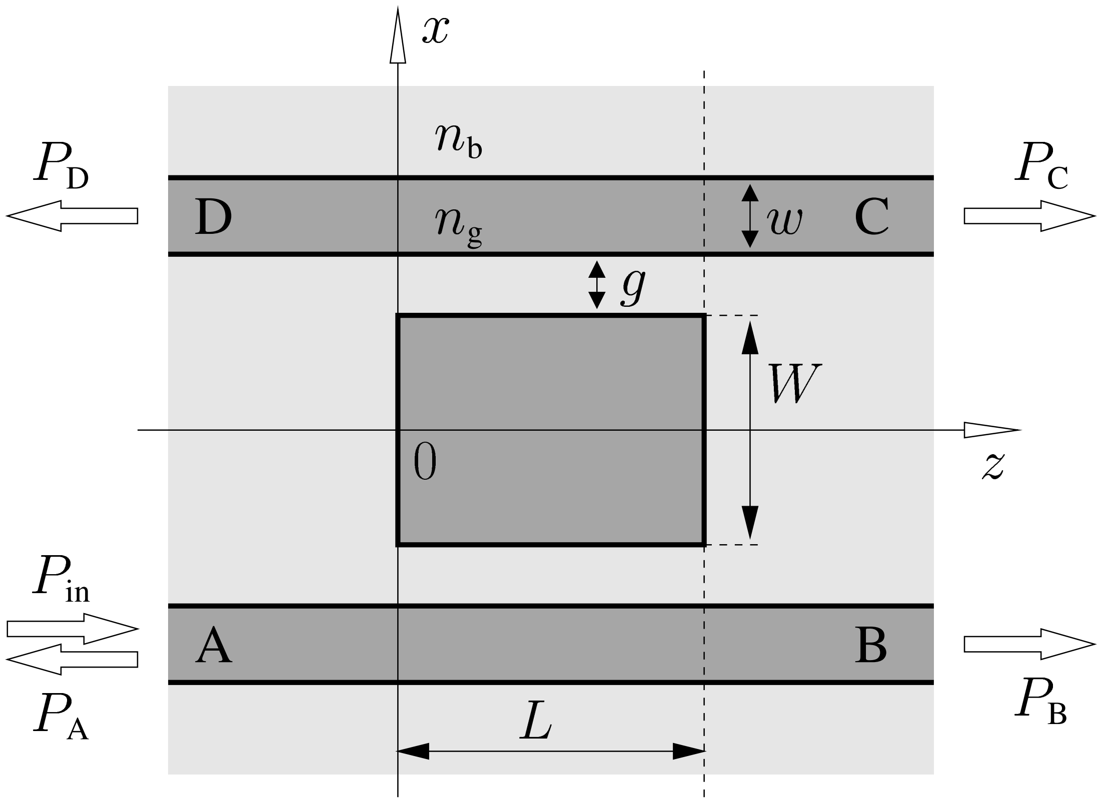
1 *Rectangular microresonators*

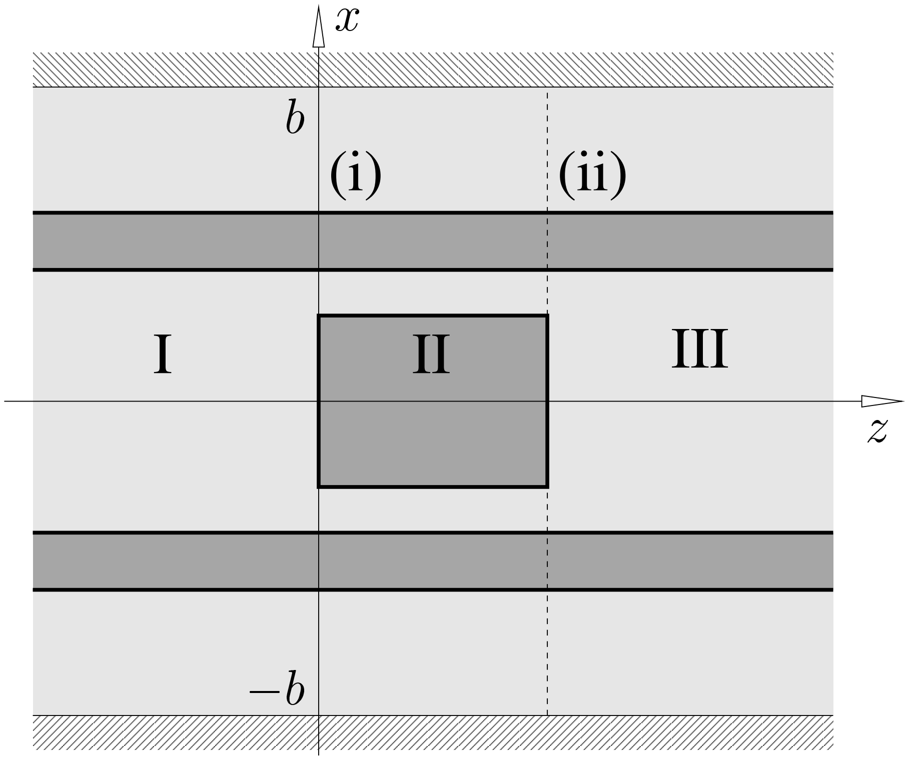
Two dielectric waveguides that are evanescently coupled to a square or rectangular region of increased refractive index can serve as a compact integrated optical microresonator. We consider these devices in a spatial two-dimensional setting, where a rigorous mode expansion technique enables accurate and quite efficient numerical simulations.

If properly dimensioned, a single microcavity shows a spectral response with a few somewhat irregular, narrow resonances. At resonance, one observes a standing electromagnetic wave of high intensity inside the cavity, that distributes the input power almost equally among the four waveguide ports. Two rectangular resonators in line at a suitable distance form a configuration that can be employed as a wavelength filter. Here the coupling of the single cavities via the adjacent waveguides leads to resonant field patterns that drop the input power into only one port.



