Analytical approaches to the description of optical microresonator devices

Manfred Hammer*, Kiran Hiremath, Remco Stoffer

MESA$^+$ Research Institute
University of Twente, The Netherlands

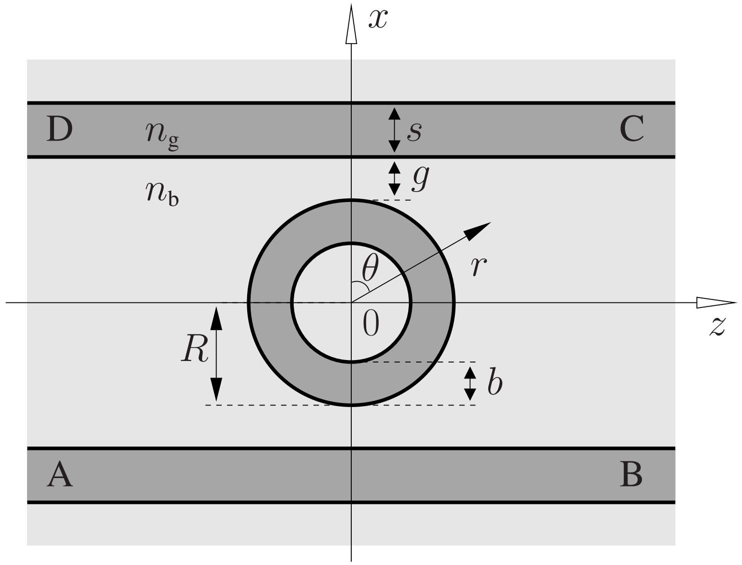
International School of Quantum Electronics "Microresonators as Building Blocks for VLSI Photonics"
Erice, Sicily, Italy, 18th-25th October 2003

* *Department of Applied Mathematics, University of Twente* *P.O. Box 217, 7500 AE Enschede, The Netherlands*
Phone: +31/53/489-3448 *Fax: +31/53/489-4833* *E-mail: m.hammer@math.utwente.nl*

Outline

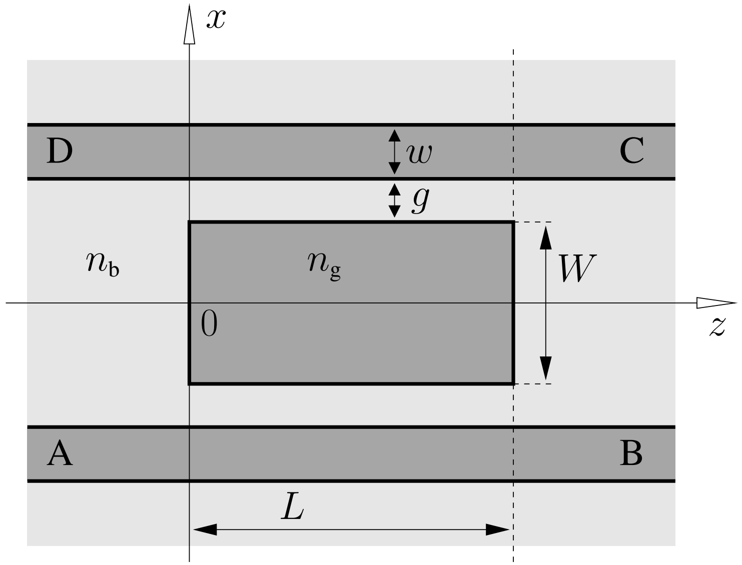
Circular traveling wave resonators

- Abstract model & implications
- Bent slab waveguides & examples
- Coupler modeling & examples

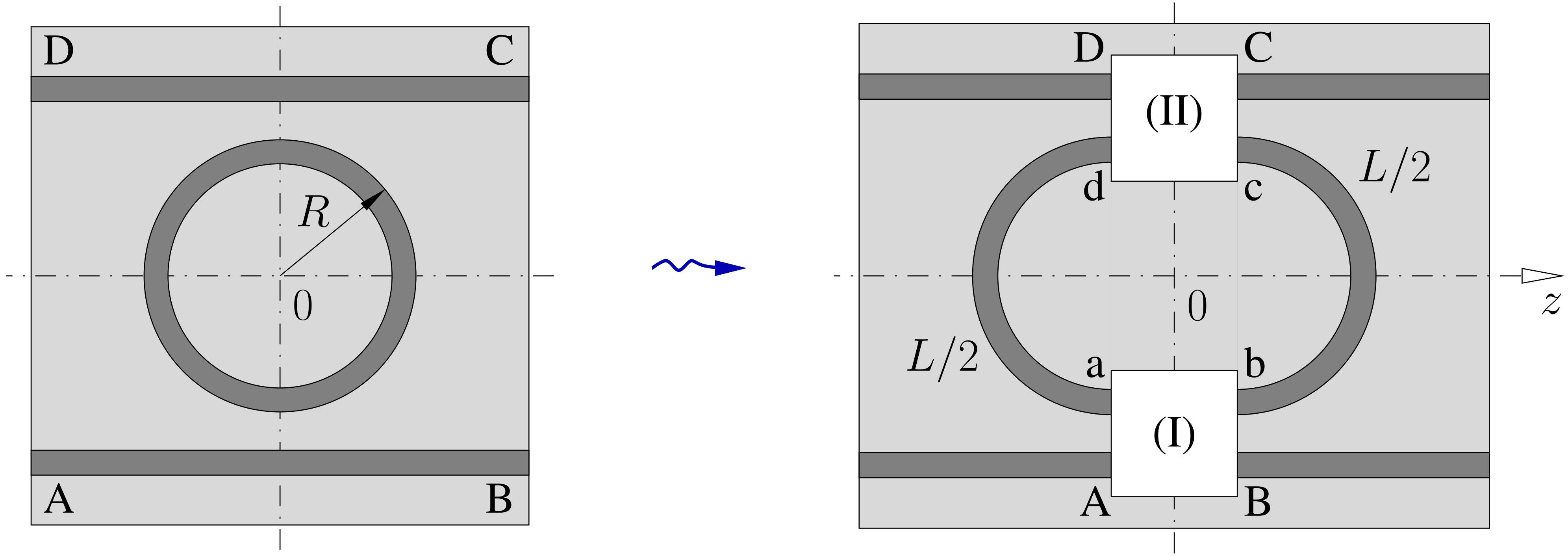


Rectangular standing wave resonators

- Abstract model & examples
- Waveguide facets
- Slab mode resonances
- Extensions

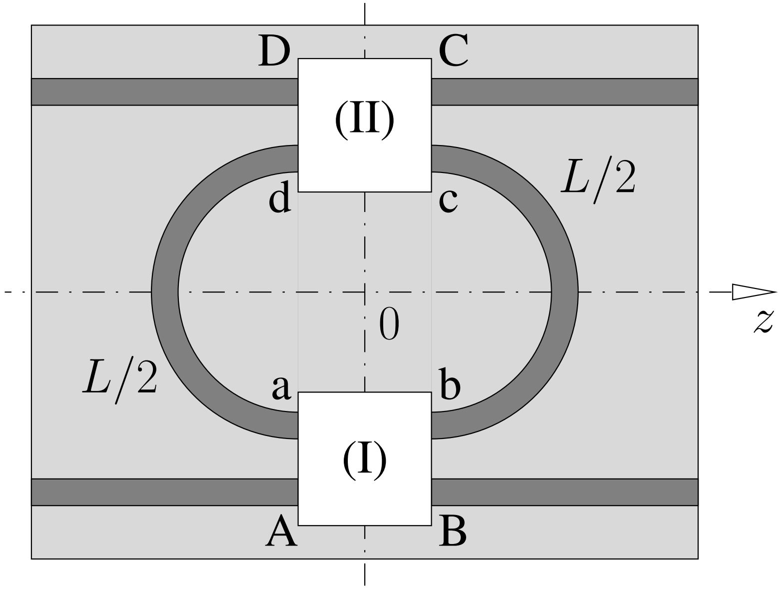


Ringresonator: Abstract model



- Ringresonator $\approx$ 2 couplers + 2 cavity segments
- CW description : $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$, $\omega = k\,\mathrm{c}$, $k = 2\pi/\lambda$.

Couplers: Scattering matrices



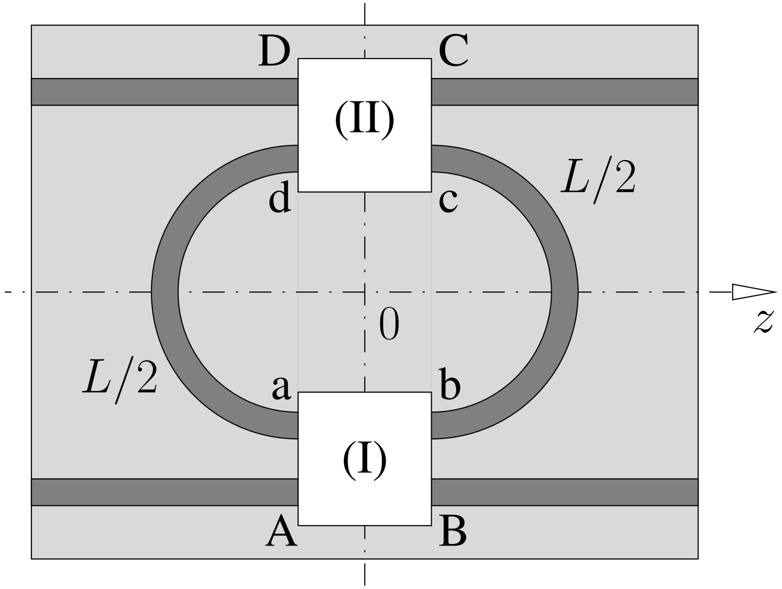
- Uniform polarization, single mode waveguides.
- Linear, nonmagnetic (attenuating) elements.
- Backreflections are negligible.
- Interaction restricted to the couplers $\leftrightarrow$ "port" definition.

↪ *Symmetric* coupler scattering matrices :

$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \chi & \tau \\ \rho & \chi & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$A_\pm,\ B_\pm,\ a_\pm,\ b_\pm$: Amplitudes of waves traveling in $\pm z$-direction.

Coupler symmetries



Symmetry $z \longrightarrow -z$:

$$A_+ \rightarrow b_+ \stackrel{!}{=} B_- \rightarrow a_-$$

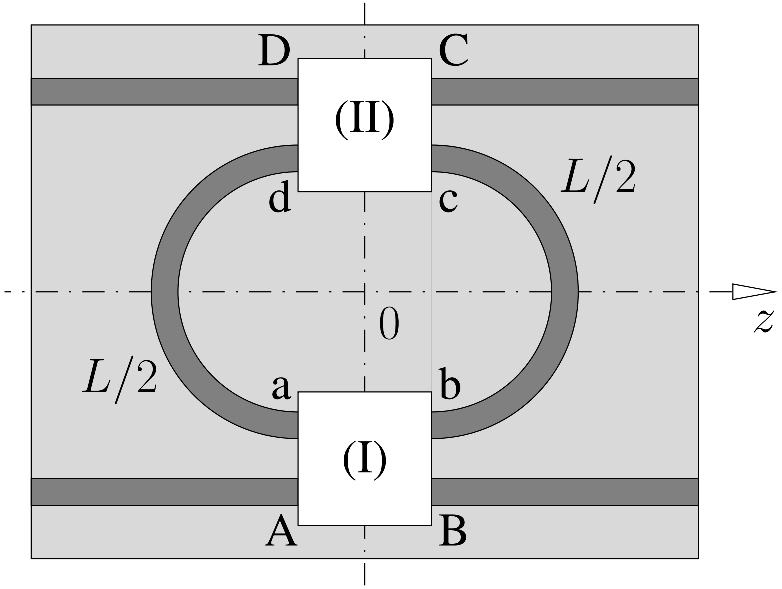
$$\begin{pmatrix} A_- \\ a_- \\ B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & \rho & \kappa \\ 0 & 0 & \kappa & \tau \\ \rho & \kappa & 0 & 0 \\ \kappa & \tau & 0 & 0 \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \\ B_- \\ b_- \end{pmatrix}$$

$$\begin{pmatrix} A_- \\ a_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} B_- \\ b_- \end{pmatrix}, \qquad \begin{pmatrix} B_+ \\ b_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} A_+ \\ a_+ \end{pmatrix}.$$

Symmetry $x \longrightarrow -x$, (I) = (II) :

$$\begin{pmatrix} D_- \\ d_- \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} C_- \\ c_- \end{pmatrix}, \qquad \begin{pmatrix} C_+ \\ c_+ \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ \kappa & \tau \end{pmatrix} \begin{pmatrix} D_+ \\ d_+ \end{pmatrix}.$$

Cavity segments



Field evolution $\sim \mathrm{e}^{-\mathrm{i}\gamma s}$
along the cavity core,
propagation distance s.

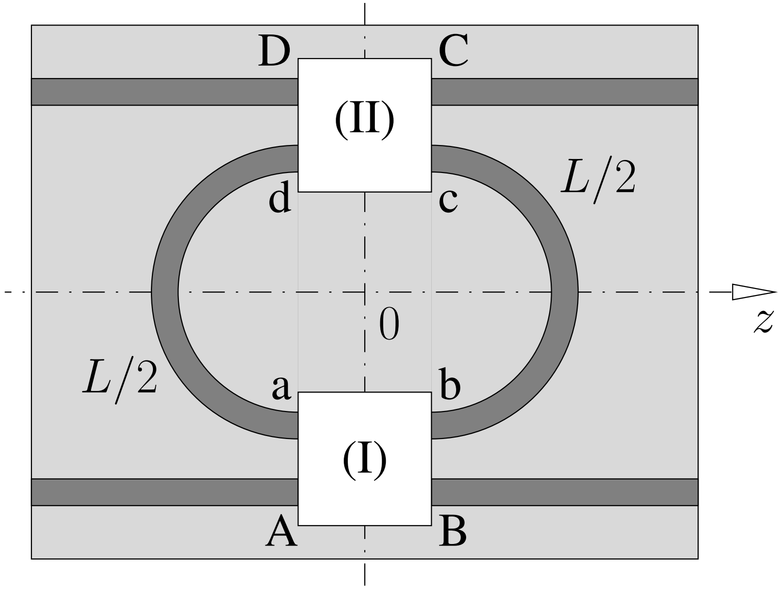
$\gamma = \beta - \mathrm{i}\alpha$,

β : phase propagation constant,
α : attenuation constant.

Relations of amplitudes at the ends of the cavity segments :

$$c_- = b_+\, \mathrm{e}^{-\mathrm{i}\beta L/2}\ \ \mathrm{e}^{-\alpha L/2}, \qquad a_+ = d_-\, \mathrm{e}^{-\mathrm{i}\beta L/2}\ \ \mathrm{e}^{-\alpha L/2},$$
$$b_- = c_+\, \mathrm{e}^{-\mathrm{i}\beta L/2}\ \ \mathrm{e}^{-\alpha L/2}, \qquad d_+ = a_-\, \mathrm{e}^{-\mathrm{i}\beta L/2}\ \ \mathrm{e}^{-\alpha L/2}.$$

Output amplitudes



Coupler scattering matrices

+ Cavity field evolution

+ External input amplitudes

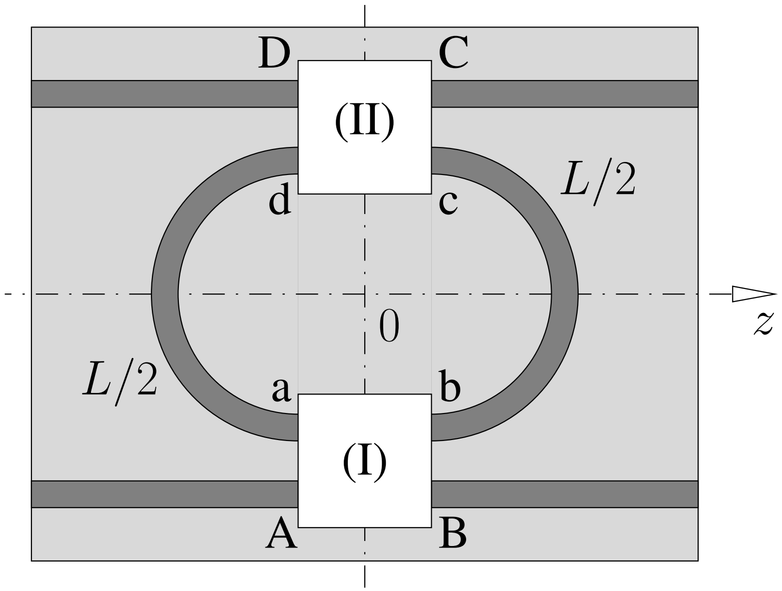
$$A_+ = \sqrt{P_{\text{in}}}\,,$$
$$B_- = C_- = D_+ = 0$$

External output amplitudes :

$$A_- = 0, \quad C_+ = 0, \quad D_- = \frac{\kappa^2 p}{1-\tau^2 p^2} A_+\,, \quad B_+ = \left(\rho + \frac{\kappa^2 \tau p^2}{1-\tau^2 p^2}\right) A_+\,,$$

$$p = \mathrm{e}^{-\mathrm{i}\beta L/2}\ \mathrm{e}^{-\alpha L/2}.$$

Power transfer



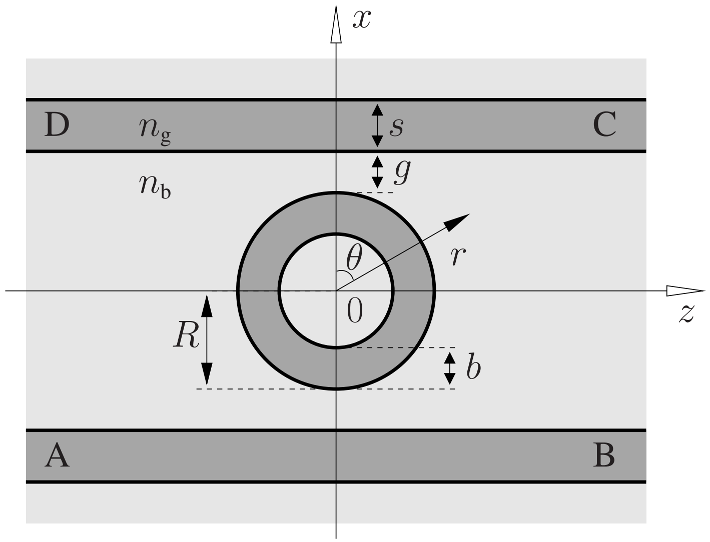
$$\text{Power drop}: \quad P_\mathrm{D} = |D_-|^2,$$
$$\text{Transmission}: \quad P_\mathrm{T} = |B_-|^2.$$

$$P_\mathrm{D} = P_\mathrm{in} \frac{|\kappa|^4 \, \mathrm{e}^{-\alpha L}}{1 + |\tau|^4 \, \mathrm{e}^{-2\alpha L} - 2|\tau|^2 \, \mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$P_\mathrm{T} = P_\mathrm{in} \frac{|\rho|^2 (1 + |\tau|^2 d^2 \, \mathrm{e}^{-2\alpha L} - 2|\tau| d \, \mathrm{e}^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 \, \mathrm{e}^{-2\alpha L} - 2|\tau|^2 \, \mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

