



TE,
$n_g = 1.5$, $n_b = 1.0$.

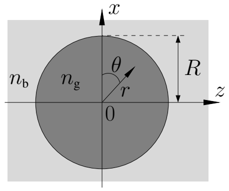


WGM(n, m):

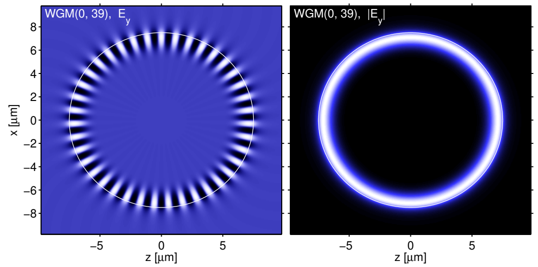
$\omega^{c}_{n,m} \in \mathbb{C},\ \ m \in \mathbb{Z},$

$$\psi^{c}_{n,m}(r,\theta) = \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}^{c}_{n,m}(r)\, \mathrm{e}^{-\mathrm{i}m\theta}.$$

Micro-disk, resonances

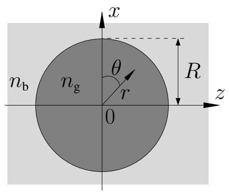


TE, $R = 7.5\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.

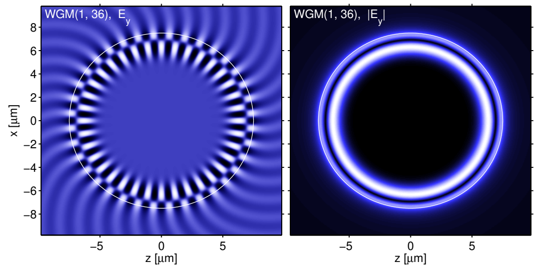


WGM(0, 39): $\lambda_r = 1.6025\,\mu$m, $Q = 5.7 \cdot 10^5$, $\Delta\lambda = 2.8 \cdot 10^{-6}\,\mu$m.

Micro-disk, resonances

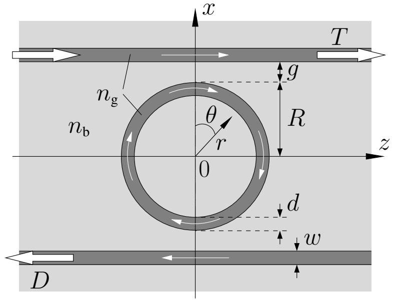


TE, $R = 7.5\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



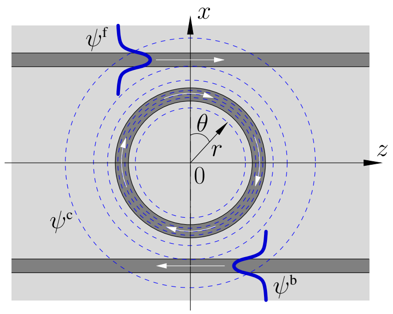
WGM(1, 36): $\lambda_r = 1.5367\,\mu$m, $Q = 2.2 \cdot 10^4$, $\Delta\lambda = 7.0 \cdot 10^{-4}\,\mu$m.

Ringresonator



TE, $R = 7.5\,\mu\mathrm{m}$, $w = 0.6\,\mu\mathrm{m}$, $d = 0.75\,\mu\mathrm{m}$, $g = 0.3\,\mu\mathrm{m}$, $n_{\mathrm{g}} = 1.5$, $n_{\mathrm{b}} = 1.0$, $\lambda \approx 1.55\,\mu\mathrm{m}$.

Ringresonator, field template

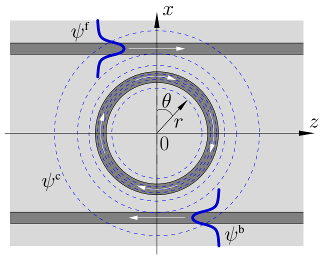


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$.
- Bus channels:
  $\boldsymbol{\psi}^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z}$.
- Cavity, WGMs:
  $\boldsymbol{\psi}^{\mathrm{c}}_j(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{c}}_j(r)\, \mathrm{e}^{-\mathrm{i}m_j\theta}$,
  $m_j \in \mathbb{Z}$.
- Further terms:
  bidirectional propagation, higher order modes, other channels, etc..