$$P_{\mathrm{in}}(\lambda) = 1. \quad P_{\mathrm{A}},\, P_{\mathrm{B}},\, P_{\mathrm{C}},\, P_{\mathrm{D}} \;=\; ?$$

2 Mode expansion simulations



Modeling steps:

- Spatial 2D setting, fixed angular frequency ω / vacuum wavelength λ.
- Division into z-homogeneous segments I, II, III.
- Artificial boundaries $\to$ discretization of the mode spectrum.
- Mode expansion ansatz, $s = \mathrm{I}, \mathrm{II}, \mathrm{III}$:

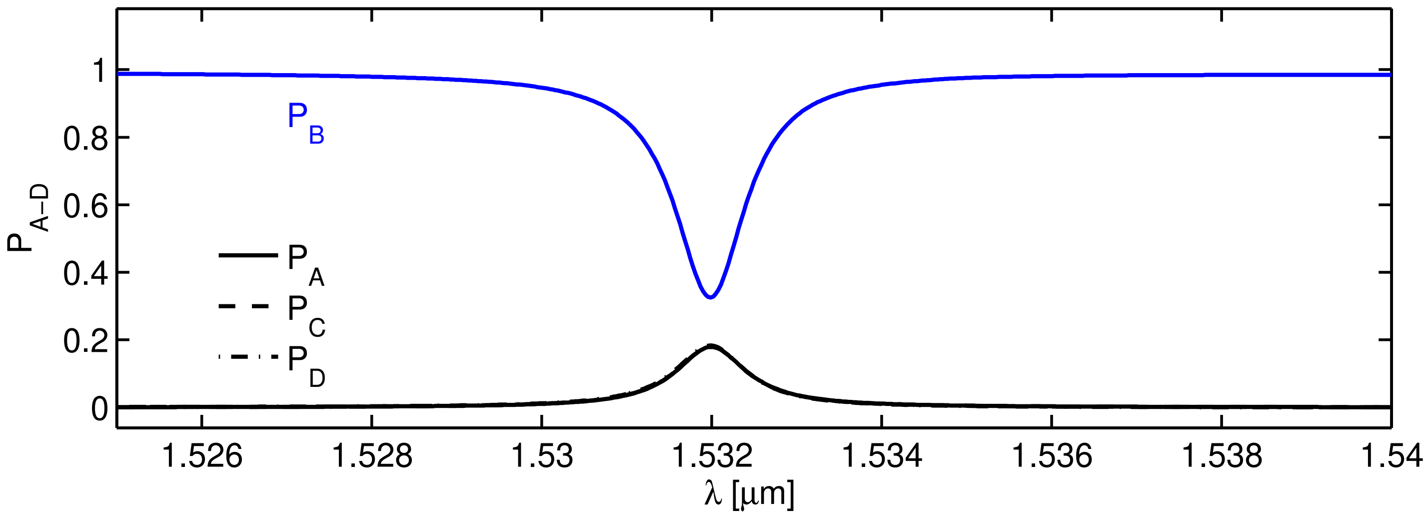
$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}^{s}(x,z,t) = \sum_m F_m^s \boldsymbol{\psi}_m^{s,\mathrm{f}}(x)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta_m^s z} + \sum_m B_m^s \boldsymbol{\psi}_m^{s,\mathrm{b}}(x)\, \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}\beta_m^s z}.$$