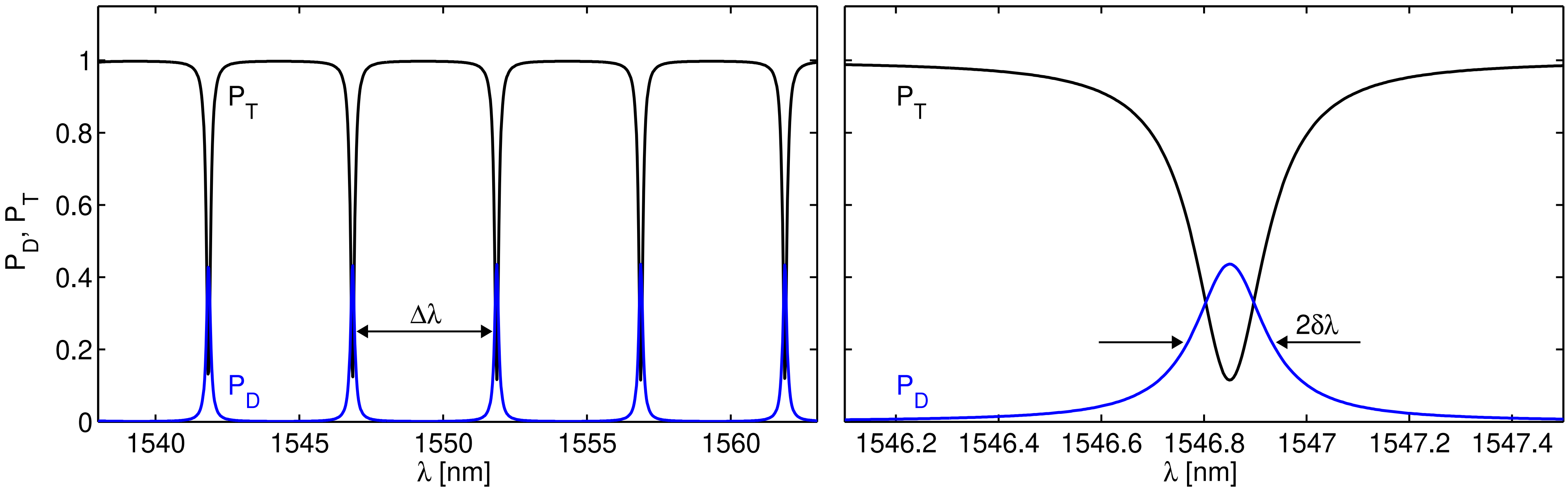
$d \, \mathrm{e}^{\mathrm{i}\psi} := \tau - \kappa^2/\rho, \quad L \neq 2\pi R.$

Spectral response



$R = 50\,\mu\text{m},\ b = s = 1.0\,\mu\text{m},\ g = 0.9\,\mu\text{m},$
$n_\text{b} = 1.45,\ n_\text{g} = 1.60;\ \text{2D},\ \text{TE}.$

$\Delta\lambda = 5.0\,\text{nm},\ 2\delta\lambda = 0.17\,\text{nm},$
$F = 30,\ Q = 9400,\ P_{\text{D,res}} = 0.44.$



Resonances

$$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4 \,\mathrm{e}^{-\alpha L}}{1 + |\tau|^4 \,\mathrm{e}^{-2\alpha L} - 2|\tau|^2 \,\mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

$$P_{\mathrm{T}} = P_{\mathrm{in}} \frac{|\rho|^2 (1 + |\tau|^2 d^2 \,\mathrm{e}^{-2\alpha L} - 2|\tau| d \,\mathrm{e}^{-\alpha L} \cos(\beta L - \varphi - \psi))}{1 + |\tau|^4 \,\mathrm{e}^{-2\alpha L} - 2|\tau|^2 \,\mathrm{e}^{-\alpha L} \cos(\beta L - 2\varphi)}$$

- $\approx$ Resonances :
  Singularities in the denominators of P_{D}, P_{T}, origin : $\beta(\lambda)$.
- Correction for finite coupler length l :
  $\beta L - 2\varphi = \beta L_{\mathrm{cav}} - \phi, \quad \phi = 2\beta l + 2\varphi, \quad L_{\mathrm{cav}} = 2\pi R, \quad \partial_\lambda \phi \approx 0.$
- Resonance condition : $\cos(\beta L_{\mathrm{cav}} - \phi) = 1$, or

$$\beta = \frac{2m\pi + \phi}{L_{\mathrm{cav}}} =: \beta_m \quad \text{integer } m; \qquad P_{\mathrm{D}}|_{\beta=\beta_m} = P_{\mathrm{in}} \frac{|\kappa|^4 \,\mathrm{e}^{-\alpha L}}{(1 - |\tau|^2 \,\mathrm{e}^{-\alpha L})^2}\,.$$

Free spectral range

- Resonance next to β_m :

$$\beta_{m-1} = \frac{2(m-1)\pi + \phi}{L_{\text{cav}}} = \beta_m - \frac{2\pi}{L_{\text{cav}}} \approx \beta_m + \left.\frac{\partial \beta}{\partial \lambda}\right|_m \Delta\lambda$$

- $\partial_\lambda \beta$ = ?
  q_j : waveguide parameters with dimension length,
  $\beta(a\lambda, aq_j) = \beta(\lambda, q_j)/a, \qquad \partial_a \, |_{a=1}$

$$\frac{\partial \beta}{\partial \lambda} = -\frac{1}{\lambda}\left(\beta + \sum_j q_j \frac{\partial \beta}{\partial q_j}\right) \approx -\frac{\beta}{\lambda}\,.$$

FSR : $$\Delta\lambda = -\frac{2\pi}{L_{\text{cav}}}\left(\left.\frac{\partial \beta}{\partial \lambda}\right|_m\right)^{-1} \approx \left.\frac{\lambda^2}{n_{\text{eff}}\, L_{\text{cav}}}\right|_m, \qquad n_{\text{eff}} = \beta/k.$$

Spectral width of the resonances

- $$P_{\mathrm{D}} = P_{\mathrm{in}} \frac{|\kappa|^4 \,\mathrm{e}^{-\alpha L}}{1 + |\tau|^4 \,\mathrm{e}^{-2\alpha L} - 2|\tau|^2 \,\mathrm{e}^{-\alpha L} \cos(\beta L_{\mathrm{cav}} - \phi)} ,$$

  $$P_{\mathrm{D}}\big|_{\beta_m} = P_{\mathrm{D,res}} .$$

- $P_{\mathrm{D}}\big|_{\beta_m + \delta\beta} = P_{\mathrm{D,res}}/2 .$ $\quad \delta\beta = ?$

- Expansion of cos-terms

  $$\hookrightarrow \quad \delta\beta = \pm \frac{1}{L_{\mathrm{cav}}} \left(\frac{1}{|\tau|} \,\mathrm{e}^{\alpha L/2} - |\tau| \,\mathrm{e}^{-\alpha L/2} \right) \approx -\frac{\beta_m}{\lambda} \,\delta\lambda$$

FWHM :

$$2\delta\lambda = \frac{\lambda^2}{\pi L_{\mathrm{cav}} n_{\mathrm{eff}}}\bigg|_m \left(\frac{1}{|\tau|} \,\mathrm{e}^{\alpha L/2} - |\tau| \,\mathrm{e}^{-\alpha L/2} \right) .$$

Finesse & Q-factor

Finesse : $$F = \frac{\Delta\lambda}{2\delta\lambda} = \pi \frac{|\tau|\,\mathrm{e}^{-\alpha L/2}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}}\,.$$

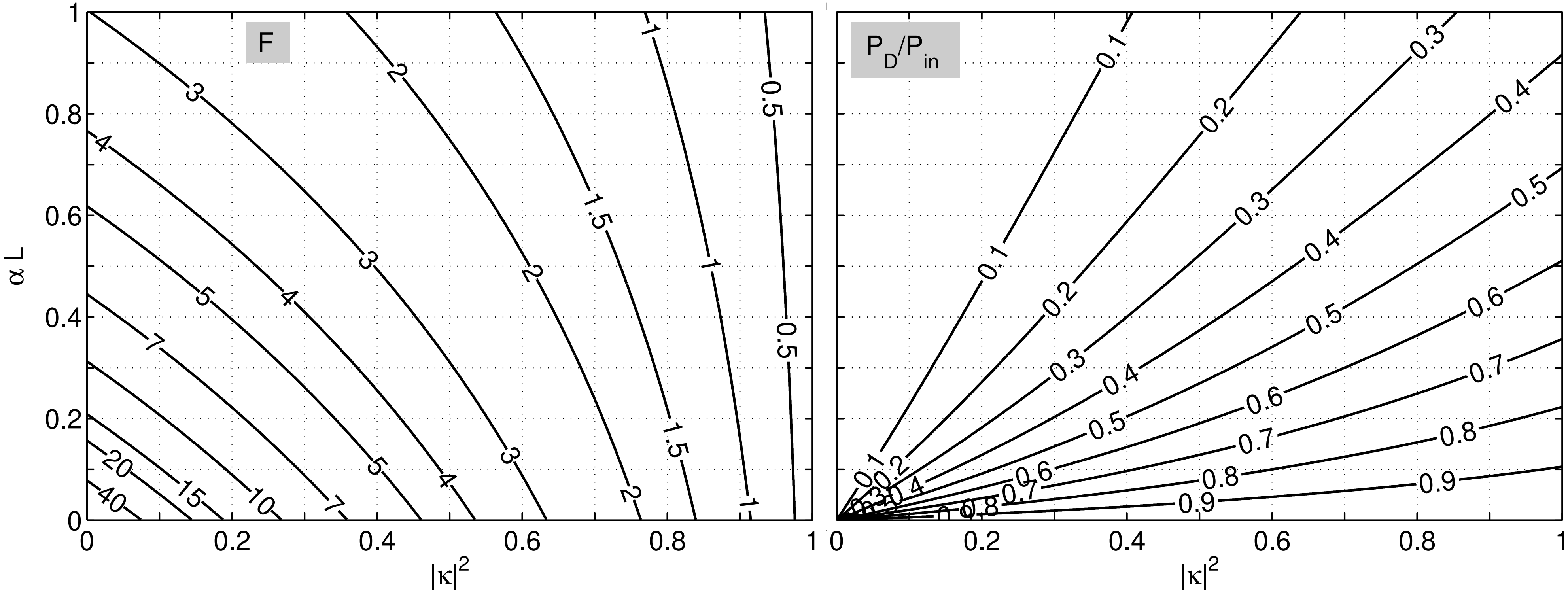
Q-factor : $$Q = \frac{\lambda}{2\delta\lambda} = \pi \frac{n_{\text{eff}} L_{\text{cav}}}{\lambda}\,\frac{|\tau|\,\mathrm{e}^{-\alpha L/2)}}{1 - |\tau|^2\,\mathrm{e}^{-\alpha L}} = \frac{n_{\text{eff}} L_{\text{cav}}}{\lambda} F\,.$$

or $$Q = kRn_{\text{eff}}F \qquad \text{for} \qquad L_{\text{cav}} = 2\pi R\,.$$

Performance versus coupling length & losses

Assumption: Lossless coupler elements, $|\rho|^2 = |\tau|^2 = 1 - |\kappa|^2$.

$$F = \pi \frac{(\sqrt{1-|\kappa|^2})\,\mathrm{e}^{-\alpha L/2}}{1-(1-|\kappa|^2)\,\mathrm{e}^{-\alpha L}}\,, \qquad P_{\mathrm{D}}|_{\mathrm{res}} = P_{\mathrm{in}} \frac{|\kappa|^4\,\mathrm{e}^{-\alpha L}}{(1-(1-|\kappa|^2)\,\mathrm{e}^{-\alpha L})^2}\,.$$



Tuning

- Resonance condition : $\beta = (2\pi m + \phi)/L_{\text{cav}} = \beta_m$,
  a peak at λ_m : $\beta(\lambda_m) = \beta_m$.
- Influence of an external (small) parameter p : $\beta(p, \lambda)$; $\beta(0, \lambda_m) = \beta_m$.
- Tuning $0 \to p$: Resonance shifts to $\tilde{\lambda}_m$ with $\beta(p, \tilde{\lambda}_m) = \beta_m$.

$$\beta(p, \tilde{\lambda}_m) \approx \beta(0, \lambda_m) + p \left. \frac{\partial \beta}{\partial p} \right|_{0,\lambda_m} + (\tilde{\lambda}_m - \lambda_m) \left. \frac{\partial \beta}{\partial \lambda} \right|_{0,\lambda_m} = \beta_m \,.$$

Wavelength shift effected by the perturbation :

$$\Delta_p \lambda_m = p \frac{\partial \beta}{\partial p} \frac{\lambda_m}{\beta_m} \qquad \text{or} \qquad \Delta_p \lambda_m = p \frac{\partial \beta}{\partial p} \frac{\lambda_m^2}{2\pi\, n_{\text{eff},m}} \,.$$

Tuning, specific

- Electrooptic tuning,
  an external field strength E_t changes the cavity permittivity :

$$\hat{\epsilon}(E_\mathrm{t}) = \hat{\epsilon}(0) + E_\mathrm{t}\hat{e}.$$

- Approximation : Bend $\longrightarrow$ straight waveguide,
  mode profile $\quad \boldsymbol{E} = (E_x, E_y, \mathrm{i}\,E_z), \quad \boldsymbol{H} = (H_x, H_y, \mathrm{i}\,H_z)$

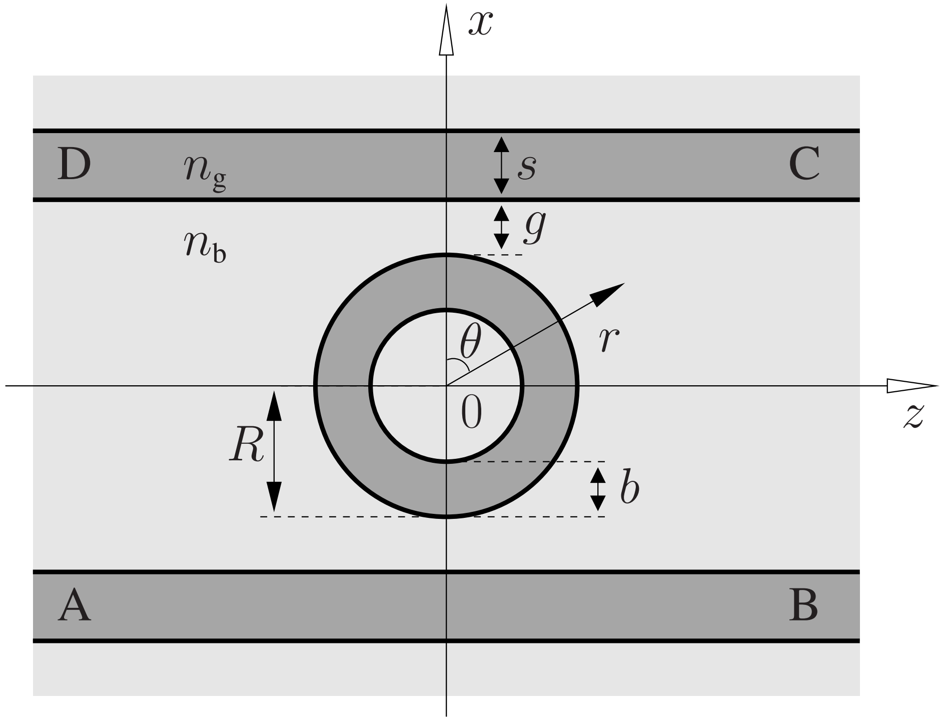
$$\frac{\partial\beta}{\partial E_\mathrm{t}} = \frac{\omega\,\epsilon_0}{2}\,\frac{\displaystyle\iint \boldsymbol{E}^*\hat{e}\boldsymbol{E}\,\mathrm{d}x\mathrm{d}y}{\displaystyle\iint (E_xH_y - E_yH_x)\,\mathrm{d}x\mathrm{d}y}\,.$$

Wavelength shift effected by the electrooptic tuning :

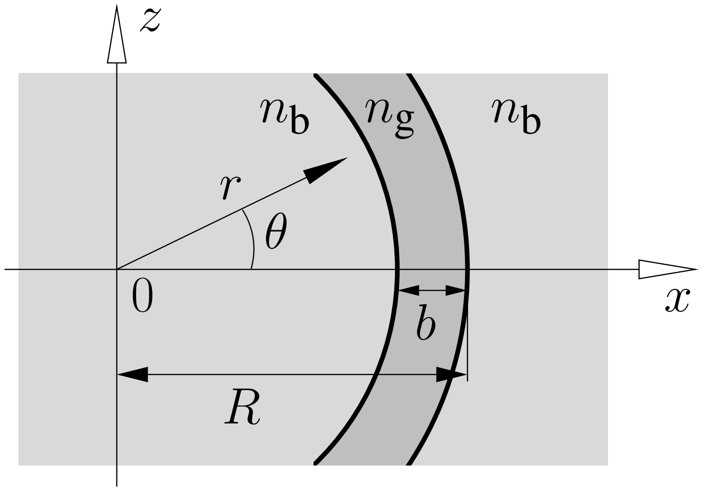
$$\Delta_{E_\mathrm{t}}\lambda = E_\mathrm{t}\frac{\lambda}{2\,n_\mathrm{eff}}\sqrt{\frac{\epsilon_0}{\mu_0}}\,\frac{\displaystyle\iint \boldsymbol{E}^*\hat{e}\boldsymbol{E}\,\mathrm{d}x\mathrm{d}y}{\displaystyle\iint (E_xH_y - E_yH_x)\,\mathrm{d}x\mathrm{d}y}\,.$$

Abstract model & implications

Bent slab waveguides & examples



Bend mode properties



Homogeneity along θ $\rightsquigarrow$ bend mode ansatz

$$\begin{pmatrix}\boldsymbol{\mathcal{E}}\\ \boldsymbol{\mathcal{H}}\end{pmatrix}(r,\theta,t) = \frac{1}{2}\mathrm{Re}\begin{pmatrix}\boldsymbol{E}_0^\mathrm{b}\\ \boldsymbol{H}_0^\mathrm{b}\end{pmatrix}(r)\,\mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\gamma R\theta},$$

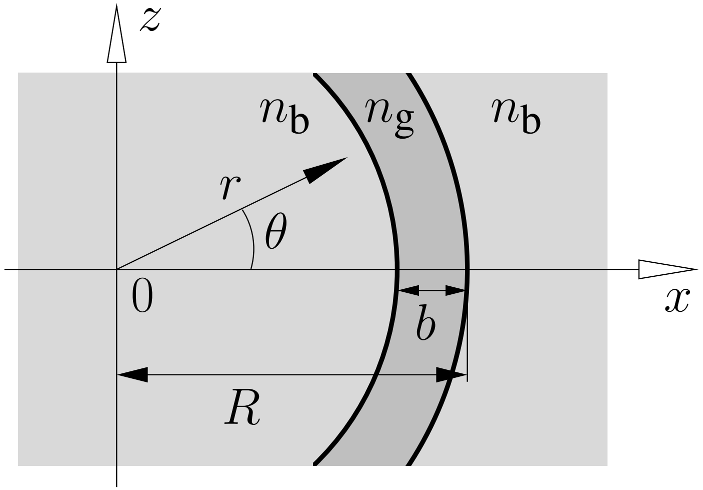
profile $\boldsymbol{E}_0^\mathrm{b}$, $\boldsymbol{H}_0^\mathrm{b}$,
propagation constant $\gamma = \beta - \mathrm{i}\alpha$.

$$\hookrightarrow \qquad \frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} + \left(k^2n^2 - \frac{\gamma^2R^2}{r^2}\right)\phi = 0\,,$$

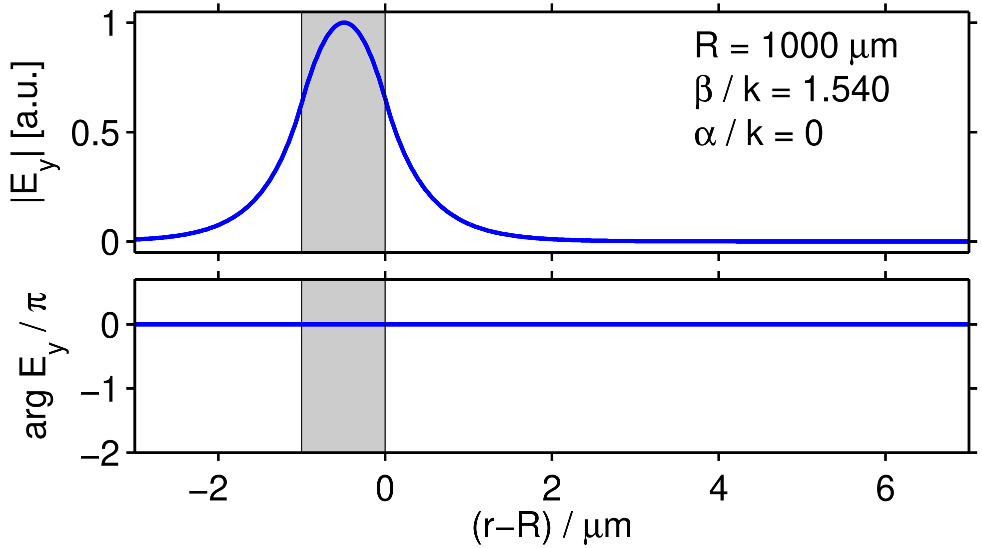
$n(r)$: piecewise constant, $\phi = E_{0,y}^\mathrm{b}$ (TE), $\phi = H_{0,y}^\mathrm{b}$ (TM).