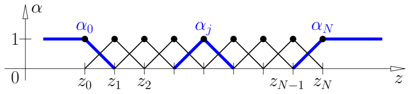
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}^{\mathrm{c}}_j(r,\theta),$$

$r = r(x,z),\ \theta = \theta(x,z)$. $\qquad f,\, b,\, c_j:$?

Ringresonator, HCMT procedure



Channels: 1-D FEM discretization,



$$f(z) = \sum_{j=0}^{N} f_j\,\alpha_j(z),$$

analogously $b(z) \to \{b_j\}$.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_j f_j\,(\alpha_j \boldsymbol{\psi}^{\mathrm{f}}_j)(x,z) + \sum_j b_j\,(\alpha_j \boldsymbol{\psi}^{\mathrm{b}}_j)(x,z) + \sum_j c_j\,\boldsymbol{\psi}'^{\mathrm{c}}_j(x,z)$$

$$=: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{\text{channels, modes, elements, resonances}\}$, $\quad a_k \in \{f_j, b_j, c_j\}$. $\quad a_k:$?

Galerkin procedure

$$\begin{array}{r} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \quad \iint_{\text{comp. domain}}$$

$$\iint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z$.

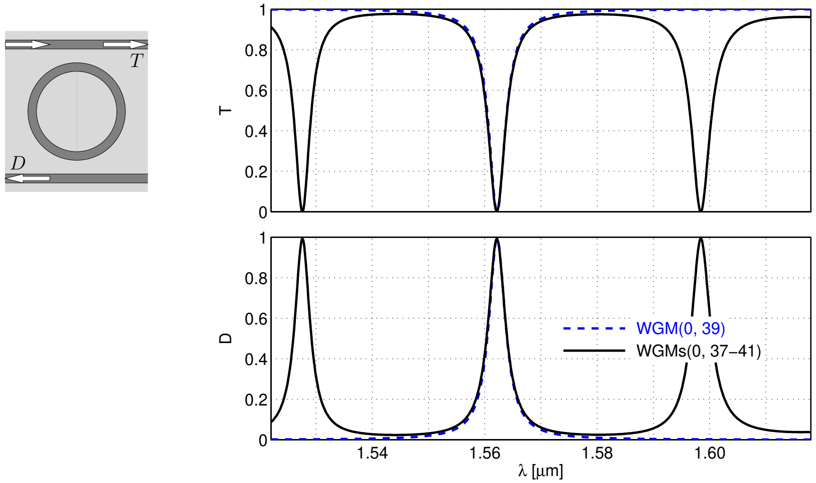
$$\sum_{k \in \{\mathrm{u},\mathrm{g}\}} K_{lk} a_k = 0\,, \quad l \in \{\mathrm{u}\}\,,$$

$$(\mathsf{K}_{\mathrm{u\,u}}\ \mathsf{K}_{\mathrm{u\,g}}) \begin{pmatrix} \boldsymbol{a}_\mathrm{u} \\ \boldsymbol{a}_\mathrm{g} \end{pmatrix} = 0\,, \quad \text{or} \quad \mathsf{K}_{\mathrm{u\,u}} \boldsymbol{a}_\mathrm{u} = -\mathsf{K}_{\mathrm{u\,g}} \boldsymbol{a}_\mathrm{g}\,.$$

