A vectorial solver for the reflection of semi-confined waves at slab waveguide discontinuities for non-perpendicular incidence

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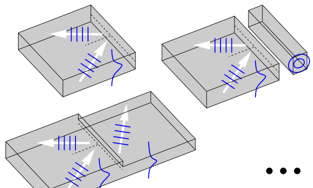
1 Oblique incidence at a slab discontinuity

The effects of transitions between regions with different layering on thin-film guided, in-plane unguided light form the basis for a series of classical integrated optical components [1, 2]. Concepts have been discussed for lenses [3, 4], mirrors [5], prisms [6], but also for complex lens-systems [7], or, more recently, for entire spectrometers [8]. The relevant interfaces are either straight, or merely slightly curved, permitting a description of the in-plane wave propagation in terms of geometrical optics. Hence we take a closer look at what happens to vertically guided, laterally plane waves at *straight* interfaces, facets, or transition regions with other cross section shapes.



While standard scalar TE / TM Helmholtz equations apply for perpendicular incidence, for non-normal incidence one is led to a vectorial problem [9] that is formally identical to that for the modes of 3-D channel waveguides. Here, however, it needs to be solved as a parametrized, inhomogeneous system on a 2-D computational window with transparent-influx boundary conditions. As a step beyond the scalar approximation [9], and older bidirectional approaches [10, 11], we here report on a dedicated vectorial solver for — in principle — arbitrary rectangular cross section geometries, based on simultaneous expansions into slab modes along *two* orthogonal coordinate axes [12].

2 Vectorial quadridirectional eigenmode propagation

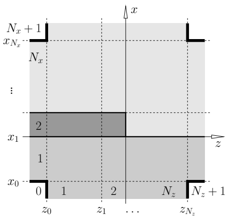
- Homogeneous Maxwell equations, frequency domain, linear dielectric media,

$$\mathrm{curl}\,\tilde{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\tilde{\boldsymbol{H}}, \quad \mathrm{curl}\,\tilde{\boldsymbol{H}} = \mathrm{i}\omega\epsilon\epsilon_0\tilde{\boldsymbol{E}}$$

  for electric and magnetic fields $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$, oscillating $\sim \exp(\mathrm{i}\omega t)$ in time with frequency $\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$, for vacuum wavenumber k, wavelength λ, speed of light c, permittivity ϵ_0, and permeability μ_0.

- Relative permittivity $\epsilon = n^2$ with $\partial_y\epsilon = 0$ everywhere

$$\begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}(x,y,z) = \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z)\,\mathrm{e}^{-\mathrm{i}k_y y}, \qquad k_y:\ \text{a given parameter.}$$



- Cross-shaped computational domain $(x,z) \in [x_0, x_{N_x}] \times [z_0, z_{N_z}]$ $\cup$ {external slab regions}.
- Rectangular, piecewise constant, permittivity $\epsilon(x,z)$.
- Division into slices $z \in [z_j, z_j+1]$ with z-constant ϵ.
- Division into layers $x \in [x_j, x_j+1]$ with x-constant ϵ.
- Establish separate expansion bases for each slice / layer.

- "Vectorial" slab modes, solutions where $\partial_y\epsilon = 0$ & $\partial_z\epsilon = 0$, $\epsilon = \epsilon(x)$

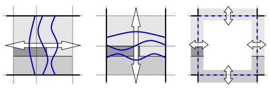
$$\partial_x^2\psi + \left(k^2\epsilon - \beta^2\right)\psi = 0, \qquad \beta^2 = k_y^2 + k_z^2, \qquad \text{(TE)}$$

$$\boldsymbol{E}(x,z) = \begin{pmatrix}0\\ k_z\psi(x)/\beta^2\\ -k_y\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}, \qquad \boldsymbol{H}(x,z) = \frac{1}{\omega\mu_0}\begin{pmatrix}-\psi(x)\\ \mathrm{i}k_y\partial_x\psi(x)/\beta^2\\ \mathrm{i}k_z\partial_x\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}.$$

$$\epsilon\partial_x\frac{1}{\epsilon}\partial_x\psi + \left(k^2\epsilon - \beta^2\right)\psi = 0, \qquad \beta^2 = k_y^2 + k_z^2, \qquad \text{(TM)}$$

$$\boldsymbol{E}(x,z) = \frac{1}{\omega\epsilon_0\epsilon}\begin{pmatrix}\psi(x)\\ -\mathrm{i}k_y\partial_x\psi(x)/\beta^2\\ -\mathrm{i}k_z\partial_x\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}, \qquad \boldsymbol{H}(x,z) = \begin{pmatrix}0\\ k_z\psi(x)/\beta^2\\ -k_y\psi(x)/\beta^2\end{pmatrix}\mathrm{e}^{-\mathrm{i}k_z z}.$$