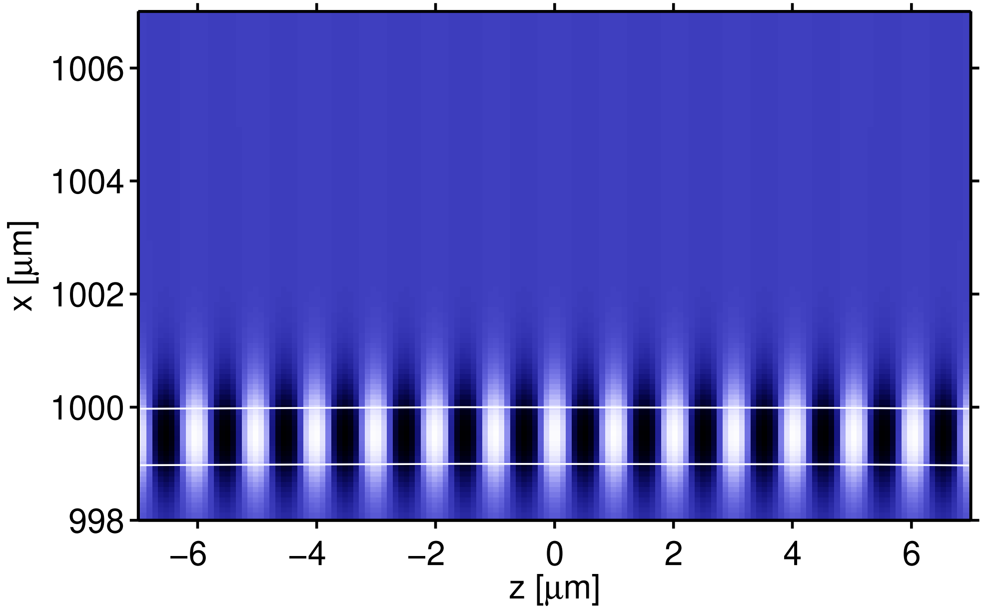
Bend modes :
- Nonzero solutions,
- bounded at the origin, $\sim J_{\gamma R}(n_\mathrm{b}kr)$ for $r < R-b$,
- outgoing exterior fields, $\sim H_{\gamma R}^{(2)}(n_\mathrm{b}kr)$ for $r > R$,
- continuity at interfaces : ϕ & $\mathrm{d}_r\phi$ (TE), ϕ & $(\mathrm{d}_r\phi)/n^2$ (TM).

Bend modes, examples

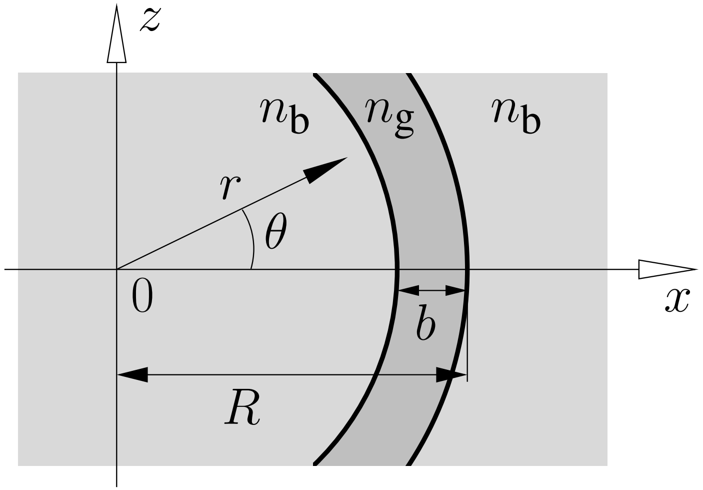


2D, TE,
$n_\mathrm{b} = 1.45,\ n_\mathrm{g} = 1.60,\ b = 1.0\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$
$R = 1000\,\mu\mathrm{m}.$

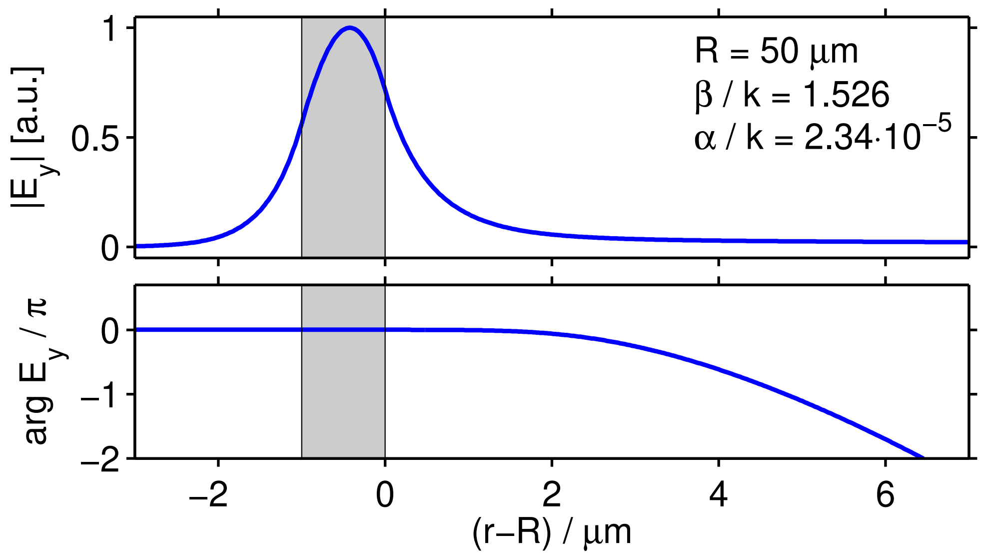


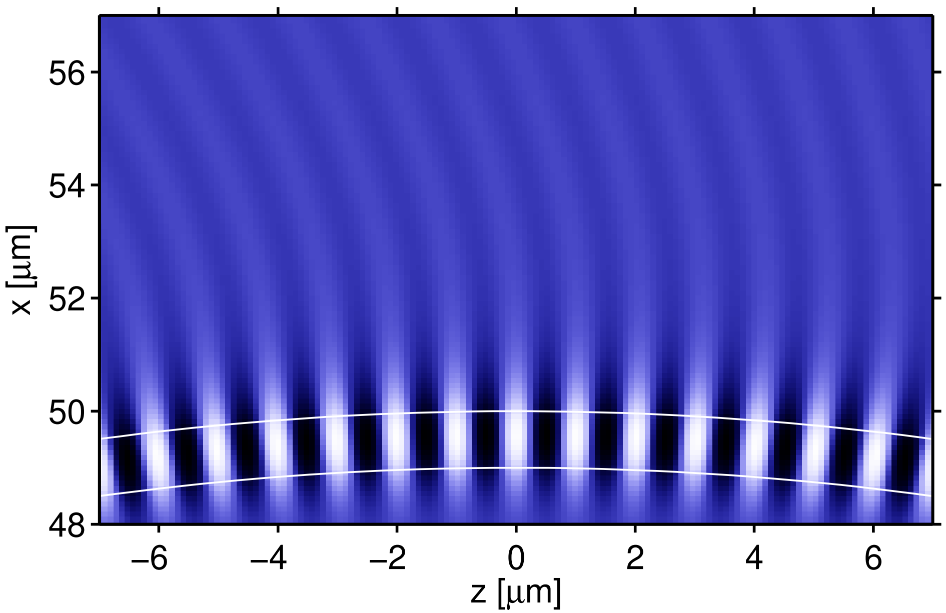


Bend modes, examples

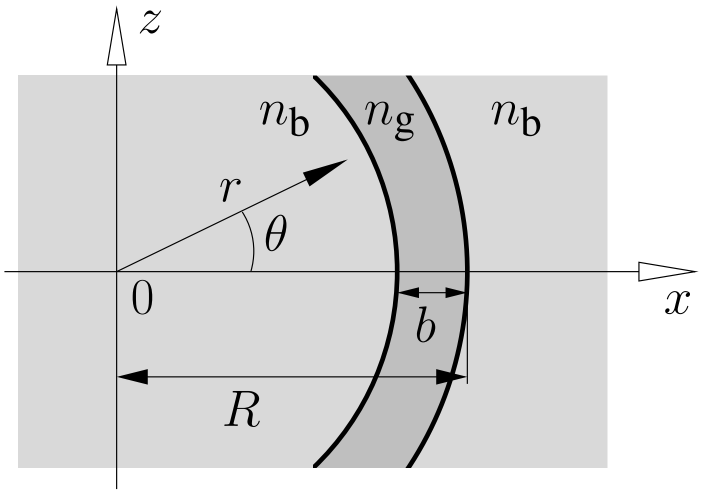


2D, TE,
$n_\mathrm{b} = 1.45$, $n_\mathrm{g} = 1.60$, $b = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m,
$R = 50\,\mu$m.

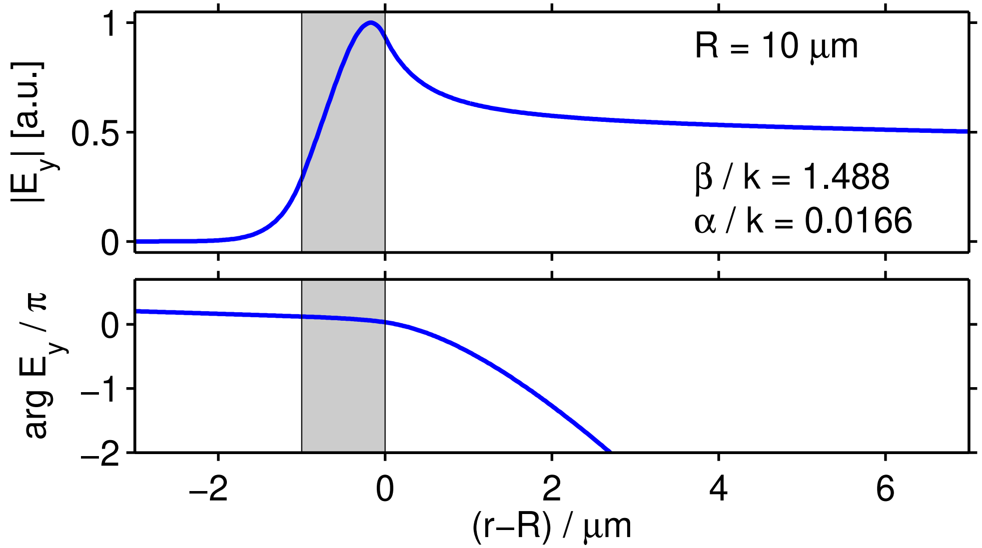


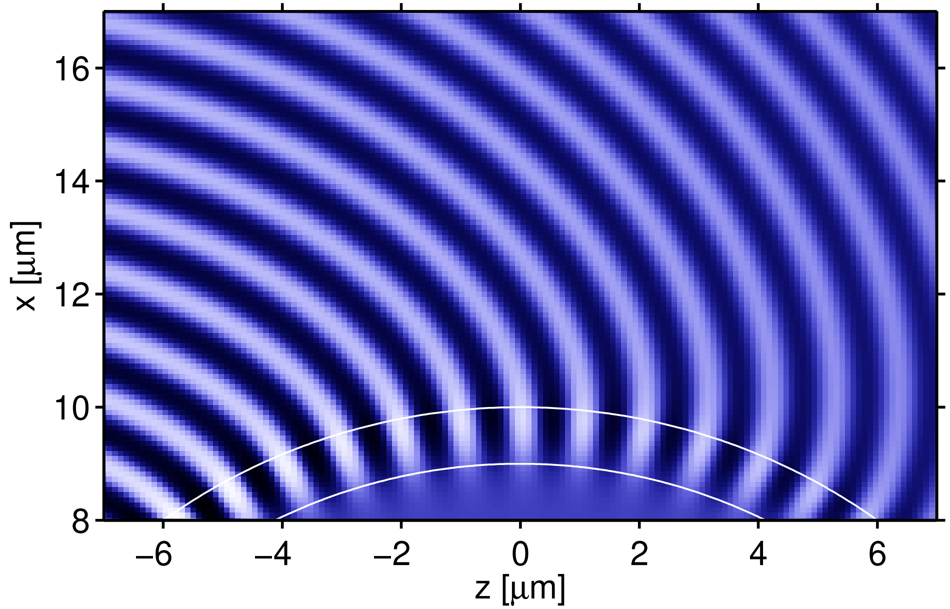


Bend modes, examples

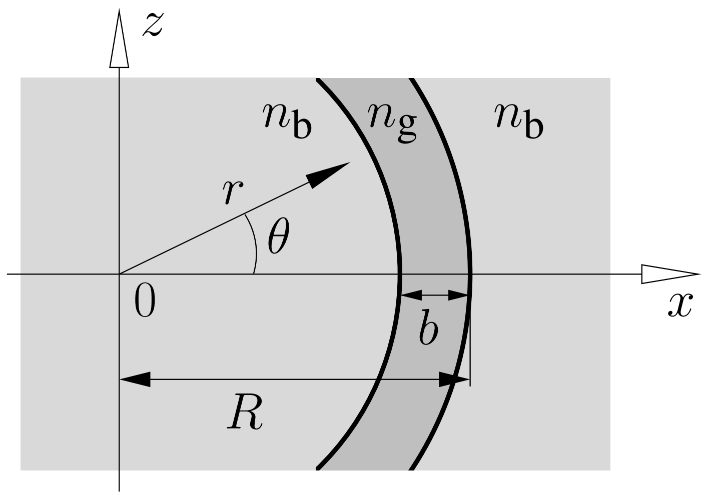


2D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m,
$R = 10\,\mu$m.

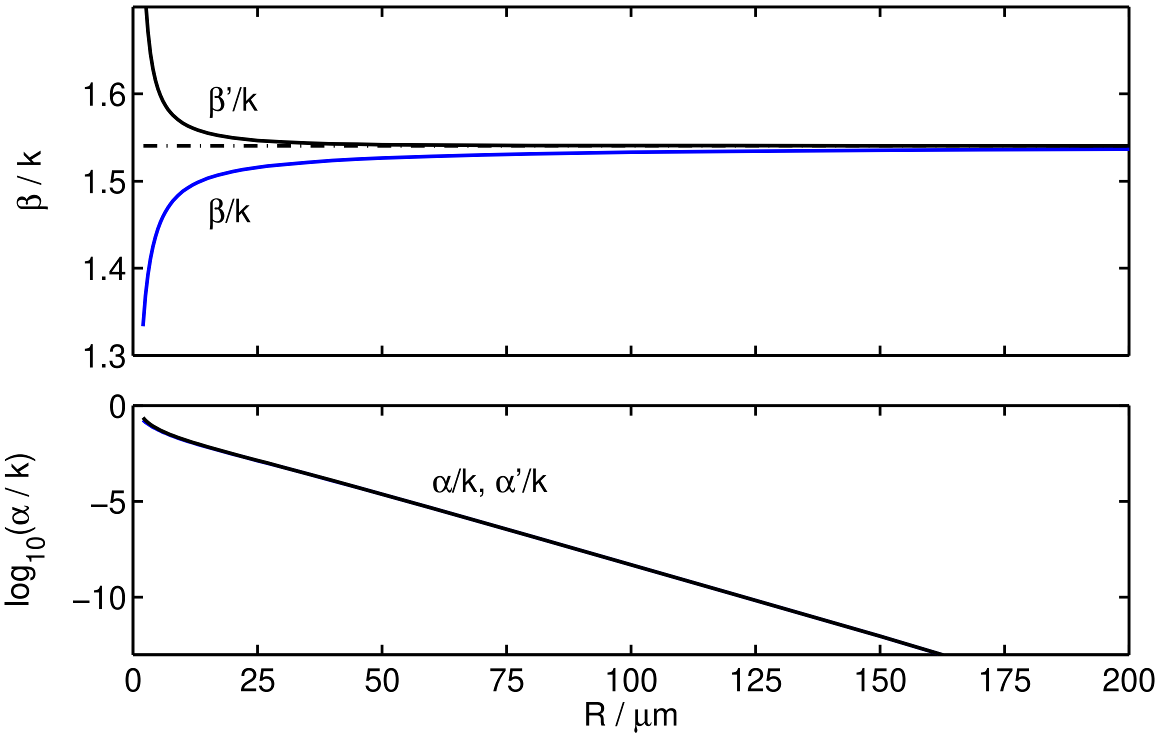




Propagation constant vs. bend radius



2D, TE,
$n_b = 1.45$, $n_g = 1.60$, $b = 1.0\,\mu$m, $\lambda = 1.55\,\mu$m,
$R \in [2, 200]\,\mu$m.