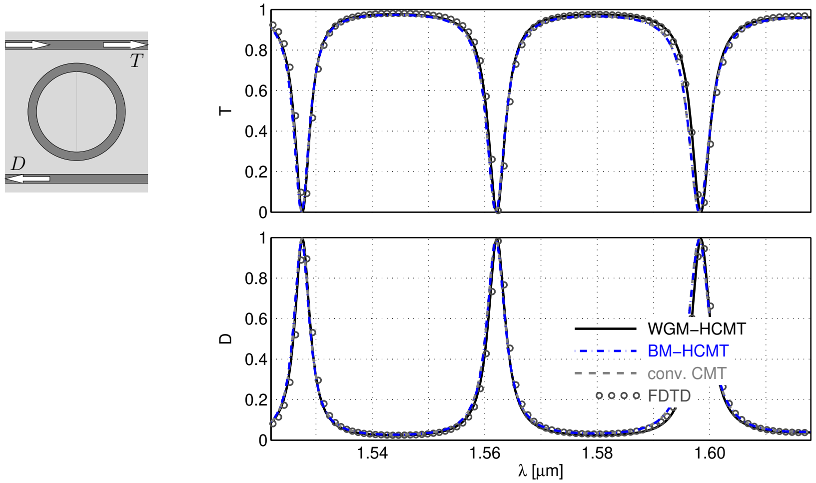
Further issues

... plenty.

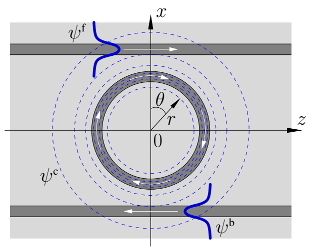
Single ring filter, spectral response

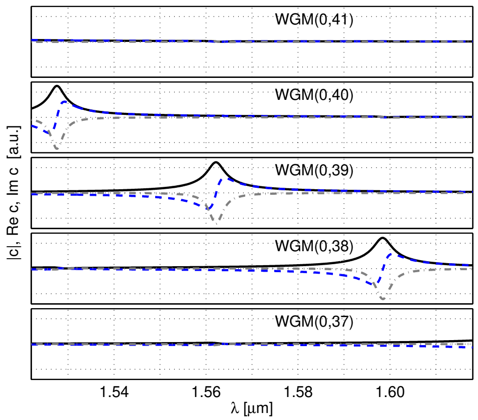


Single ring filter, benchmark



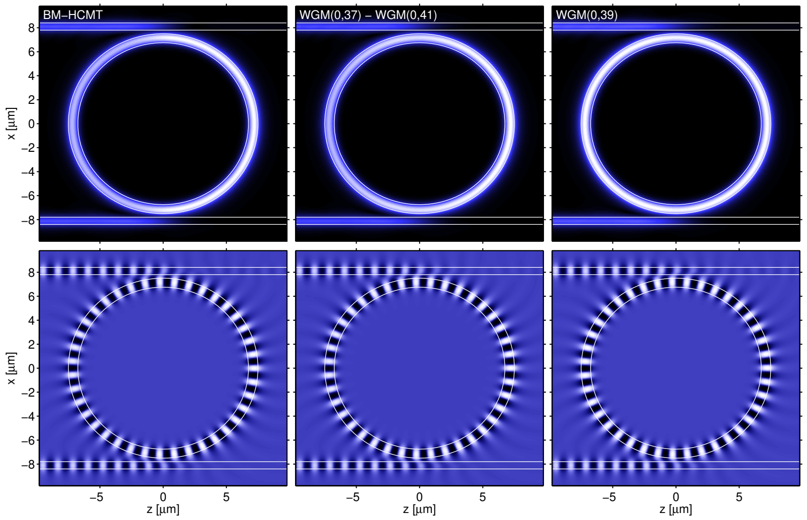
Single ring filter, WGM amplitudes



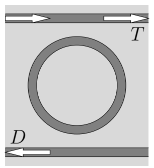


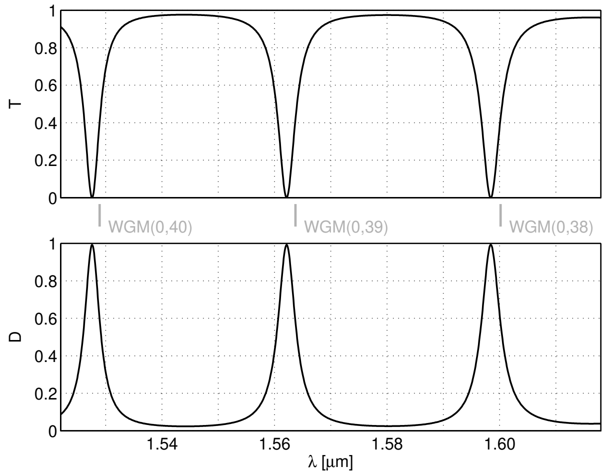
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\prime\mathrm{c}}(x,z)$$