  $\{\boldsymbol{\psi}_m^{s,\mathrm{f}}, \boldsymbol{\psi}_m^{s,\mathrm{b}}\}$: forward + backward, propagating + evanescent terms.

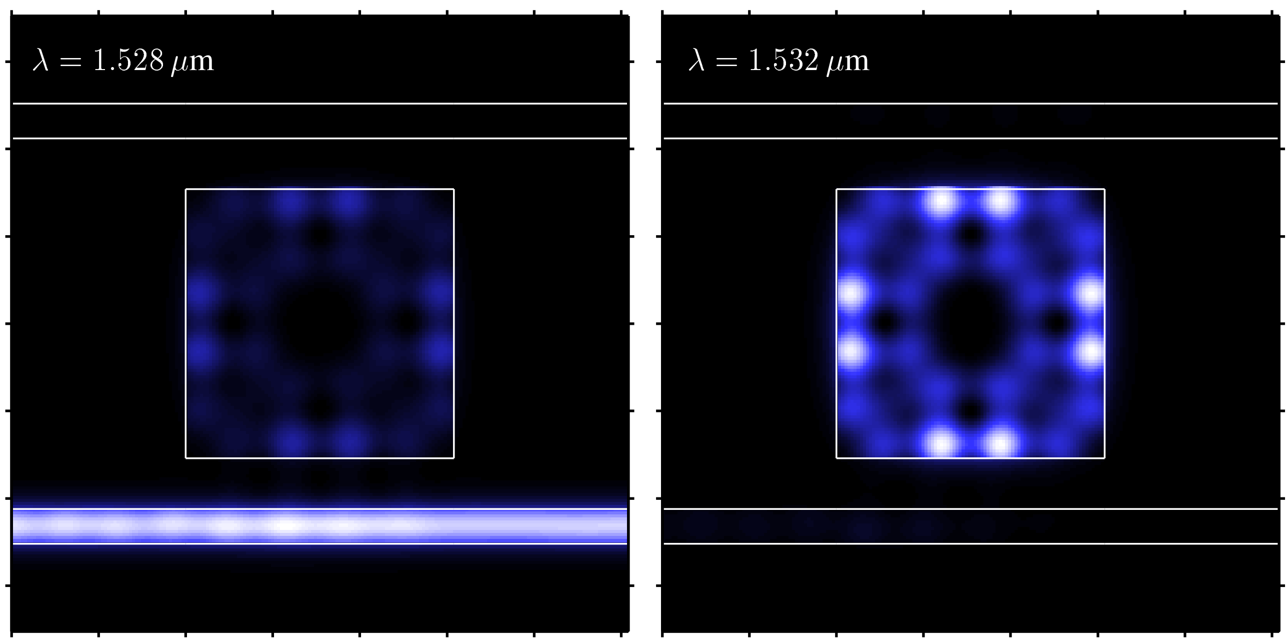
- Inner product for overlap computations + orthogonality relations
  $\to$ bidirectional field expansions at the junctions (i), (ii),
  $\to$ linear set of equations for $\{F_m^s, B_m^s\}$.
- $F_0^{\mathrm{I}} = 1, \quad F_j^{\mathrm{I}} = 0, j \neq 0, \quad B_m^{\mathrm{III}} = 0; \quad B_m^{\mathrm{I}}, F_m^{\mathrm{II}}, B_m^{\mathrm{II}}, F_m^{\mathrm{III}} = \ ?$