Alternative definition :
$R' = R - b/2.$

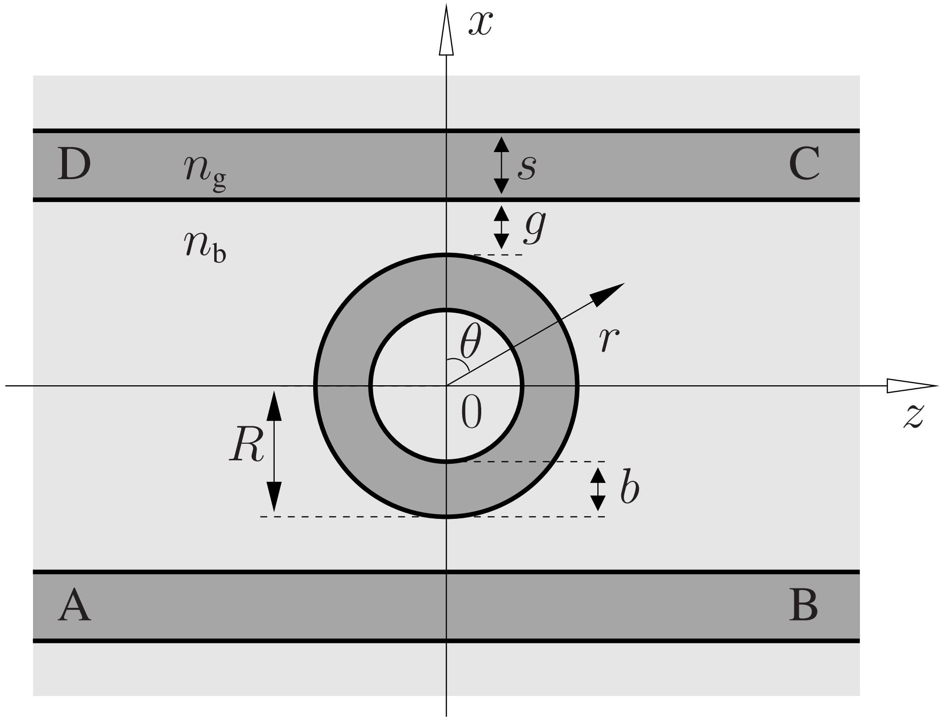
Identical physical fields

$$\gamma' R' = \gamma R,$$

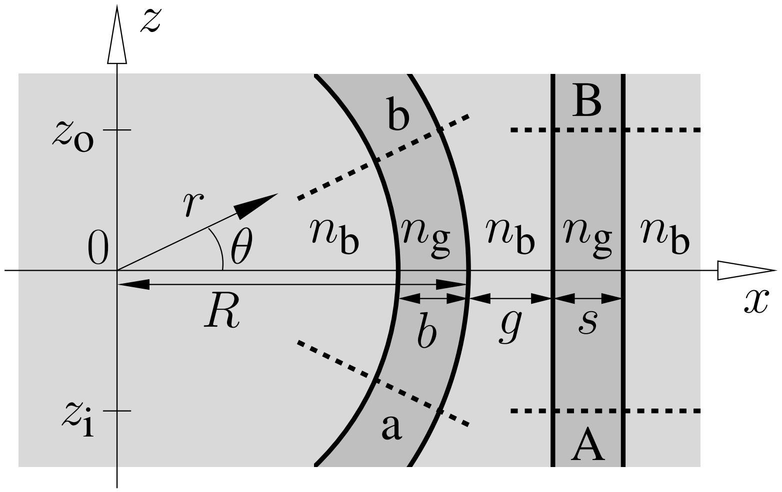
$$\gamma' = \gamma \, \frac{R}{R - b/2} \, .$$

Bent slab waveguides & examples

Coupler model & examples



Coupler modeling, CMT ansatz



Known: Modes of the bent and straight cores

$$\begin{pmatrix} \boldsymbol{E}_\mathrm{b} \\ \boldsymbol{H}_\mathrm{b} \end{pmatrix}(x,z) = \begin{pmatrix} \boldsymbol{E}_0^\mathrm{b} \\ \boldsymbol{H}_0^\mathrm{b} \end{pmatrix}(r(x,z))\,\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{b} R\theta(x,z)},$$

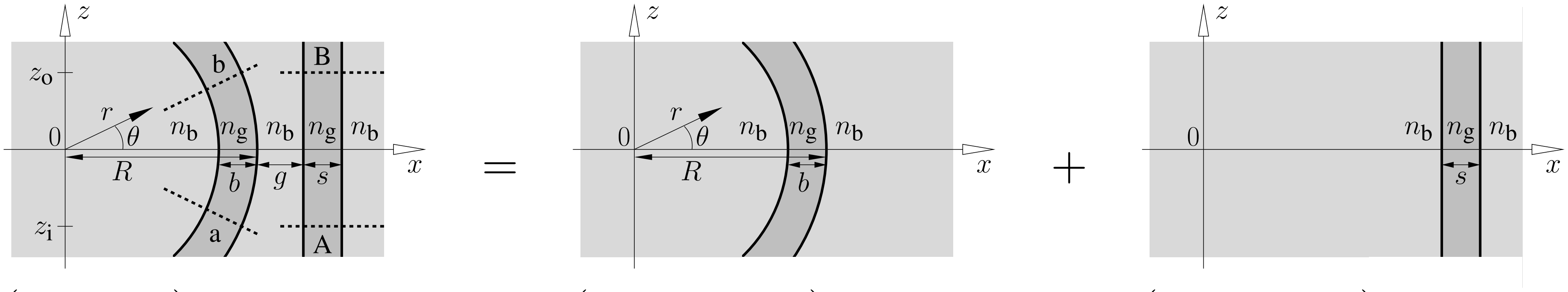
$$\begin{pmatrix} \boldsymbol{E}_\mathrm{s} \\ \boldsymbol{H}_\mathrm{s} \end{pmatrix}(x,z) = \begin{pmatrix} \boldsymbol{E}_0^\mathrm{s} \\ \boldsymbol{H}_0^\mathrm{s} \end{pmatrix}(x)\,\mathrm{e}^{-\mathrm{i}\beta_\mathrm{s} z}.$$

Coupled mode ansatz:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x,z,t) = \frac{1}{2}\mathrm{Re}\left\{ A_\mathrm{b}(z)\begin{pmatrix} \boldsymbol{E}_\mathrm{b} \\ \boldsymbol{H}_\mathrm{b} \end{pmatrix}(x,z) + A_\mathrm{s}(z)\begin{pmatrix} \boldsymbol{E}_\mathrm{s} \\ \boldsymbol{H}_\mathrm{s} \end{pmatrix}(x,z) \right\}\,\mathrm{e}^{\mathrm{i}\omega t}.$$

$A_\mathrm{b}(z)$, $A_\mathrm{s}(z)$ = ?

Coupled mode equations



$(\boldsymbol{E}, \boldsymbol{H}, \epsilon)$ $\qquad$ $(\boldsymbol{E}_\mathrm{b}, \boldsymbol{H}_\mathrm{b}, \epsilon_\mathrm{b})$ $\qquad$ $(\boldsymbol{E}_\mathrm{s}, \boldsymbol{H}_\mathrm{s}, \epsilon_\mathrm{s})$

Suitable integral form of Maxwells equations (“reciprocity theorem”)

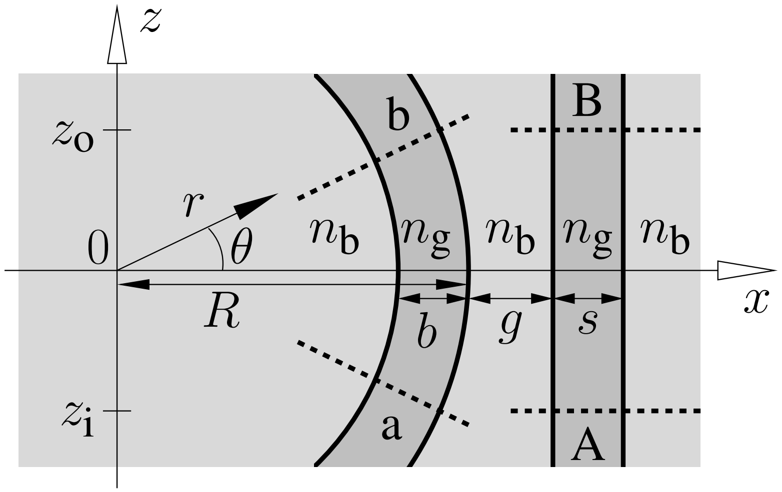
$$\begin{pmatrix} \sigma_\mathrm{bb} & \sigma_\mathrm{bs} \\ \sigma_\mathrm{sb} & \sigma_\mathrm{ss} \end{pmatrix} \frac{\mathrm{d}}{\mathrm{d}z} \begin{pmatrix} A_\mathrm{b} \\ A_\mathrm{s} \end{pmatrix} = \begin{pmatrix} c_\mathrm{bb} & c_\mathrm{bs} \\ c_\mathrm{sb} & c_\mathrm{ss} \end{pmatrix} \begin{pmatrix} A_\mathrm{b} \\ A_\mathrm{s} \end{pmatrix},$$

$$\sigma_{pq} = \frac{1}{4} \int (E^*_{px} H_{qy} - E^*_{py} H_{qx} + H^*_{py} E_{qx} - H^*_{px} E_{qy})\, \mathrm{d}x,$$

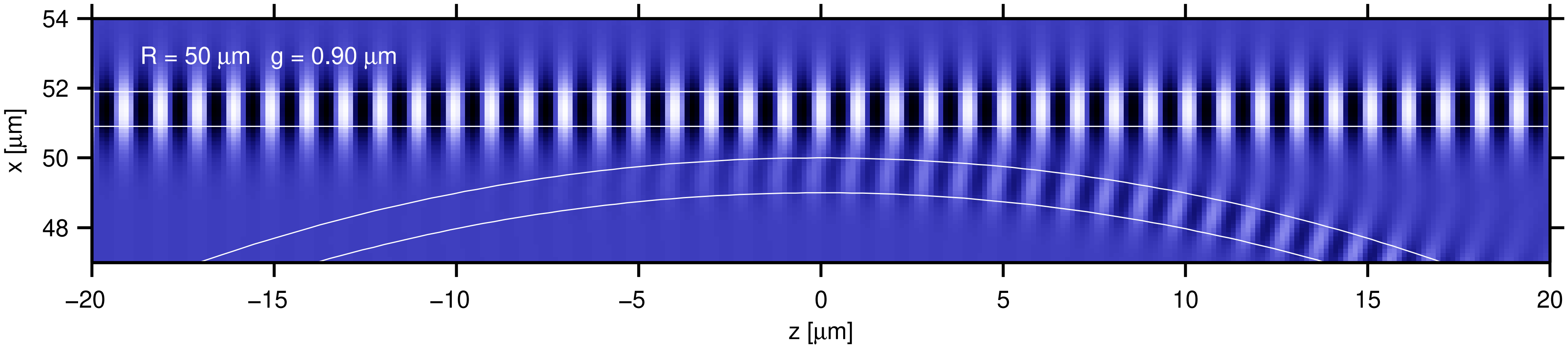
$$c_{pq} = -\mathrm{i} \frac{\omega \epsilon_0}{4} \int \boldsymbol{E}^*_\mathrm{p} (\epsilon - \epsilon_\mathrm{q}) \boldsymbol{E}_\mathrm{q}\, \mathrm{d}x, \qquad p, q = \mathrm{b}, \mathrm{s}.$$

Numerical evaluation $\rightsquigarrow$ $A_\mathrm{b}(z),\ A_\mathrm{s}(z)$ $\rightsquigarrow$ $\ldots$ $\rightsquigarrow$ $\rho,\ \kappa,\ \tau.$

CMT coupler model, examples

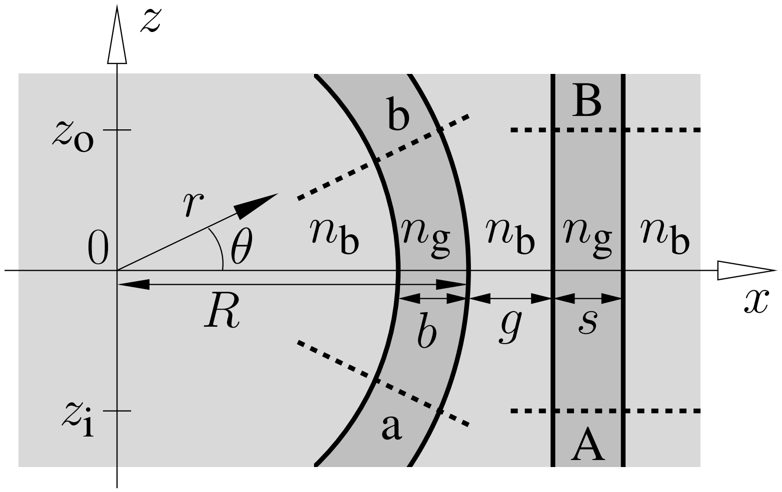


2D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = s = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 50\,\mu\text{m},\ g = 0.90\,\mu\text{m}.$

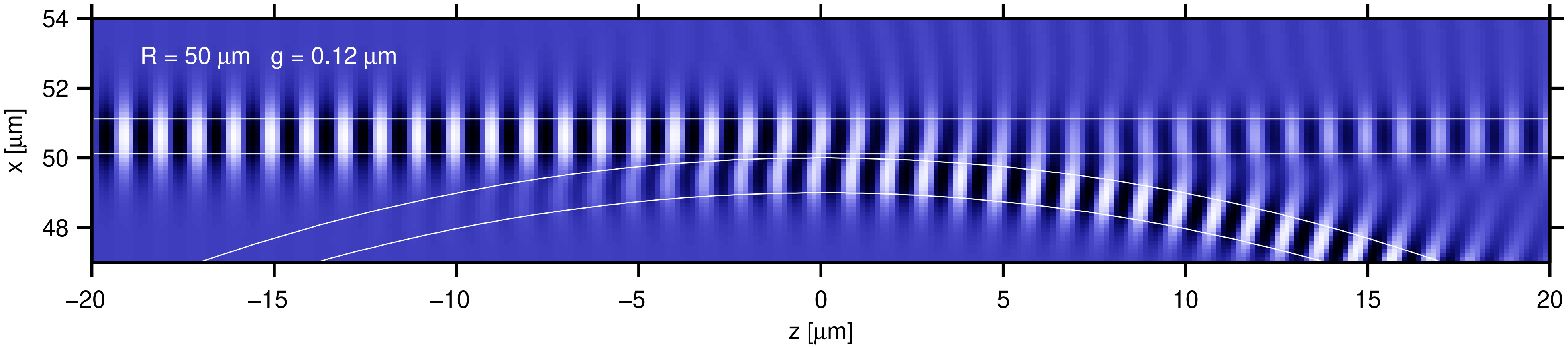


$$|\rho|^2 = 0.93,\ |\kappa|^2 = 0.07,\ |\tau|^2 = 0.92.$$

CMT coupler model, examples

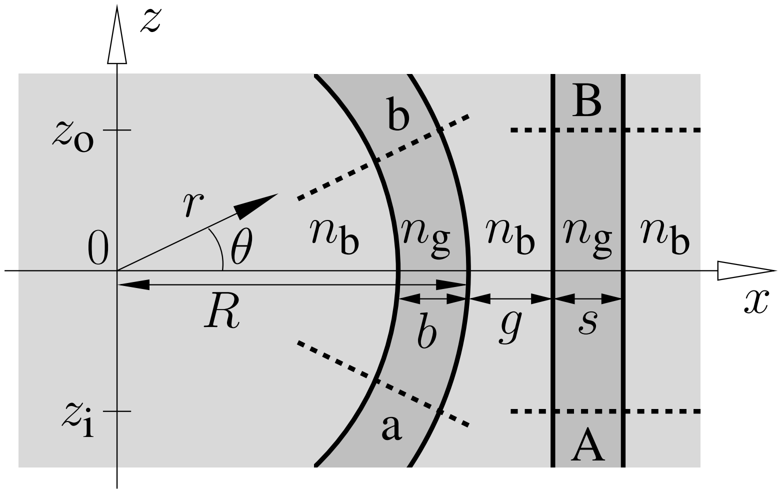


2D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = s = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 50\,\mu\text{m},\ g = 0.12\,\mu\text{m}.$

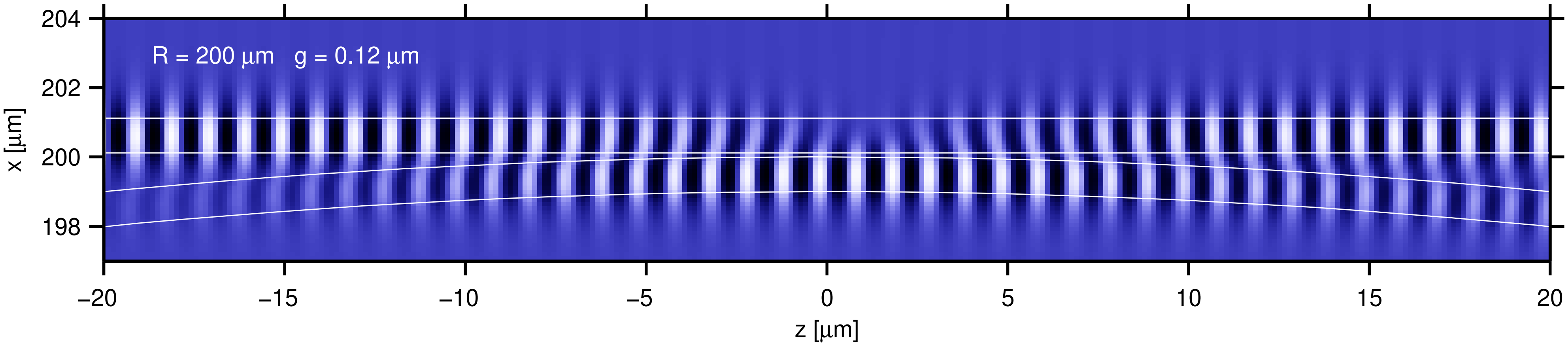


$$|\rho|^2 = 0.16,\ |\kappa|^2 = 0.83,\ |\tau|^2 = 0.16.$$

CMT coupler model, examples

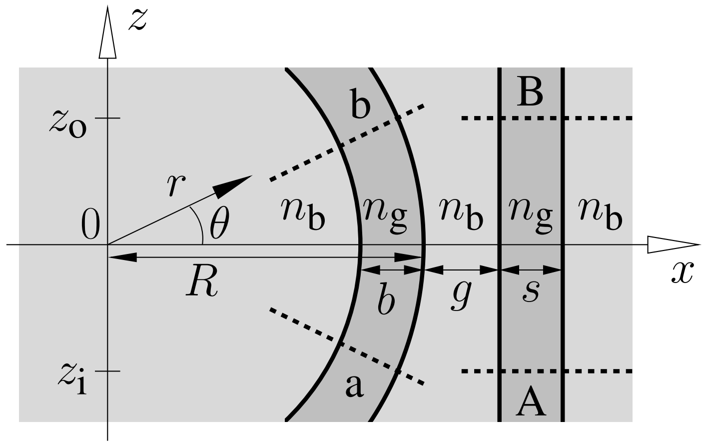


2D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = s = 1.0\,\mu\text{m},\ \lambda = 1.55\,\mu\text{m},$
$R = 200\,\mu\text{m},\ g = 0.12\,\mu\text{m}.$

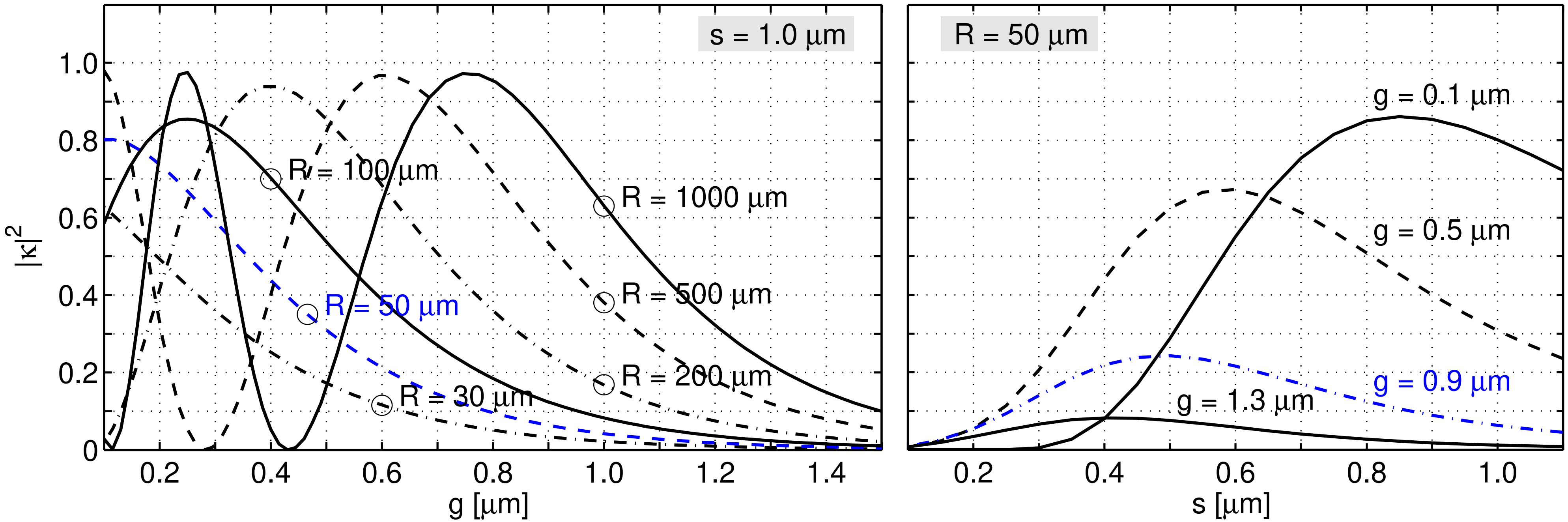


$$|\rho|^2 = 0.93,\ |\kappa|^2 = 0.07,\ |\tau|^2 = 0.93.$$

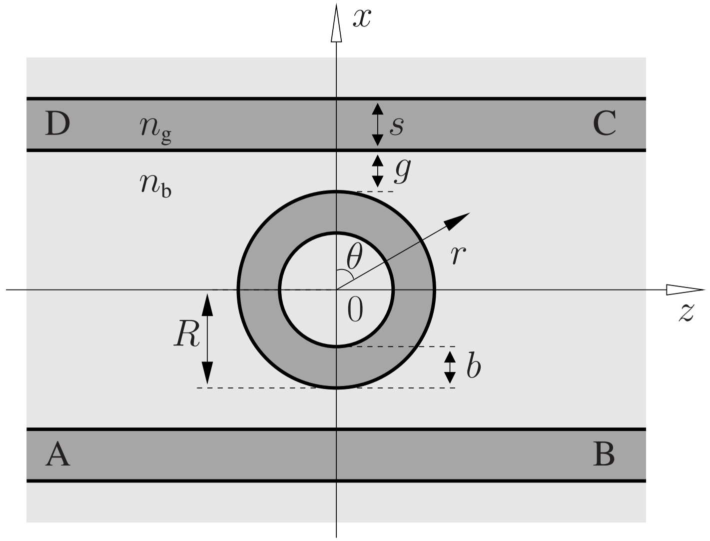
Coupling coefficients vs. coupler geometry