Single ring filter, transmission resonance



Single ring filter, resonance positions I





Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{array}{r} \boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \;\&\; \text{boundary conditions: “outgoing waves”} \right\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

$$\begin{array}{r} \boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \quad \Bigg| \quad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \qquad \iint_{\text{comp. domain}}$$

$$\iint \mathcal{A}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\boldsymbol{G},$$

where $\mathcal{A}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{G}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E})$,
$\mathcal{B}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \mathrm{i}\epsilon_0\epsilon\boldsymbol{F}^*\cdot\boldsymbol{E} + \mathrm{i}\mu_0\boldsymbol{G}^*\cdot\boldsymbol{H}$.

HCMT supermode analysis

- Insert $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}$,
- require
$$\iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \ \text{ for all } \ l\,,$$
- compute
$$A_{lk} = \iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,,$$
$$B_{lk} = \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,.$$

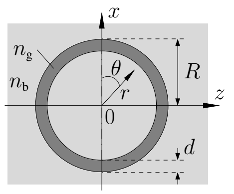
$$\sum_k A_{lk} a_k - \omega^{\mathrm{s}} B_{lk} a_k = 0 \ \text{ for all } \ l\,, \quad \text{or} \quad \mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}} \mathsf{B}\boldsymbol{a}\,.$$

$$\left\{\omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ \boldsymbol{E},\ \boldsymbol{H}\right\}^{\mathrm{s}}.$$

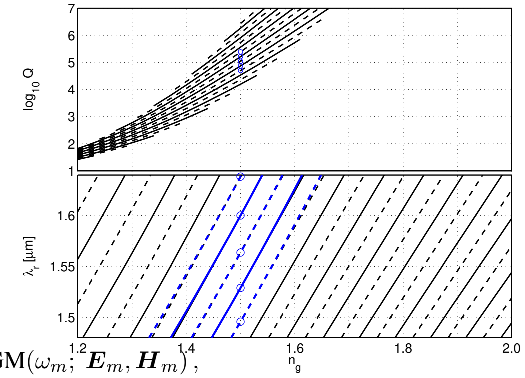
Further issues

… plenty.

WGMs, small uniform perturbations



TE, $R = 7.5\,\mu$m, $d = 0.75\,\mu$m
$n_{\mathrm{b}} = 1.0$.

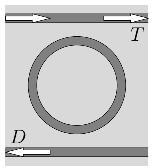


ϵ_m ↔ $\mathrm{WGM}(\omega_m; \boldsymbol{E}_m, \boldsymbol{H}_m)$,

$\epsilon = \epsilon_m + \Delta\epsilon$ ↔ $\mathrm{WGM}(\omega_m + \Delta\omega;\ \approx \boldsymbol{E}_m, \approx \boldsymbol{H}_m)\,,$

$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iint \Delta\epsilon |\boldsymbol{E}_m|^2\,\mathrm{d}x\,\mathrm{d}z}{\iint \left(\epsilon_m \epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right)\mathrm{d}x\,\mathrm{d}z}\,.$$

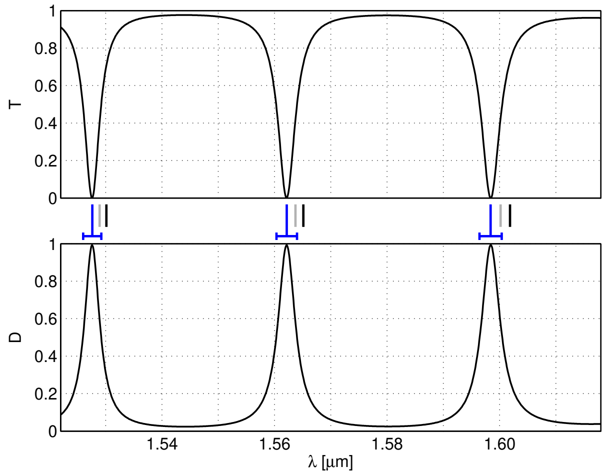
Single ring filter, resonance positions II