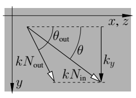
- Boundary conditions $\psi = 0$ or $\partial_x\psi = 0$ (. . .) at $x = x_0$, $x = x_{N_x}$ discretized mode spectra, complete sets.
- Exchange roles of x and z: modes travelling along $\pm x$, profiles depend on z.
- Incoming field: slab mode with $\beta_{\rm in} = kN_{\rm in}$ at angle θ $k_y(\theta) = kN_{\rm in}\sin\theta$
- Mode types (z-propagating / -evanescent) change with θ: $k_z(\theta) = \pm\sqrt{\beta^2 - k_y^2(\theta)}$.
- Expand the electromagnetic field into local sets of TE & TM eigenmodes.
- Bidirectional mode overlaps along all interfaces,
  Linear set of equations for local mode amplitudes, with incoming field as RHS.



Algebraic procedure [12]:
- horizontal vBEP, inner slices,
- vertical vBEP, inner layers,
- cross-overlaps at outer interfaces connect inner solutions and external regions (vBEP: vectorial bidirectional eigenmode propagation).

Vectorial quadridirectional eigenmode propagation (vQUEP)

3 Critical angles



Uniform $k_y = kN_{\rm in}\sin\theta$ related to incoming mode $(N_{\rm in})$ & incidence angle θ:

Outgoing modes $(N_{\rm out})$ leave at angles $\theta_{\rm out}$ with $k_y = kN_{\rm out}\sin\theta_{\rm out}$, *different* for every outgoing mode;

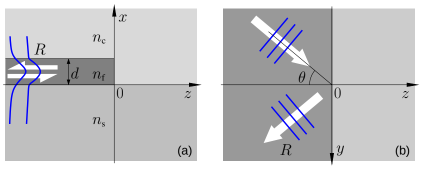
generalized Snell's law: $N_{\rm out}\sin\theta_{\rm out} = N_{\rm in}\sin\theta$,

applicable to all (reflected, transmitted, up- or downwards scattered) outgoing *propagating* modes.

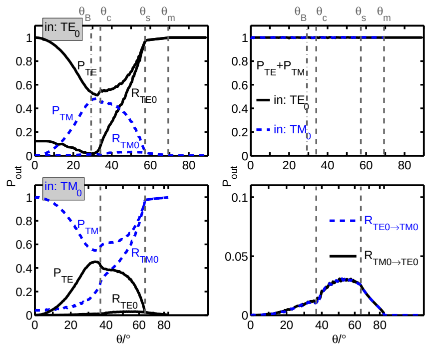
Outgoing modes with $N_{\rm out} \le N_{\rm crit}$ become evanescent for incidence angles $\theta \ge \theta_{\rm crit}$ with $\sin\theta_{\rm crit} = N_{\rm crit}/N_{\rm in}$, i.e. these modes do not carry power away.

Relevant values for the following examples:
- $\sin\theta_{\rm c} = n_{\rm c}/N_{\rm in}$, no forward / backward power loss into the cover for $\theta \ge \theta_{\rm c}$.
- $\sin\theta_{\rm s} = n_{\rm s}/N_{\rm in}$, no forward / backward power loss into the substrate for $\theta \ge \theta_{\rm s}$.
- in: TE$_0$, $\sin\theta_{\rm m} = N_{\rm TM0}/N_{\rm TE0}$, no power conversion to refl. TM$_0$ mode for $\theta \ge \theta_{\rm m}$.