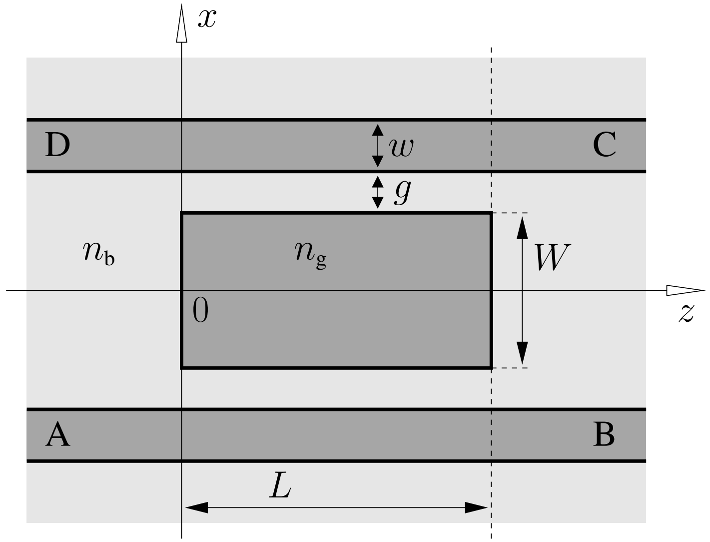
2D, TE,
$n_b = 1.45,\ n_g = 1.60,\ b = 1.0\,\mu m,\ \lambda = 1.55\,\mu m.$



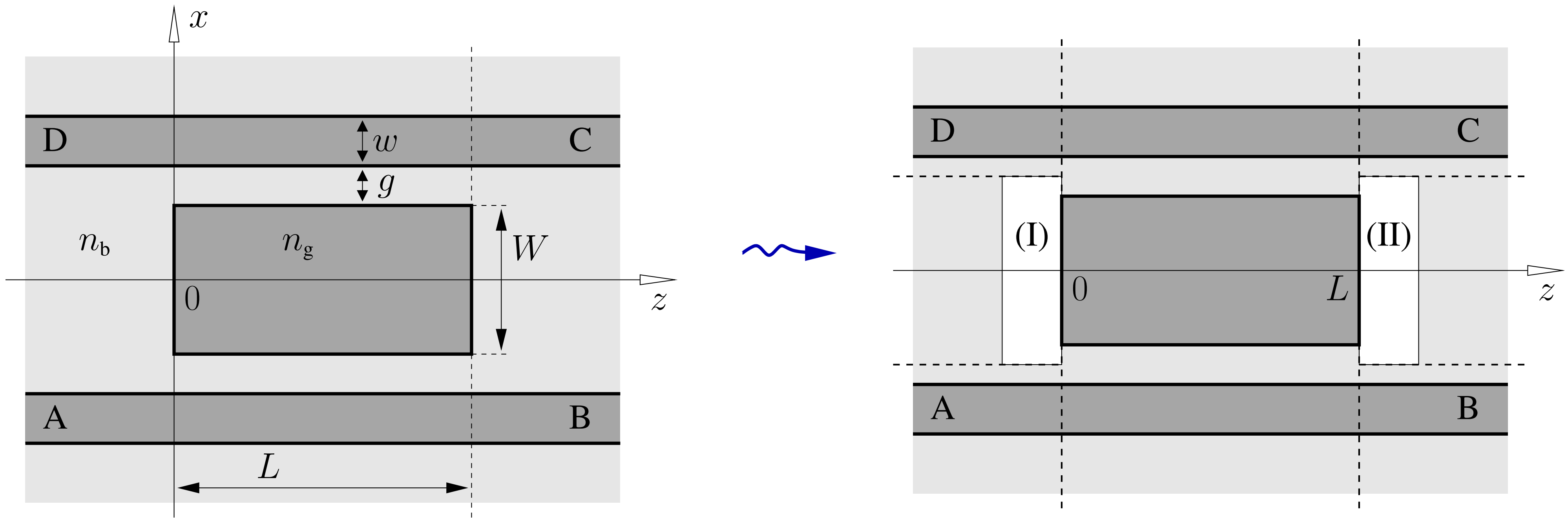
Circular traveling wave resonators



Rectangular standing wave resonators

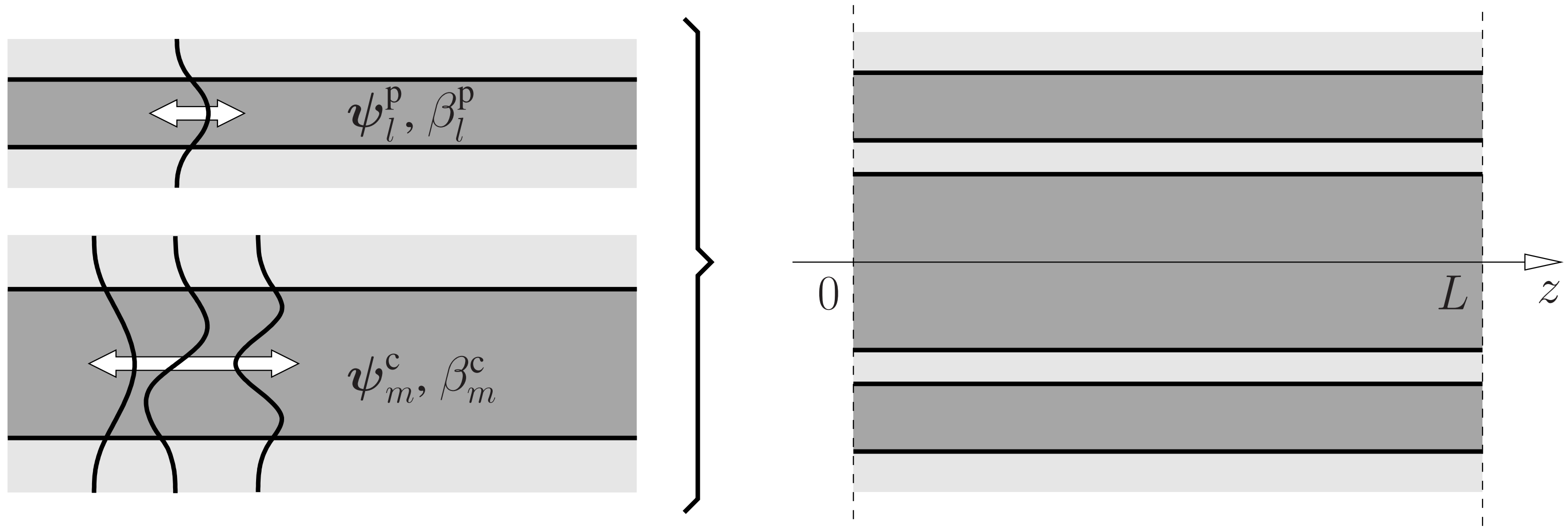


Rectangular Resonator: Abstract model



- Resonator $\approx$ central cavity segment + 2 facets
- CW description : $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$, $\omega = k\,\mathrm{c}$, $k = 2\pi/\lambda$.

Central coupler segment: Basis fields & transfer matrix

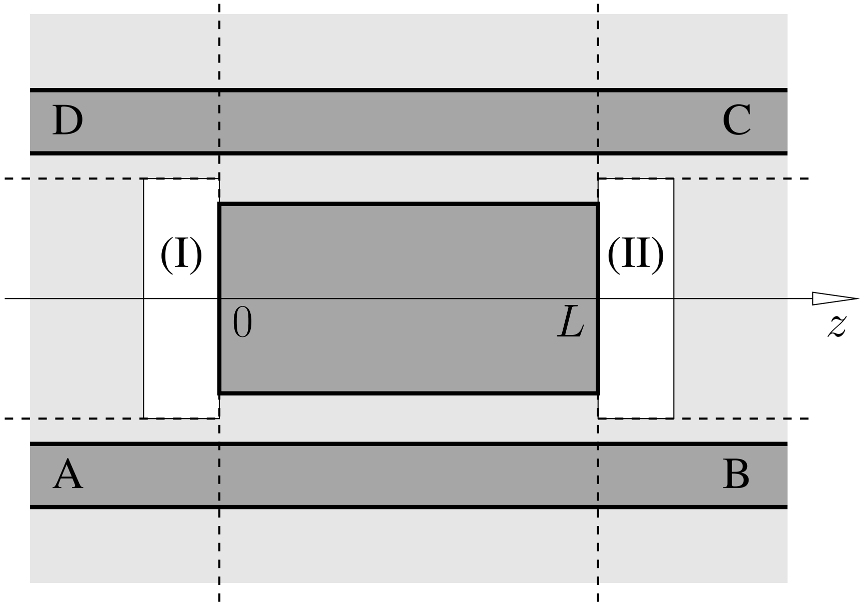


- Basis: Guided fields $\boldsymbol{\psi}_l^{\mathrm{p}}$, $\boldsymbol{\psi}_m^{\mathrm{c}}$, and propagation constants β_l^{p}, β_m^{c}, of port and cavity cores.
- $\boldsymbol{F}$, $\boldsymbol{B}$: Amplitudes of forward and backward copies of the basis modes.
- Propagation along the cavity segment:

$$\boldsymbol{F}(L) = \mathsf{T}\,\boldsymbol{F}(0), \qquad \boldsymbol{B}(0) = \mathsf{T}\,\boldsymbol{B}(L),$$

T : Cavity transfer matrix.

Facets: Multimode reflection



Facets:

$\approx$ no effect on $\boldsymbol{\psi}_l^{\mathrm{p}}$,
strong effect on $\boldsymbol{\psi}_m^{\mathrm{c}}$.

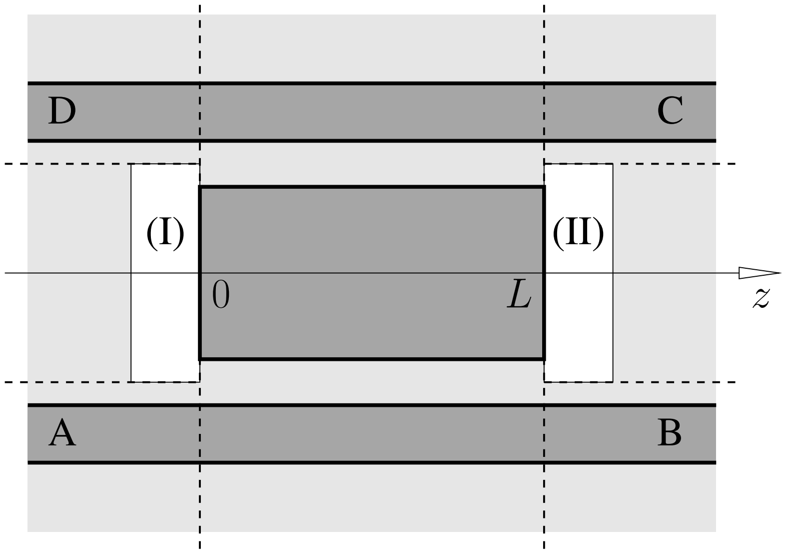
$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_{\mathrm{p}} \\ \boldsymbol{F}_{\mathrm{c}} \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} \boldsymbol{B}_{\mathrm{p}} \\ \boldsymbol{B}_{\mathrm{c}} \end{pmatrix}.$$

Guided wave reflection at the facet:

$$\boldsymbol{B}_{\mathrm{c}}(L) = \mathsf{R}\,\boldsymbol{F}_{\mathrm{c}}(L), \qquad \boldsymbol{F}_{\mathrm{c}}(0) = \mathsf{R}\,\boldsymbol{B}_{\mathrm{c}}(0),$$

R: Facet reflectivity matrix.

Output amplitudes & power



Cavity transfer matrix

$$\mathsf{T} = \begin{pmatrix} \mathsf{T}_{\rm pp} & \mathsf{T}_{\rm pc} \\ \mathsf{T}_{\rm cp} & \mathsf{T}_{\rm cc} \end{pmatrix}$$

+ Facet reflectivity matrix R

+ External input amplitudes

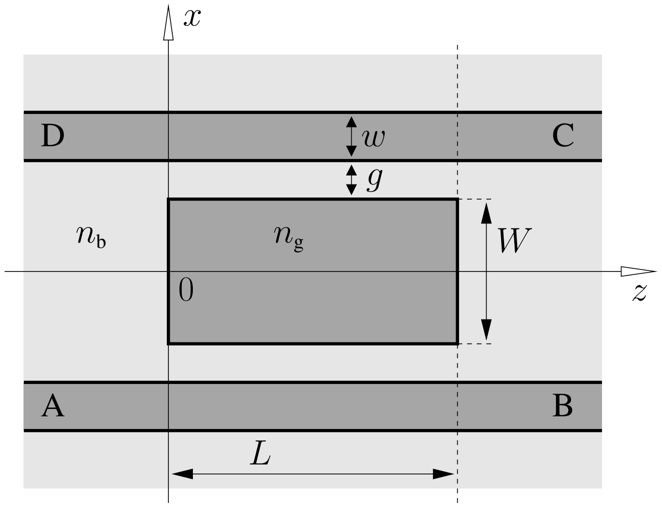
$$\boldsymbol{F}_{\rm p}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{B}_{\rm p}(L) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

External output amplitudes :

$$\boldsymbol{F}_{\rm p}(L) = (\mathsf{T}_{\rm pp} + \mathsf{T}_{\rm pc}\mathsf{R}\Omega^{-1}\mathsf{T}_{\rm cc}\mathsf{R}\mathsf{T}_{\rm cp})\boldsymbol{F}_{\rm p}(0),$$
$$\boldsymbol{B}_{\rm p}(0) = \mathsf{T}_{\rm pc}\mathsf{R}\Omega^{-1}\mathsf{T}_{\rm cp}\,\boldsymbol{F}_{\rm p}(0), \qquad \Omega = 1 - \mathsf{T}_{\rm cc}\mathsf{R}\mathsf{T}_{\rm cc}\mathsf{R}\,.$$

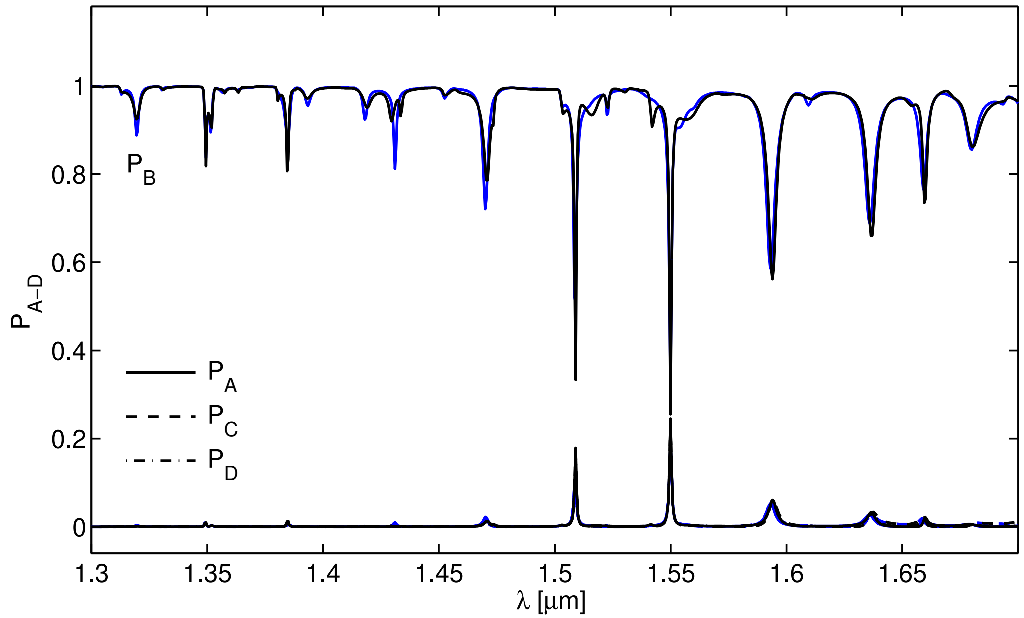
$$P_{\rm A} = |B_{{\rm p},1}(0)|^2, \quad P_{\rm B} = |F_{{\rm p},1}(L)|^2, \quad P_{\rm C} = |F_{{\rm p},2}(L)|^2, \quad P_{\rm D} = |B_{{\rm p},2}(0)|^2.$$

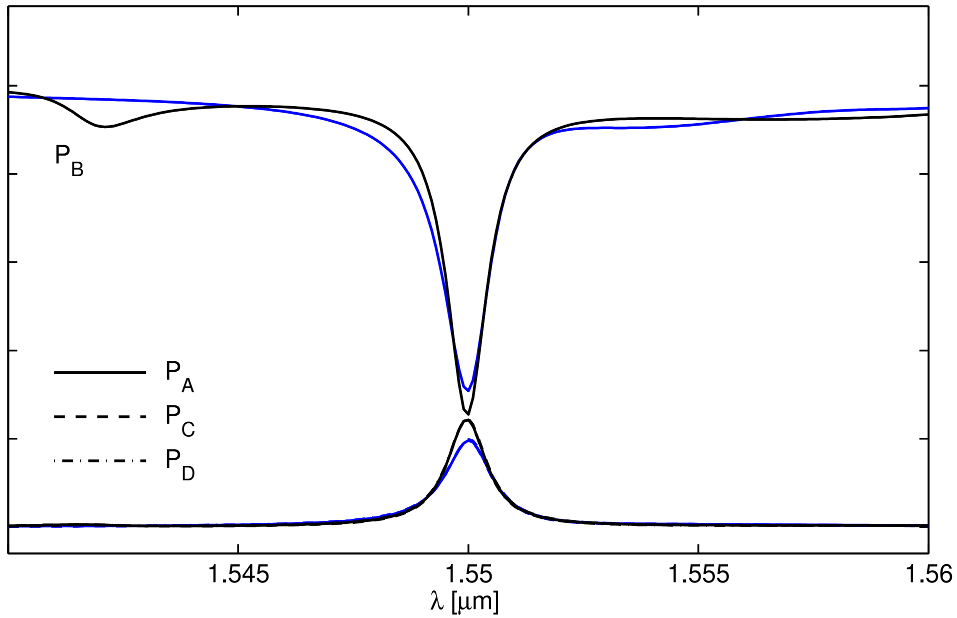
Spectral response



$W = 2.524\,\mu\text{m},\ L = 5.738\,\mu\text{m},$
$w = 0.112\,\mu\text{m},\ g = 0.450\,\mu\text{m},$
$n_{\text{b}} = 1.45,\ n_{\text{g}} = 3.40;$ TE.