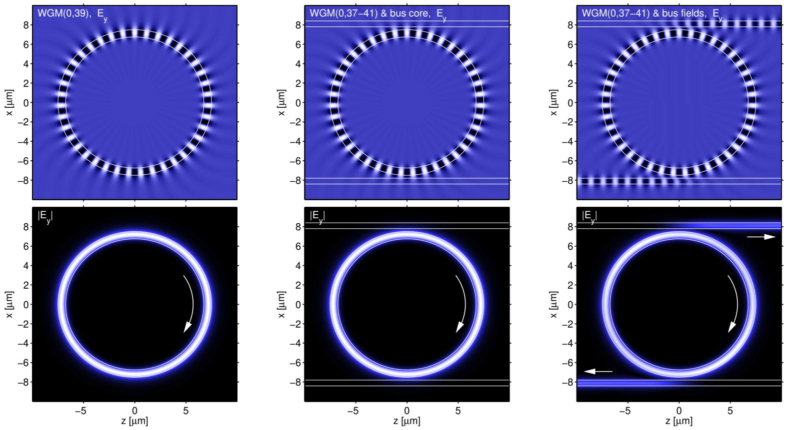
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Single ring filter, unidirectional supermodes

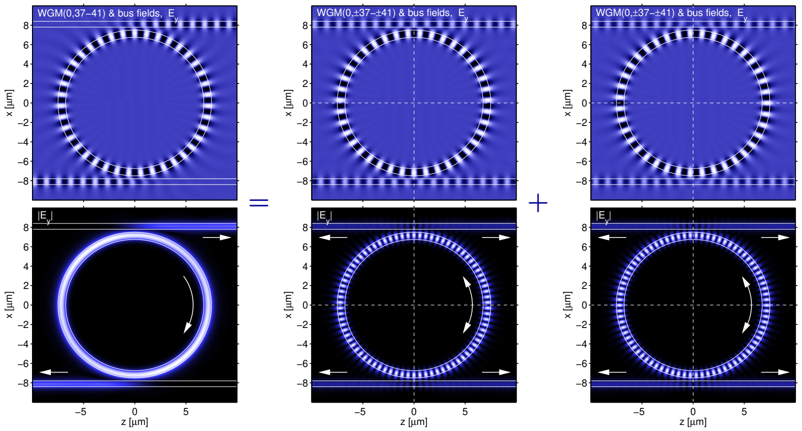


$\lambda_r = 1.5637\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

$\lambda_r = 1.5651\,\mu$m,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu$m.

$\lambda_r = 1.5622\,\mu$m,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu$m.

Single ring filter, bidirectional supermodes

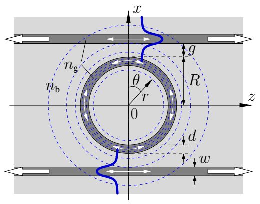


$\lambda_r = 1.56219\,\mu$m,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu$m.

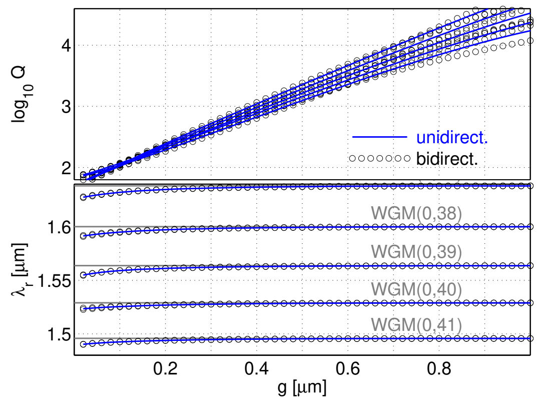
$\lambda_r = 1.56223\,\mu$m,
$Q = 4.4 \cdot 10^2$,
$\Delta\lambda = 3.5 \cdot 10^{-3}\,\mu$m.