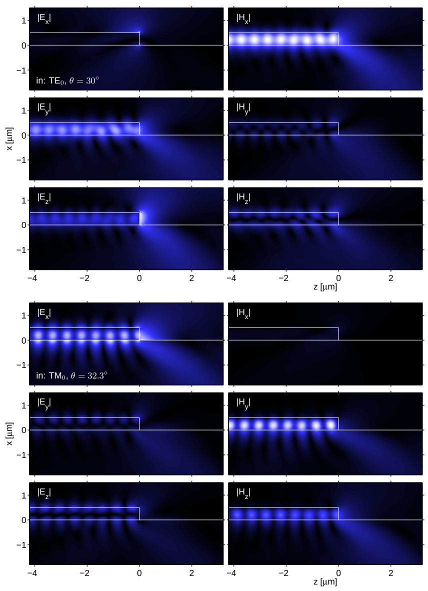
4 Rectangular slab waveguide facet



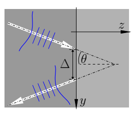
$n_{\rm c} = 1.0$, $n_{\rm f} = 2.0$, $n_{\rm s} = 1.5$, $d = 0.5\,\mu$m, $\lambda = 1.55\,\mu$m.



quasi-Brewster angle $\tan\theta_{\rm B} = n_{\rm c}/N_{\rm TE0}$

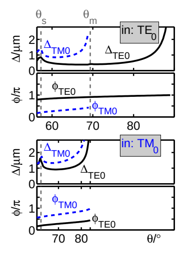


5 Beam displacement

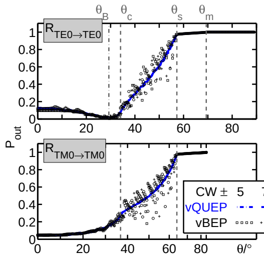


Phase change ϕ upon reflection at the interface lateral displacement Δ of incident and reflected wave bundles,

Goos-Hänchen-shift [14, 9]: $\Delta = \dfrac{1}{kN_{\rm in}\cos\theta}\dfrac{\mathrm{d}\phi}{\mathrm{d}\theta}$.



6 vQUEP versus vBEP



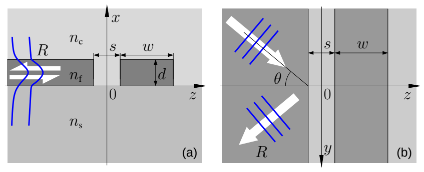
Computational domains:

$[x_0, x_{N_x}] = [-\mathrm{CW}, \mathrm{CW}]$ (vBEP)

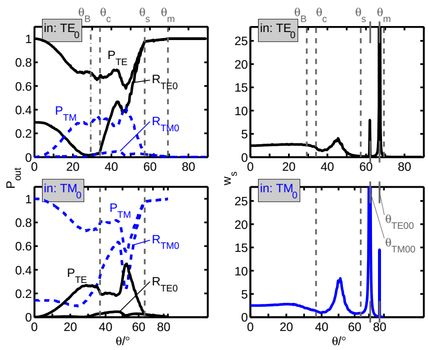
or

$[x_0, x_{N_x}] \times [z_0, z_{N_z}] = [-\mathrm{CW}, \mathrm{CW}] \times [-\mathrm{CW}, \mathrm{CW}]$ (vQUEP).

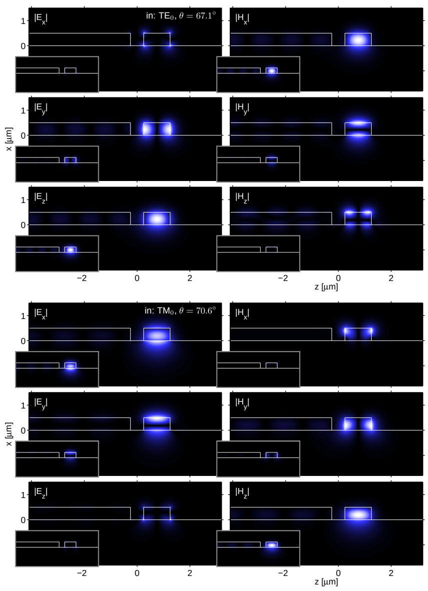
7 Strip waveguide, lateral excitation



$n_{\rm c} = 1.0$, $n_{\rm f} = 2.0$, $n_{\rm s} = 1.5$, $d = 0.5\,\mu$m, $s = 0.5\,\mu$m, $w = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m.



$w_{\rm s} = \int_{\rm Strip}(\text{energy density})\,\mathrm{d}x\,\mathrm{d}z$, $\theta_{\rm TE00}, \theta_{\rm TM00}$: strip, vectorial modes (WMM [13]).



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9 References

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