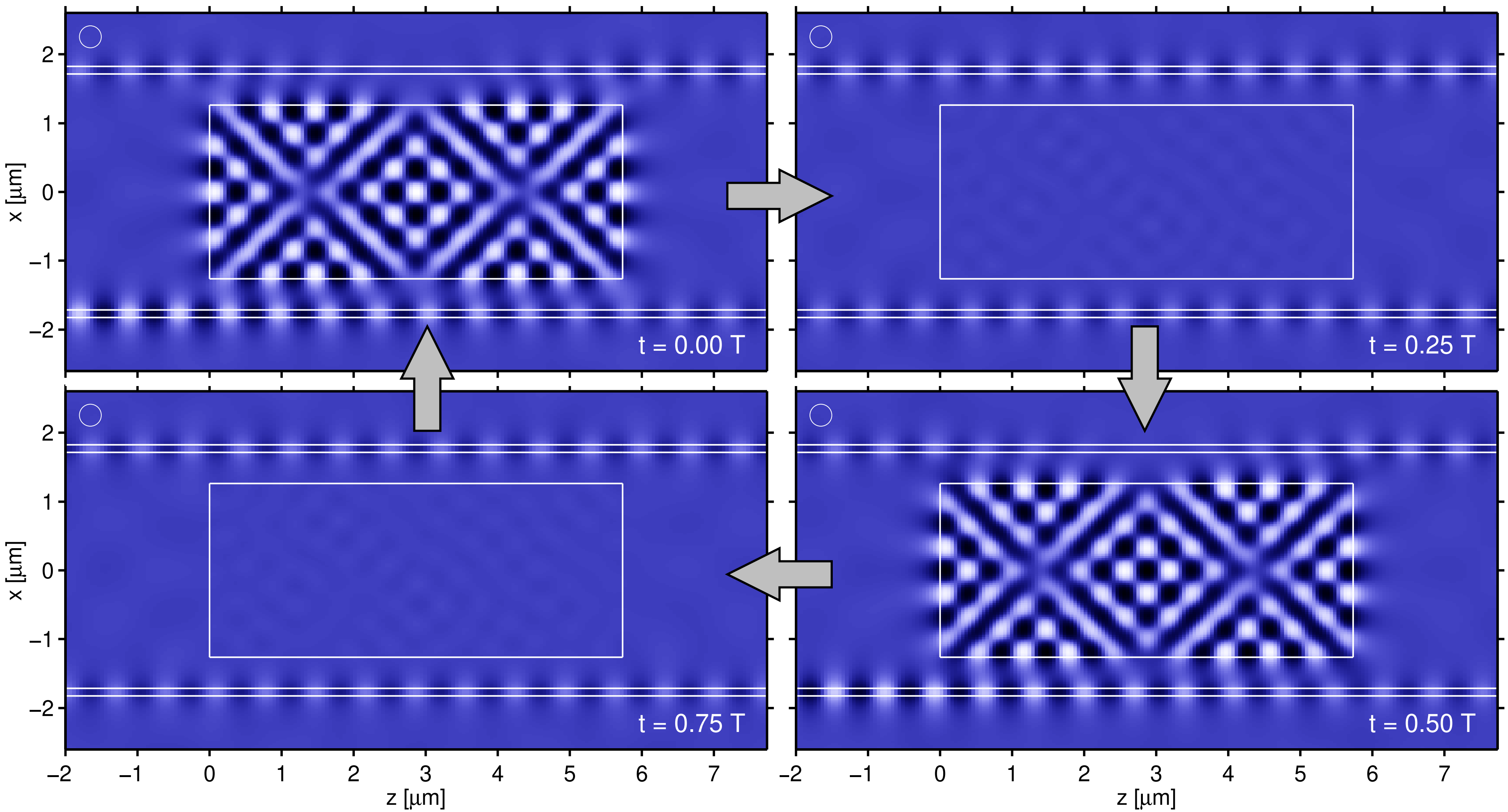
CMT,
Rigorous simulations (BEP).





Resonant field pattern

$\mathcal{E}_y(x, z, t)$:



$\lambda = 1.55\,\mu\text{m},\;\; T = 5.17\,\text{fs}.$

Detaching the cavity

- Resonances $\leftrightarrow$ Singularities in Ω.

- The cavity amplifies a field $\boldsymbol{F}_{\mathrm{c}}(L) = \boldsymbol{v}$ that corresponds to a large eigenvalue a of Ω^{-1}: $\Omega^{-1}\boldsymbol{v} = a\,\boldsymbol{v},\ \Omega\,\boldsymbol{v} = (1/a)\,\boldsymbol{v},$ amplification $A = |a|^2$.

  Resonant configurations:
  - $\Omega = 1 - \mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has a zero eigenvalue.
  - $\mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has an eigenvalue 1:
  $$\boldsymbol{F}_{\mathrm{c}}(L) \xrightarrow{\mathsf{R}} \boldsymbol{B}_{\mathrm{c}}(L) \xrightarrow{\mathsf{T}_{\mathrm{cc}}} \boldsymbol{B}_{\mathrm{c}}(0) \xrightarrow{\mathsf{R}} \boldsymbol{F}_{\mathrm{c}}(0) \xrightarrow{\mathsf{T}_{\mathrm{cc}}} \boldsymbol{F}_{\mathrm{c}}(L)\,.$$
  - $\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has an eigenvalue $\mu = +1$ or $\mu = -1$.

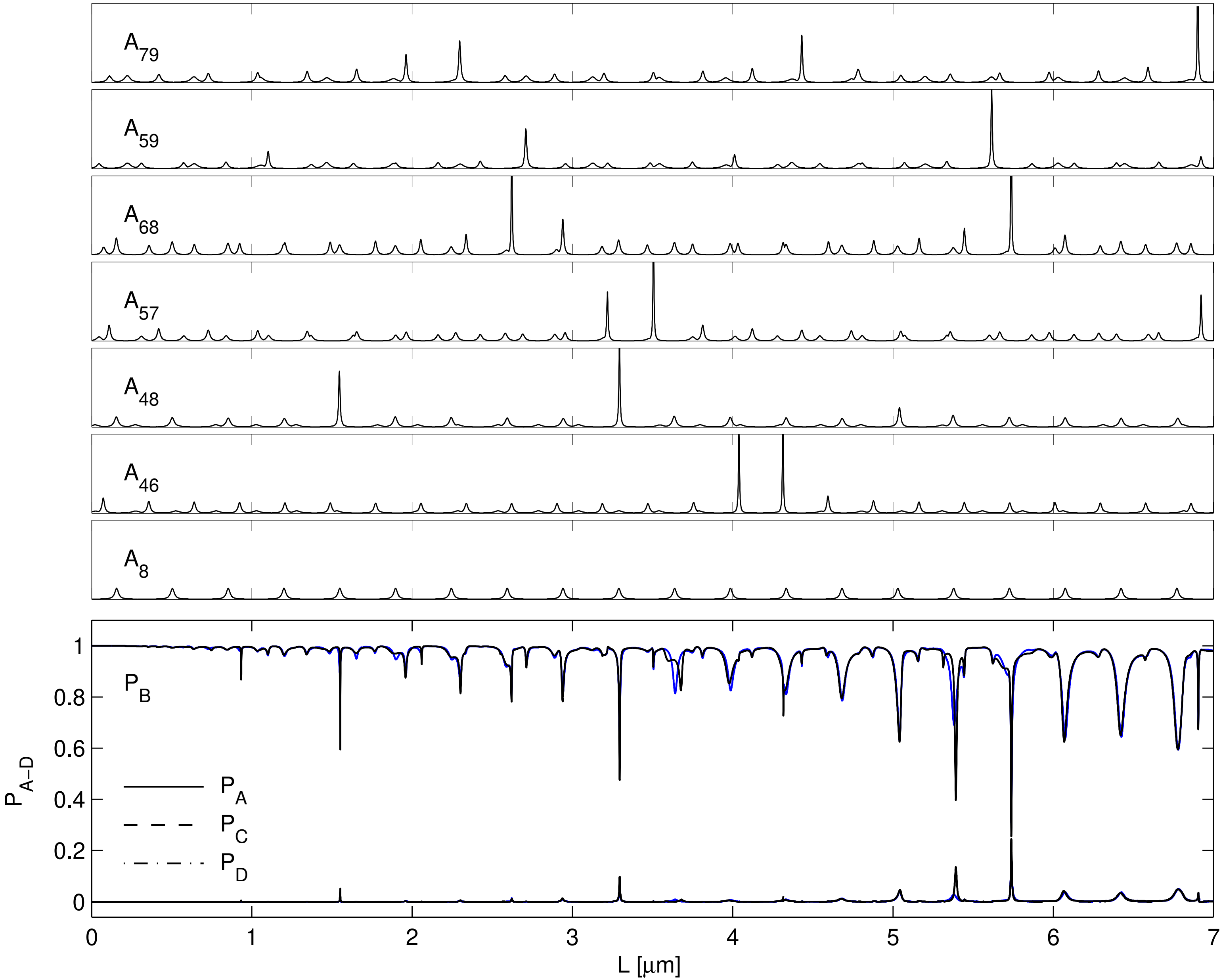
  Amplification for a field $\boldsymbol{v}$ with $\mathsf{T}_{\mathrm{cc}}\mathsf{R}\,\boldsymbol{v} = \mu\,\boldsymbol{v},\ \mu = r\,\mathrm{e}^{\mathrm{i}\chi}$:
  $$A = \frac{1}{|\mu^2 - 1|^2} = \frac{1}{1 + r^4 - 2r^2\cos(2\chi)}\,.$$

- Isolated cavity:
$$\mathsf{T}_{\mathrm{cc}} \xrightarrow{g\,\to\,\infty} \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\beta_1^{\mathrm{c}}L}, \ldots, \mathrm{e}^{-\mathrm{i}\beta_N^{\mathrm{c}}L}\right)\,.$$

Resonant configurations, CMT model

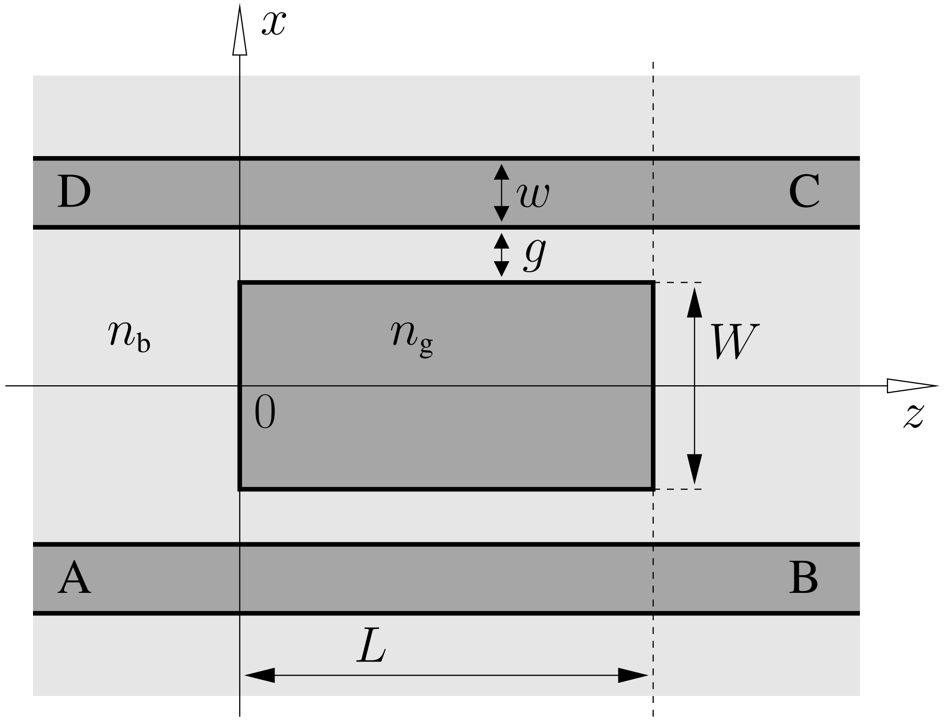
$A_m,\ A_{m,l}$: cavity modes m or m & l.

classification of resonances.

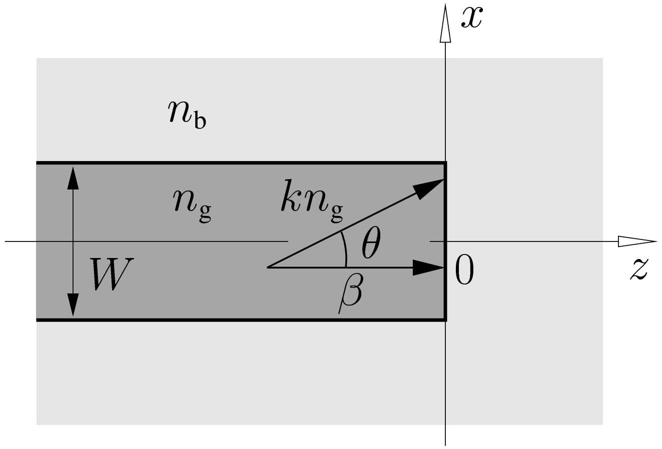


Abstract model & examples

Waveguide facets

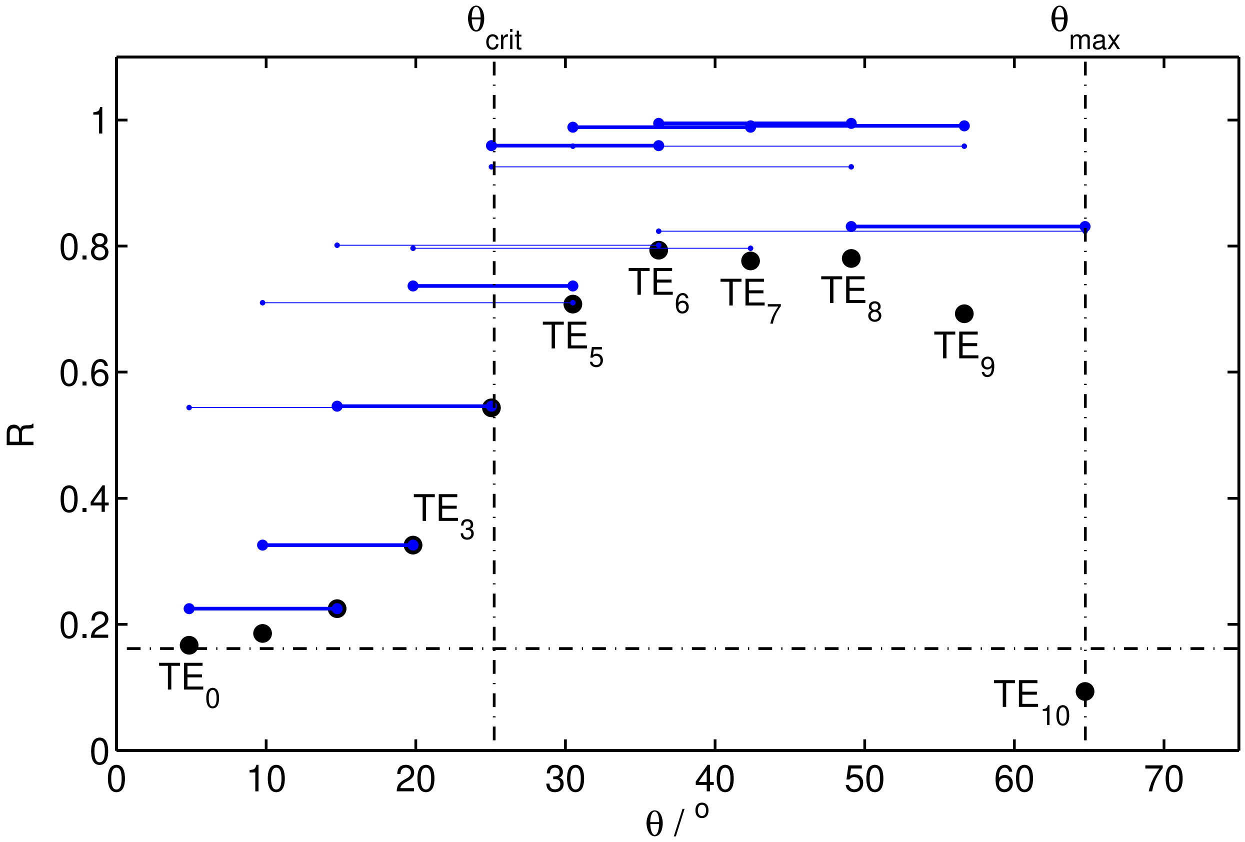


Multimode facet reflectivity



$$\sin\theta_{\mathrm{crit}} = n_{\mathrm{b}}/n_{\mathrm{g}}, \qquad \sin\theta_{\mathrm{max}} = \sqrt{1 - n_{\mathrm{b}}^2/n_{\mathrm{g}}^2}\,,$$

$$\theta > \theta_{\mathrm{crit}} \;\leftrightarrow\; n_{\mathrm{g}} > \sqrt{2}\,n_{\mathrm{b}}\,.$$

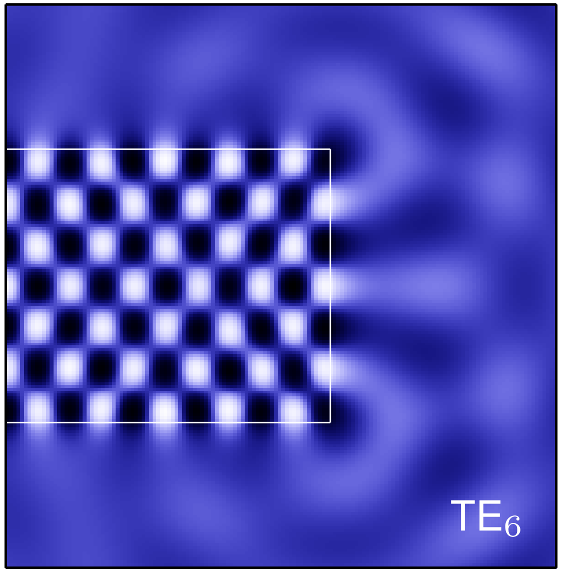


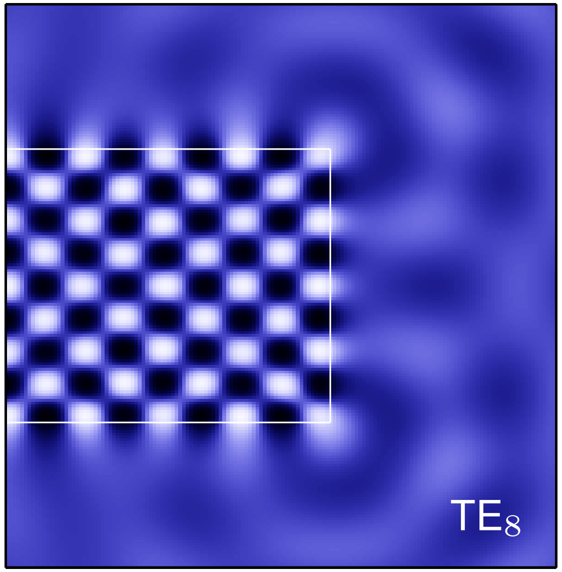
● Single mode reflectivity

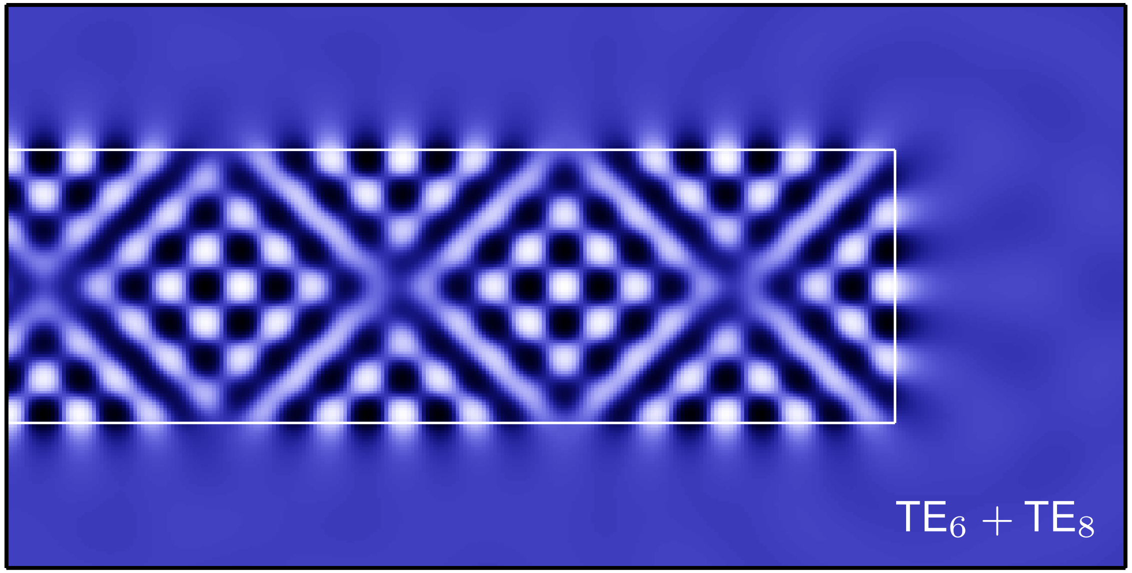
●—● Two mode superposition, maximum reflectivity

$n_{\mathrm{b}} = 1.45$,
$n_{\mathrm{g}} = 3.40$,
$W = 2.524\,\mu$m,
$\lambda = 1.55\,\mu$m.