$\lambda_r = 1.56215\,\mu$m,
$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.9 \cdot 10^{-3}\,\mu$m.

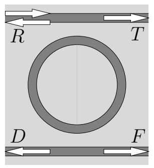
Single ring filter, supermodes vs. gap



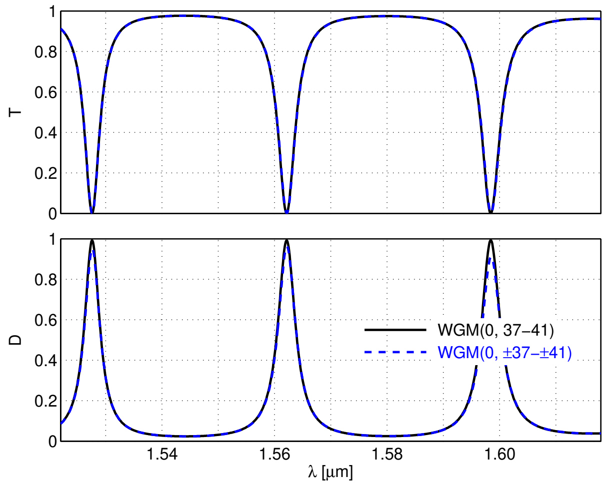
TE,
$R = 7.5\,\mu$m, $d = 0.75\,\mu$m,
$w = 0.6\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



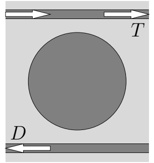
Single ring filter, transmission, bidirectional template



$R < 10^{-4}$,
$F < 10^{-4}$.



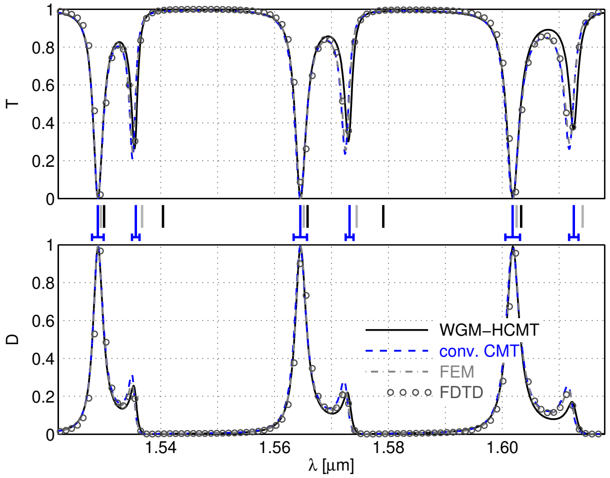
Micro-disk resonator, spectral response



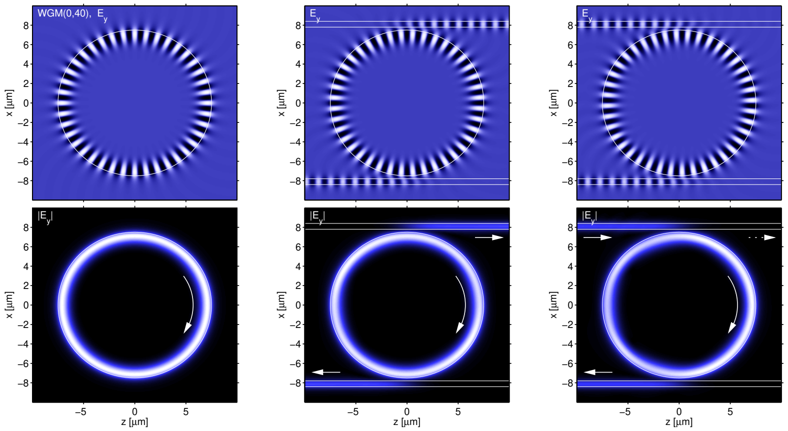
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Micro-disk, resonant fields (0)