3 *Single cavity — Resonance*

Relative power transmission $P_{\text{A-D}}$ to ports A to D versus the vacuum wavelength λ of the input light in port A:



Time averaged electromagnetic energy density:

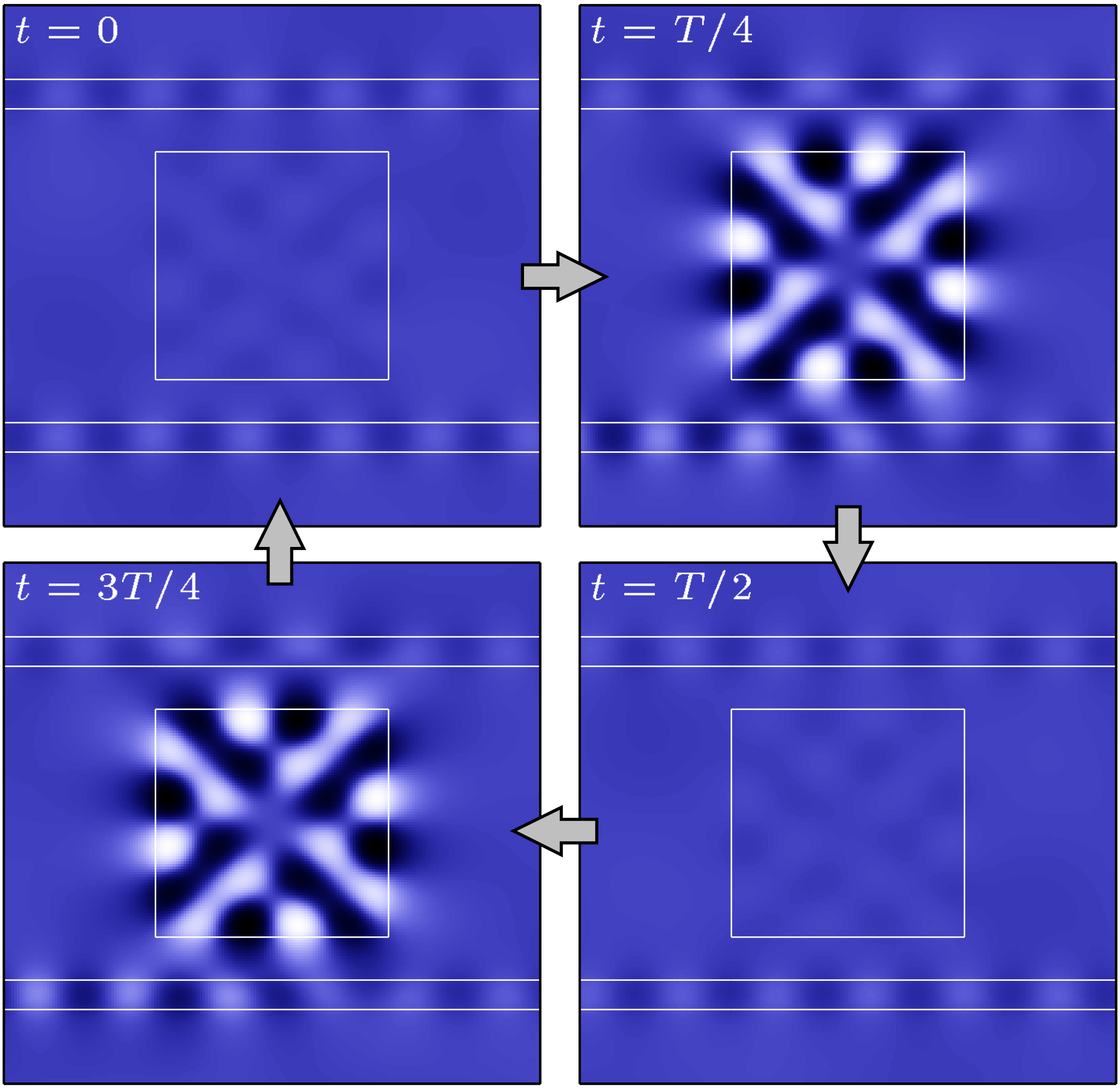


* $n_g = 3.2$, $n_b = 1.0$, $w = 0.20\,\mu$m, $g = 0.29\,\mu$m, $W = 1.54\,\mu$m, $L = 1.54\,\mu$m, $\lambda \in [1.3\,\mu\text{m}, 1.7\,\mu\text{m}]$, TE fields, $\boldsymbol{\mathcal{E}} = (0, E_y, 0)$.

* *C. Manolatou, M. J. Khan, S. Fan, P. R. Villeneuve, H. A. Haus, J. D. Joannopoulos, Coupling of Modes Analysis of Resonant Channel Add-Drop Filters, IEEE Journal of Quantum Electronics 35(9), 1322–1331, 1999*