Single- and bimodal reflections







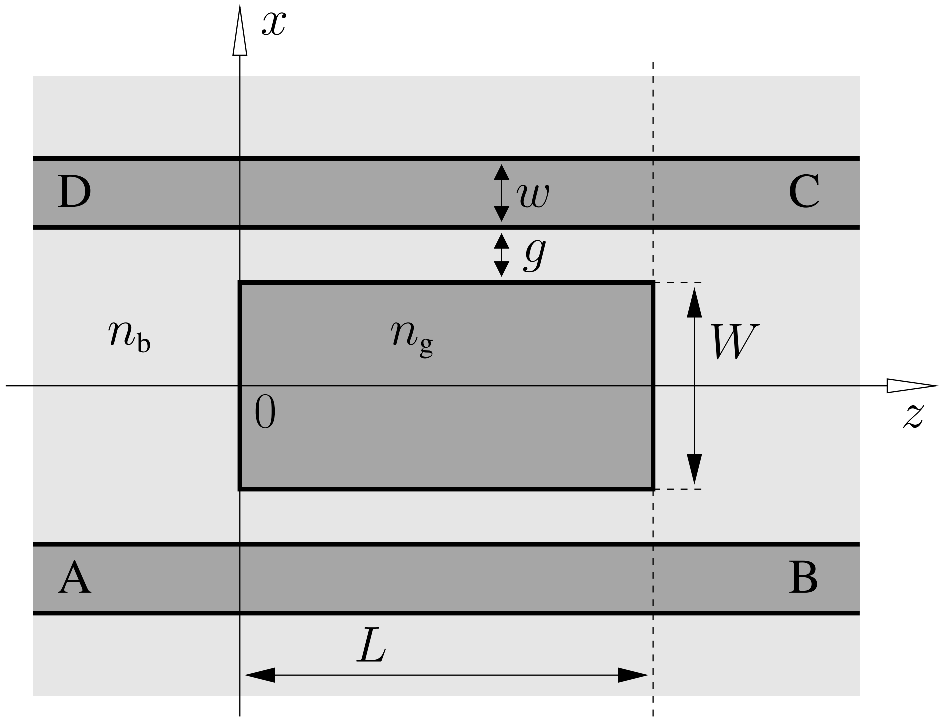
$$R_6 = 0.79$$

$$R_8 = 0.78$$

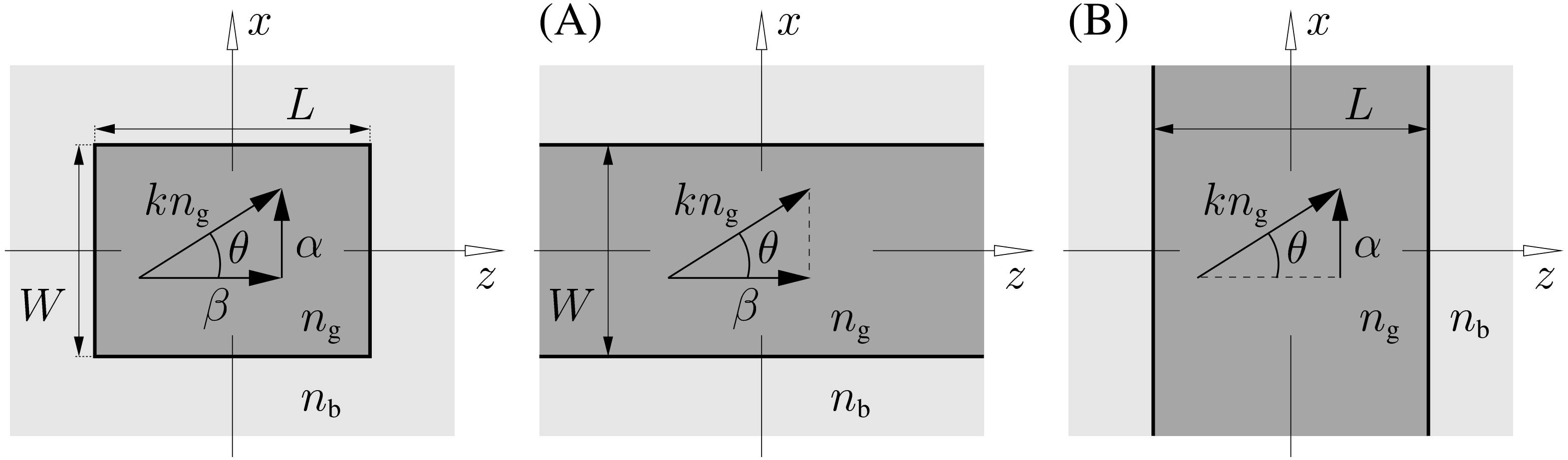
$$R_{6,8} > 0.99$$

Waveguide facets

Slab mode resonances



Resonances, slab mode reasoning



Field in $-W/2 < x < W/2,\ -L/2 < z < L/2$:

(A) $E_y(x,z) = E_0\,\phi(x)(\,\mathrm{e}^{-\mathrm{i}\beta z} + b\,\mathrm{e}^{\mathrm{i}\beta z}),\ \ \phi(x) = \mathrm{e}^{-\mathrm{i}\alpha x} \pm \mathrm{e}^{\mathrm{i}\alpha x},$

(B) $E_y(x,z) = E_0\,\psi(z)(\,\mathrm{e}^{-\mathrm{i}\alpha x} + d\,\mathrm{e}^{\mathrm{i}\alpha x}),\ \ \psi(z) = \mathrm{e}^{-\mathrm{i}\beta z} \pm \mathrm{e}^{\mathrm{i}\beta z}.$

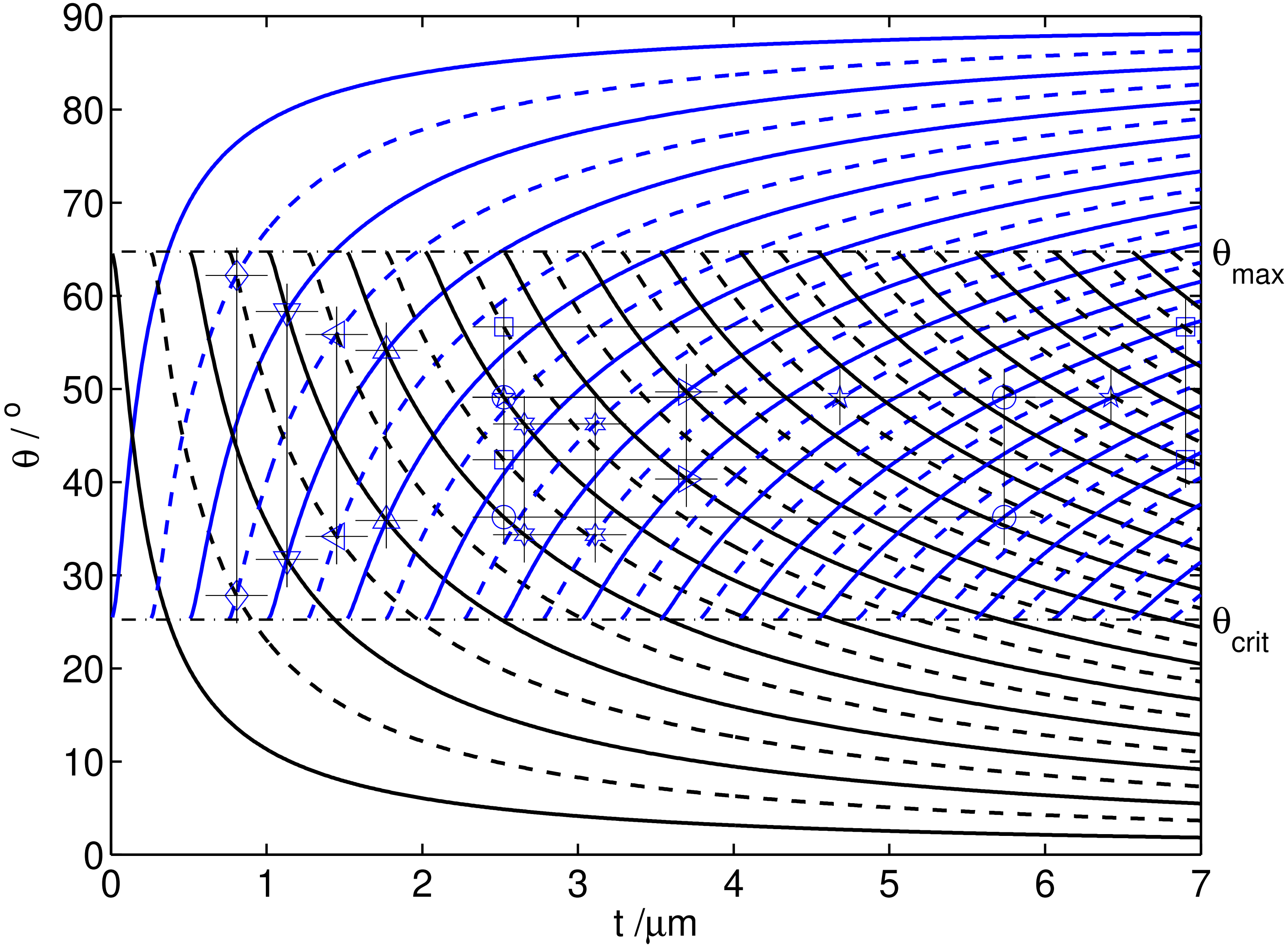
(A) = (B) : $b = \pm 1,\ d = \pm 1.$

Resonant configurations W, L for given $\lambda = 2\pi/k$, n_g, n_b:
The slab waveguide of thickness W supports a mode with angle θ,
while the slab of thickness L guides a field with angle $\pi/2 - \theta$.

Bimodal resonance: Two pairs of modes with proper symmetry ($\leftrightarrow$ facet edges) satisfy the conditions simultaneously.

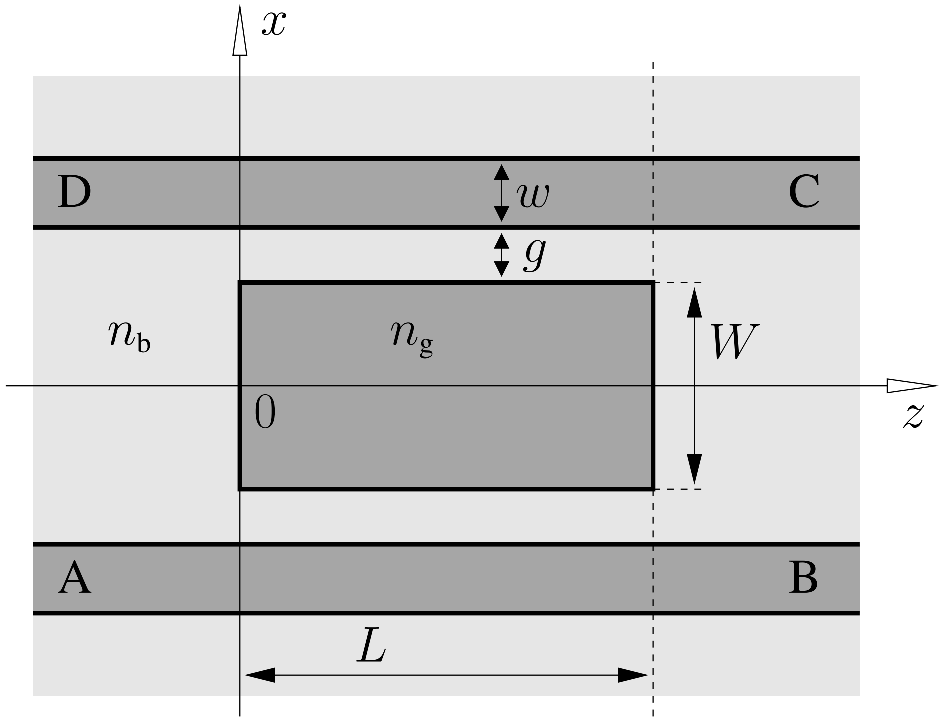
Looking up resonant configurations

. . . in a plot
mode angle θ
vs. slab thickness t :

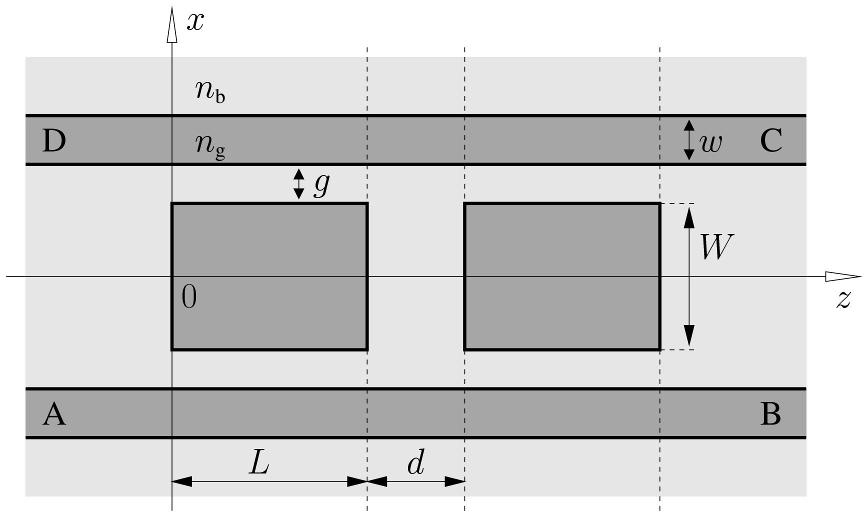


Slab mode resonances

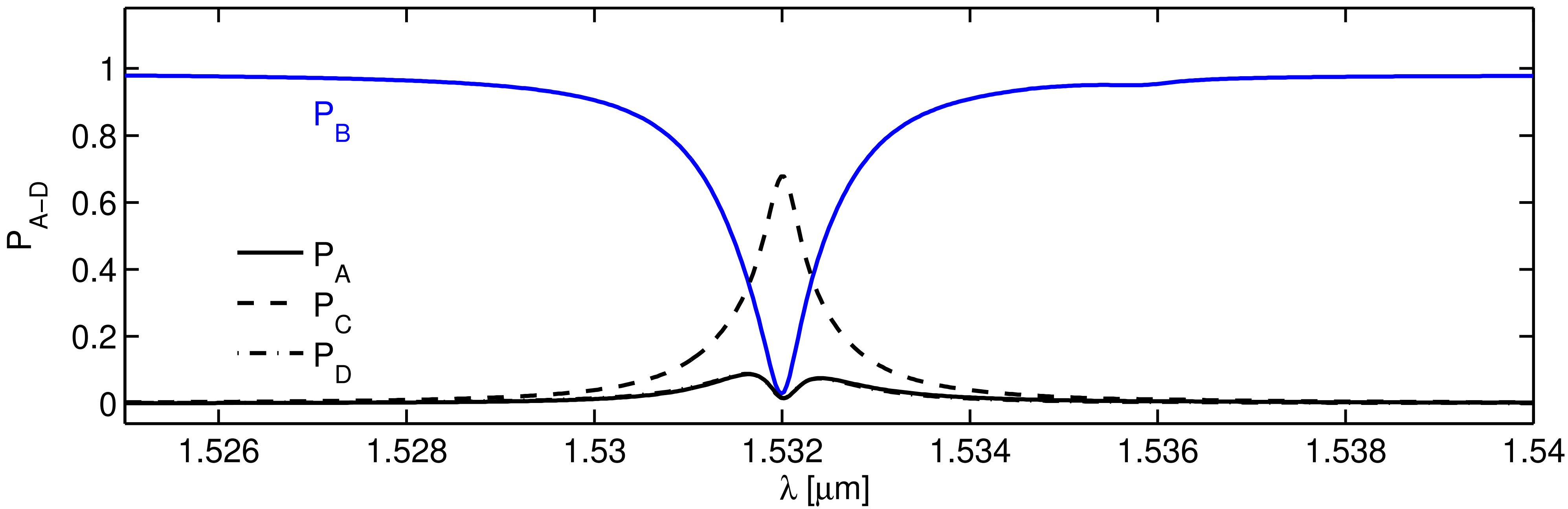
Extensions



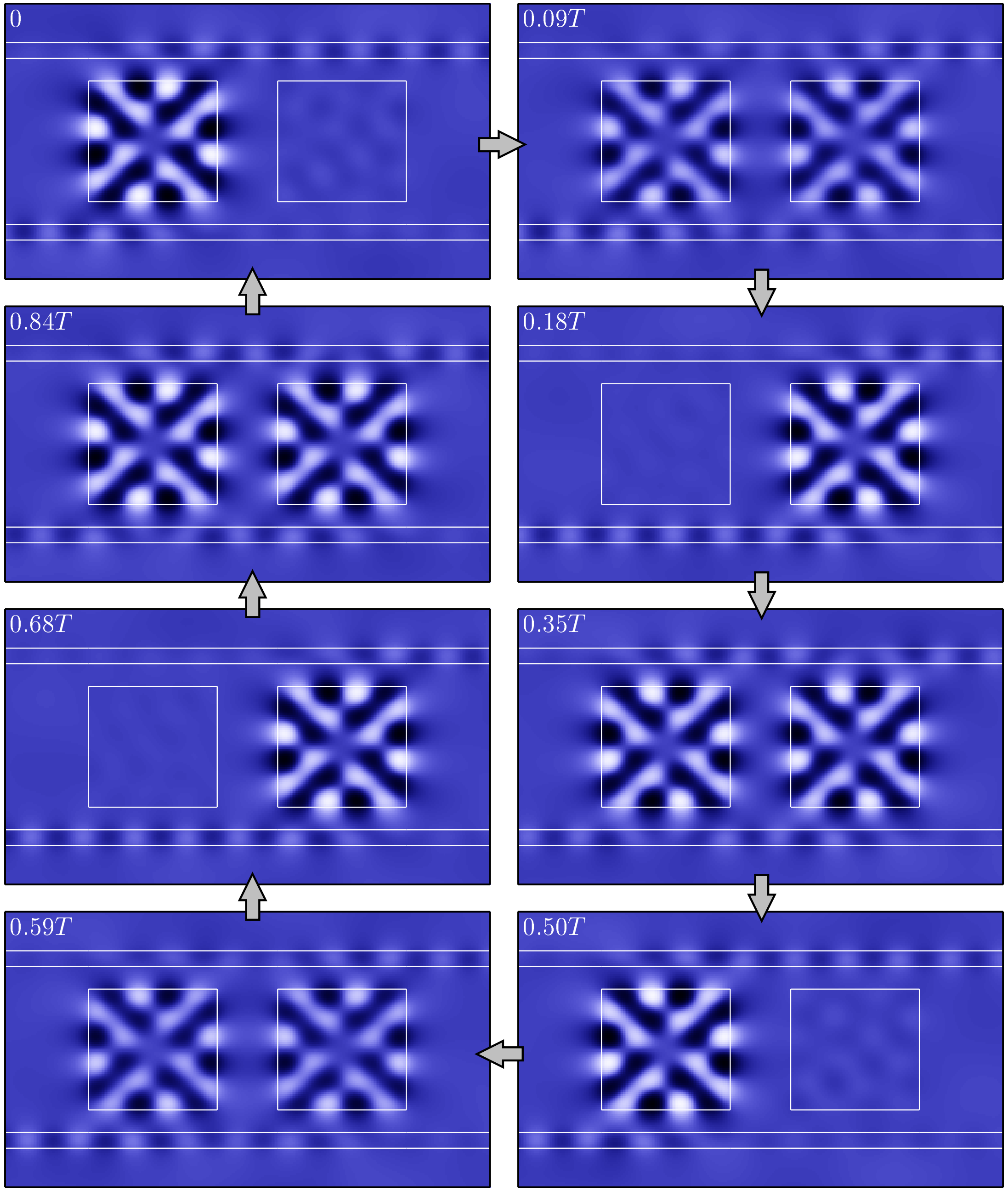
Filter device based on rectangular cavities



$w = 0.20\,\mu$m, $g = 0.29\,\mu$m,
$W = L = 1.54\,\mu$m, $d = 0.72\,\mu$m,
$n_\mathrm{b} = 1.0$, $n_\mathrm{g} = 3.2$; 2D, TE.