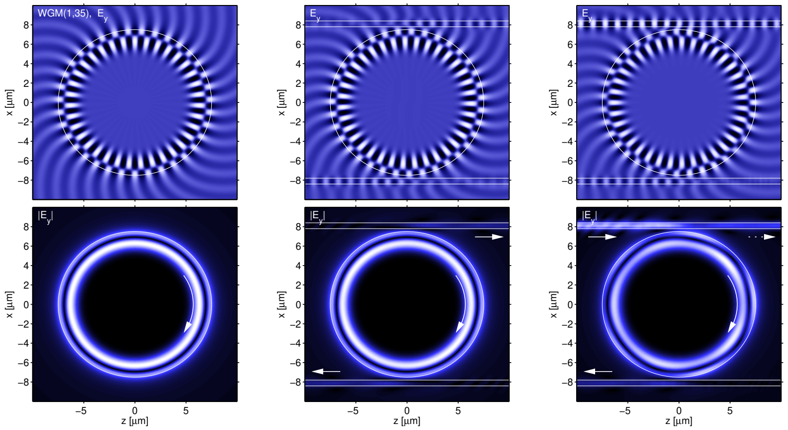


$\lambda_r = 1.56514\,\mu$m,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\,\mu$m.

$\lambda_r = 1.56454\,\mu$m,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}\,\mu$m.

$\lambda_r = 1.56456\,\mu$m.

Micro-disk, resonant fields (1)

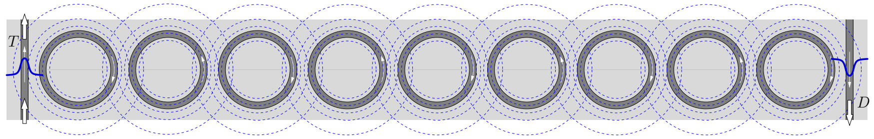


$\lambda_r = 1.57444\,\mu$m,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu$m.

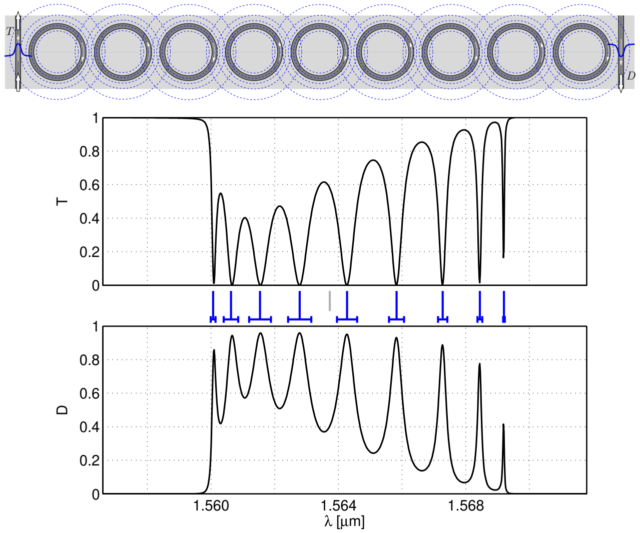
$\lambda_r = 1.57320\,\mu$m,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\,\mu$m.

$\lambda_r = 1.57306\,\mu$m.

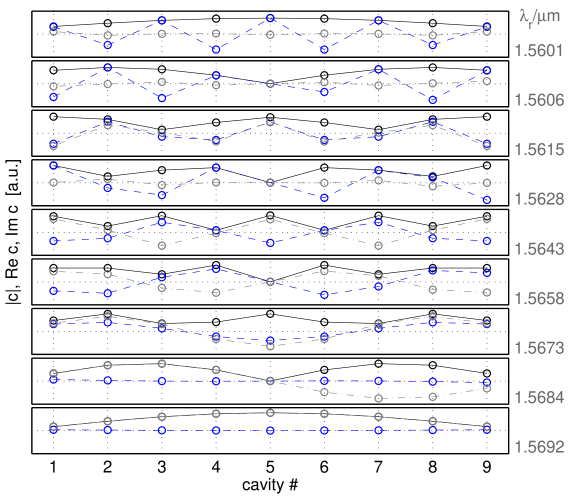
CROW



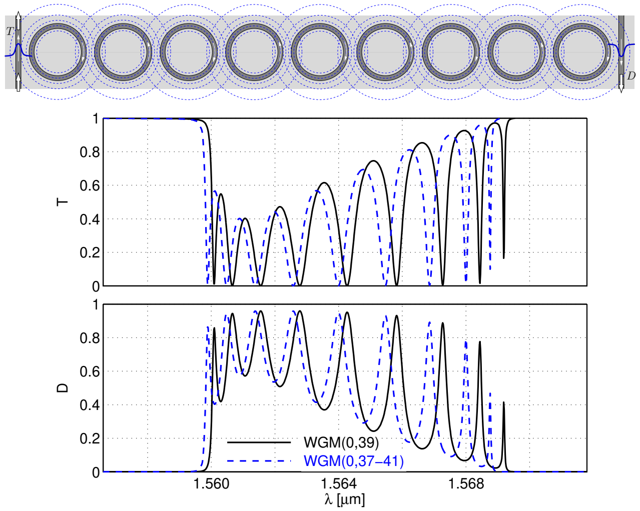
CROW, spectral response I



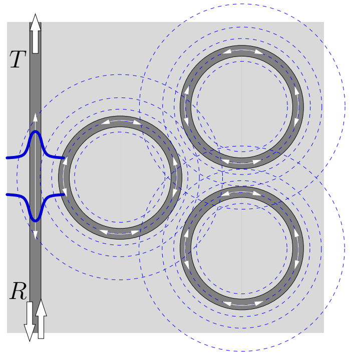
CROW, supermode pattern



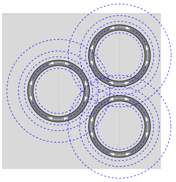
CROW, spectral response II



Three-ring molecule

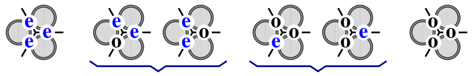


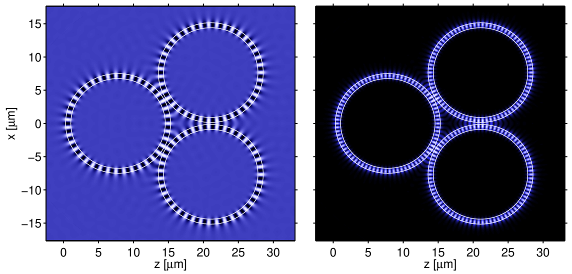
Three-ring molecule, supermodes



Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

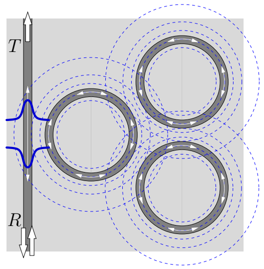
Symmetries:

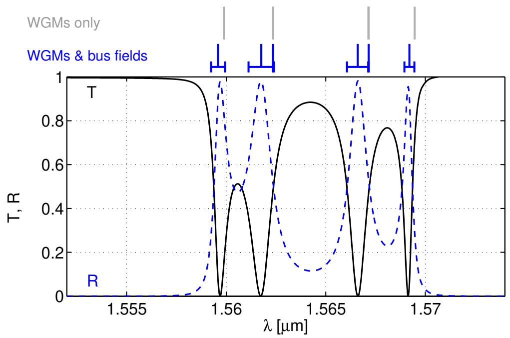




$\lambda_r = 1.56946\,\mu$m,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}\,\mu$m.

Three-ring molecule, excitation

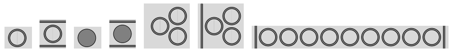




HCMT for circuits of circular resonators

WGM-HCMT:

- an ab-initio, quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- configurations with localized resonances: demonstrated,
- extension to 3-D (todo): numerical basis fields, still moderate effort,
- reasonably versatile:



...

Fast evaluation of spectral properties

Time consuming: evaluation of modal "overlaps" K_{lk} in K:

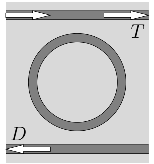
$$K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,.$$

All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

⇝ Interpolate $\mathsf{K}(\lambda)$:

- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\quad \lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b,\ \lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b$,
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}(\mathsf{K}_1 - \mathsf{K}_0)$,
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

Computational window



$R = 7.5\,\mu\text{m}$