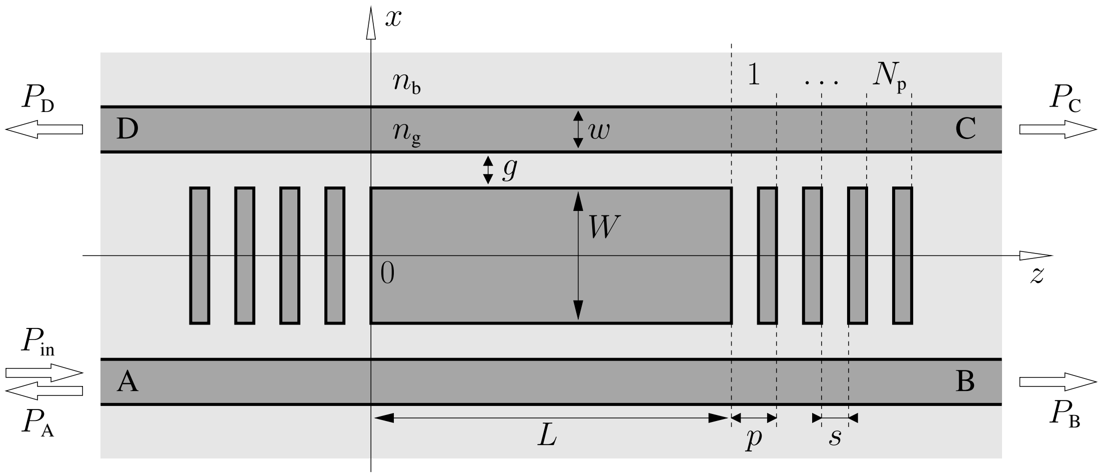
Filter: Resonant field pattern



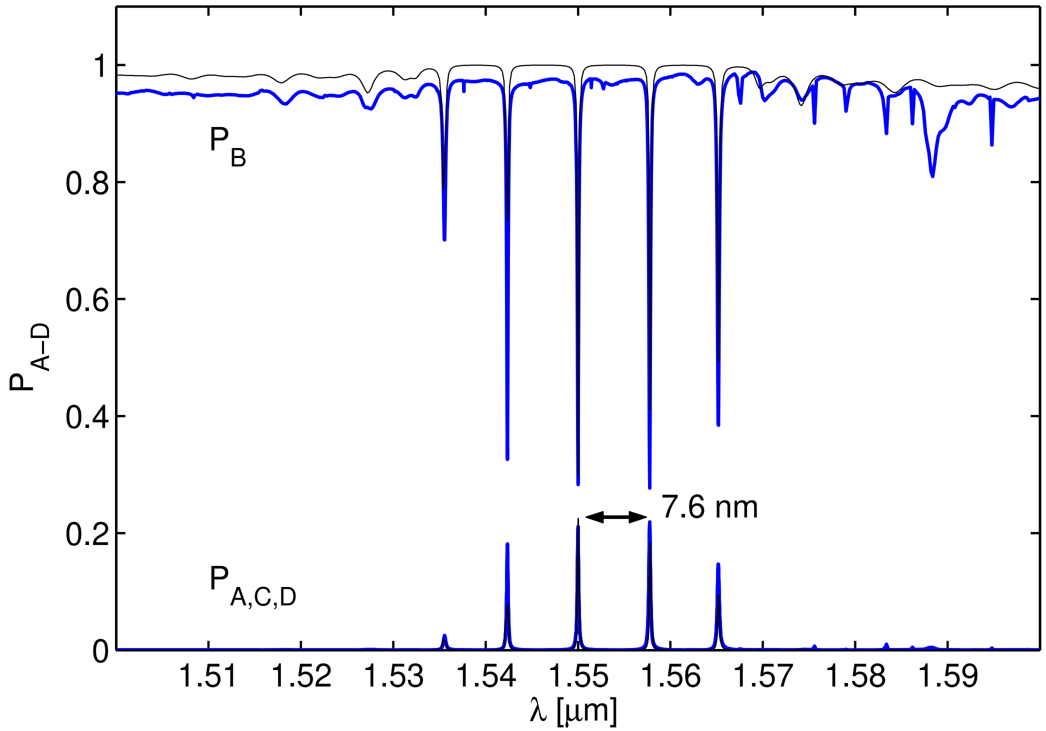
$\lambda = 1.532\,\mu\text{m}$,
$T = 5.11\,\text{fs}$.

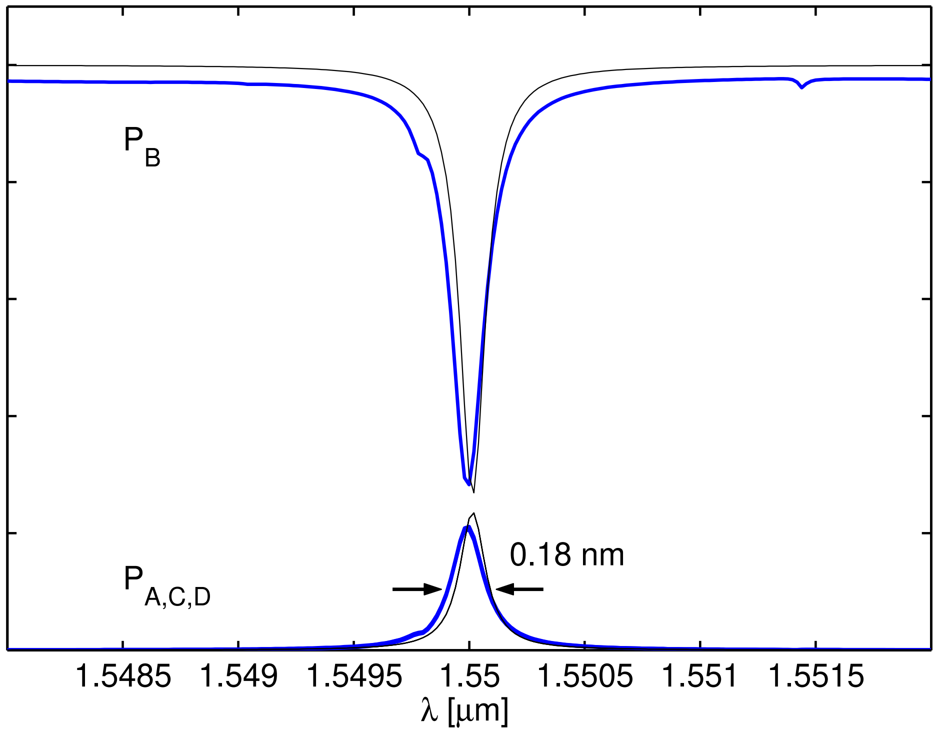
$P_\text{D} = 0.02$, $P_\text{C} = 0.68$,
$P_\text{A} = 0.02$, $P_\text{B} = 0.03$.

Grating assisted resonator

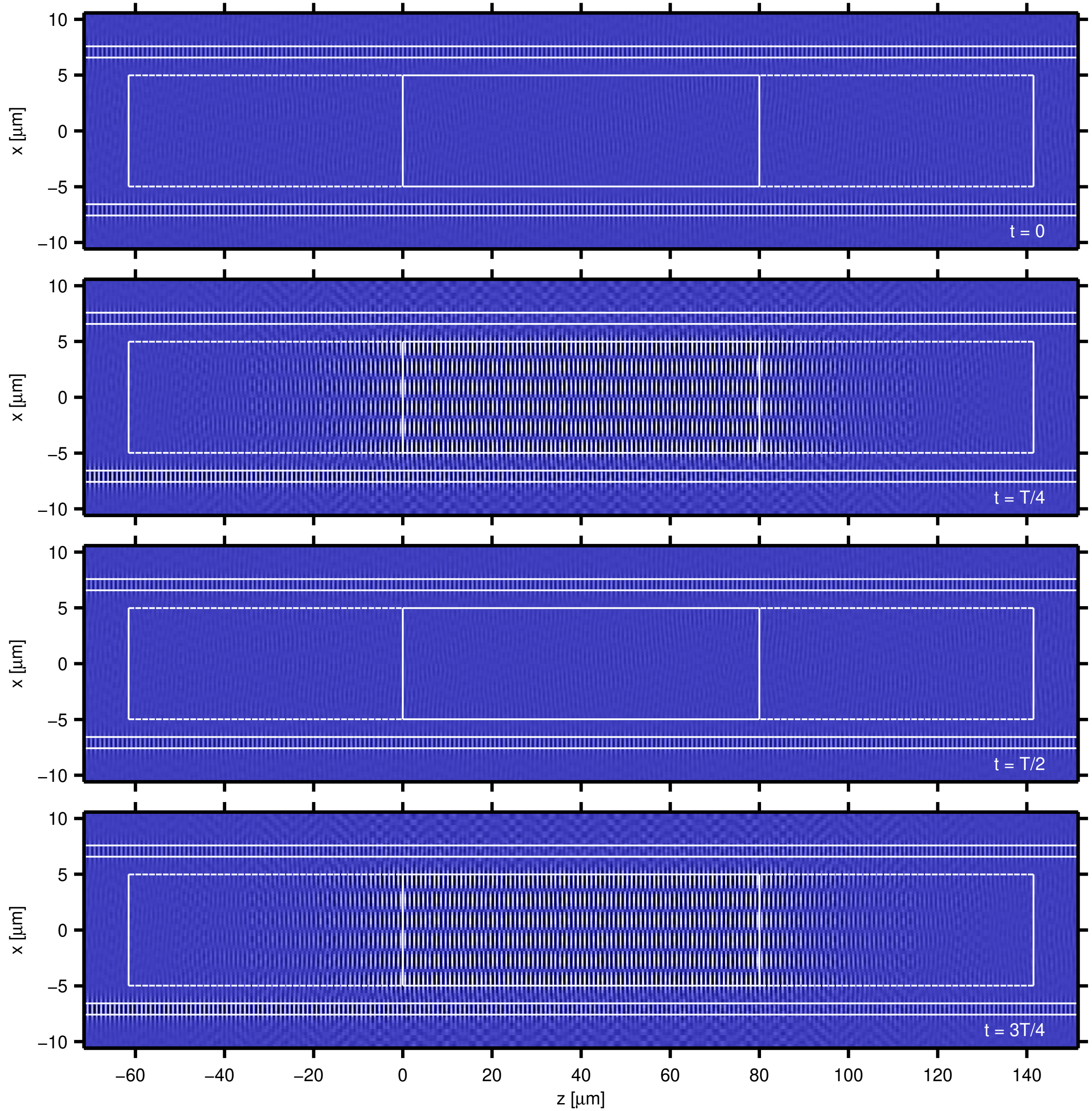


$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m, $L = 79.985\,\mu$m,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m, $N_p = 40$,
$n_\mathrm{b} = 1.45$, $n_\mathrm{g} = 1.60$; 2D, TE.





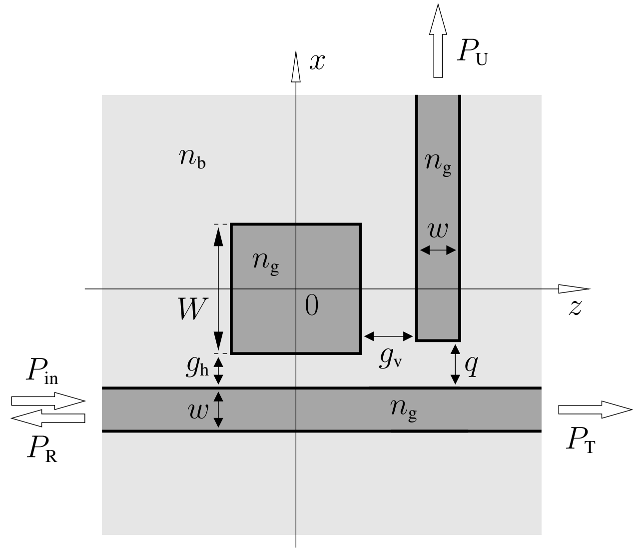
Grating assisted resonator: Resonant field pattern



$\lambda = 1.55\,\mu\mathrm{m}$,
$T = 5.17\,\mathrm{fs}$.

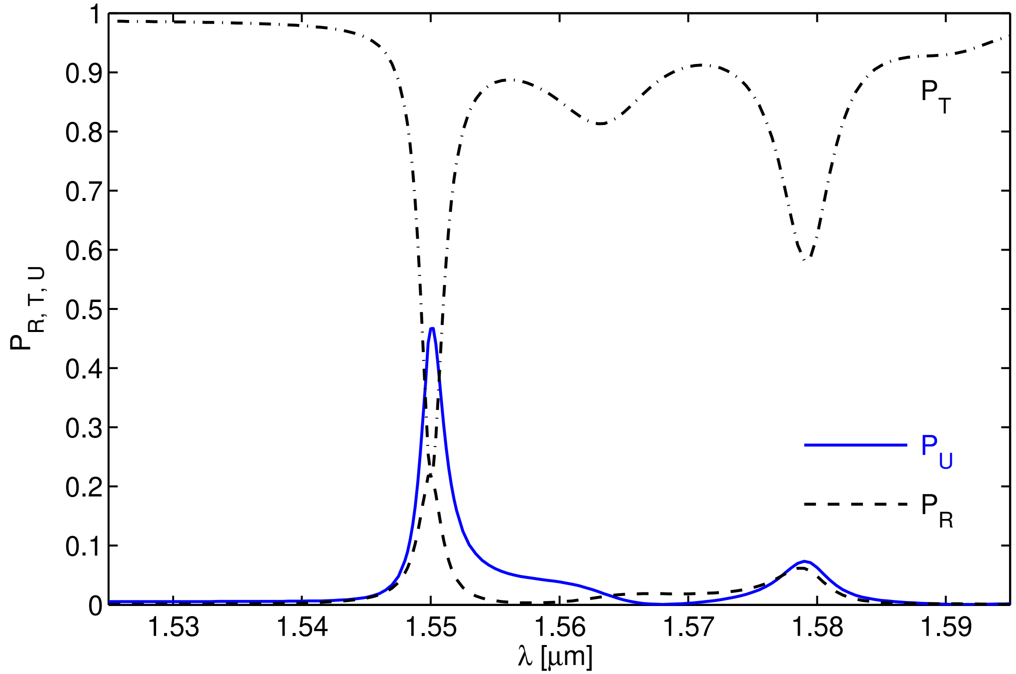
$P_\mathrm{D} = 0.21,\ P_\mathrm{C} = 0.21,$
$P_\mathrm{A} = 0.21,\ P_\mathrm{B} = 0.28.$

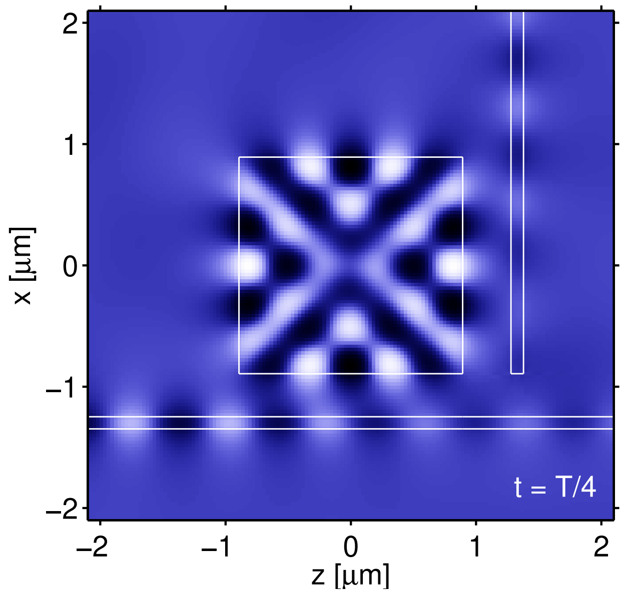
Resonator with perpendicular ports



$W = 1.786\,\mu\text{m},\ w = 1.0\,\mu\text{m},$
$g_\text{h} = q = 0.355\,\mu\text{m},\ g_\text{v} = 0.385\,\mu\text{m},$
$n_\text{b} = 1.0,\ n_\text{g} = 3.4;$ 2D, TE.

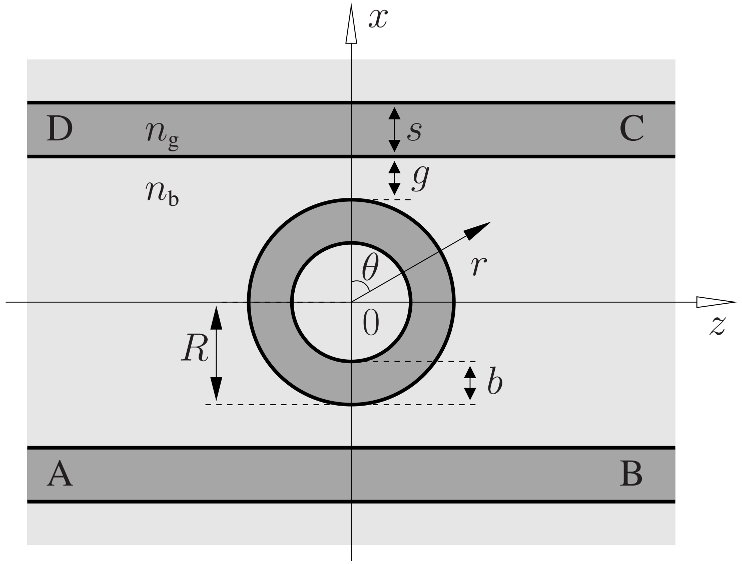
$\lambda = 1.55\,\mu\text{m}: \quad P_\text{R} = 0.22,\ P_\text{T} = 0.22,\ P_\text{U} = 0.46.$





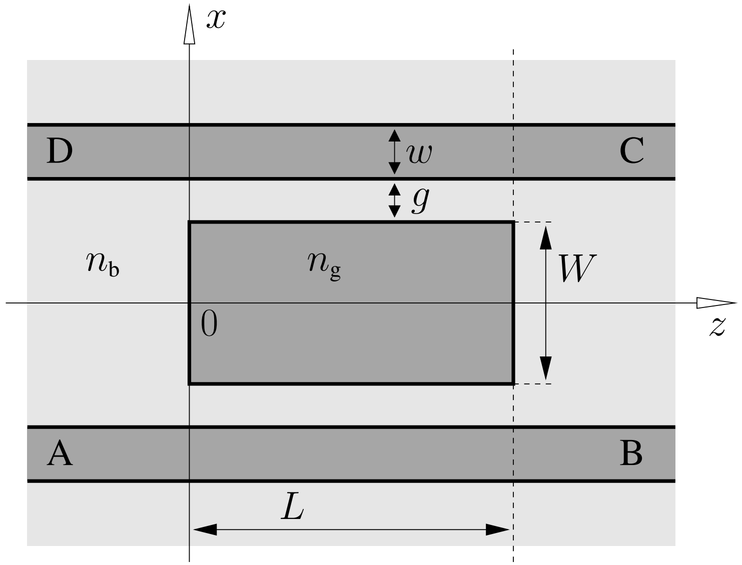
Concluding remarks

Analytical approaches to optical microresonators:



Circular traveling wave resonators

- Alternative viewpoint:
  Time-domain gallery modes
- Extension to 3D:
  Straightforward, if . . .



Rectangular standing wave resonators

- Speculative.
  So far 2D models exist.
- Standing wave phenomena
  . . . in small rings ?