4 *Single cavity — Field patterns at resonance*

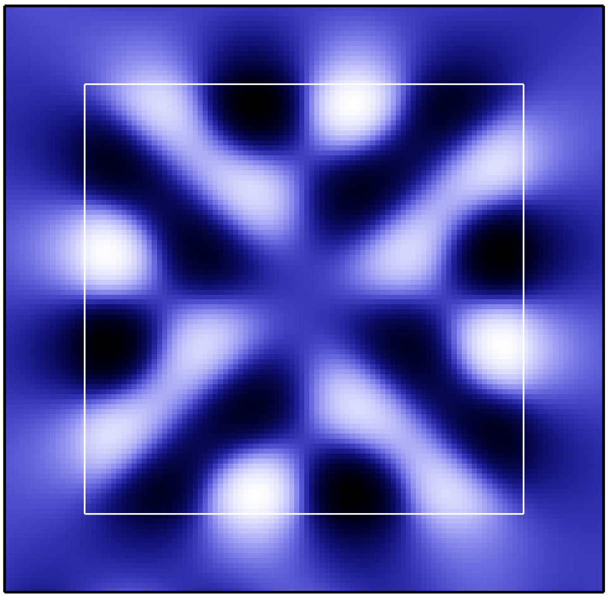
Time evolution of the electric field $E_y(x, z, t)$ in the square resonator:



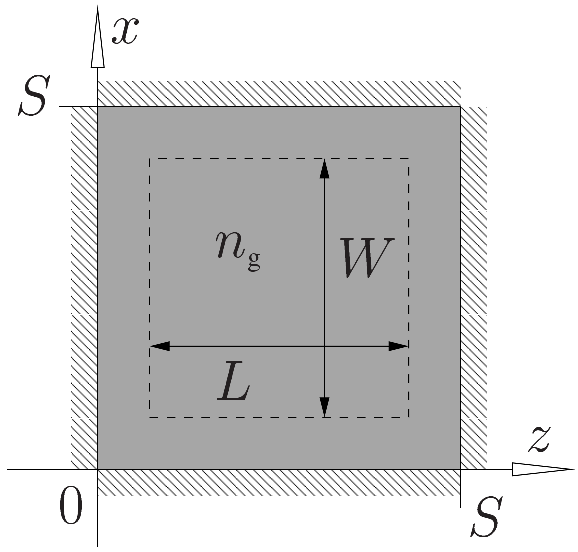
Snapshots over one time period $T = 5.11\,\mathrm{fs}$, $\lambda = 1.532\,\mu\mathrm{m}$, $f = 196\,\mathrm{THz}$.
Relative output power: $P_\mathrm{A} = P_\mathrm{C} = P_\mathrm{D} = 0.18$, $P_\mathrm{B} = 0.33$.

5 Single cavity — Pattern identification

Electric field around the cavity at resonance:



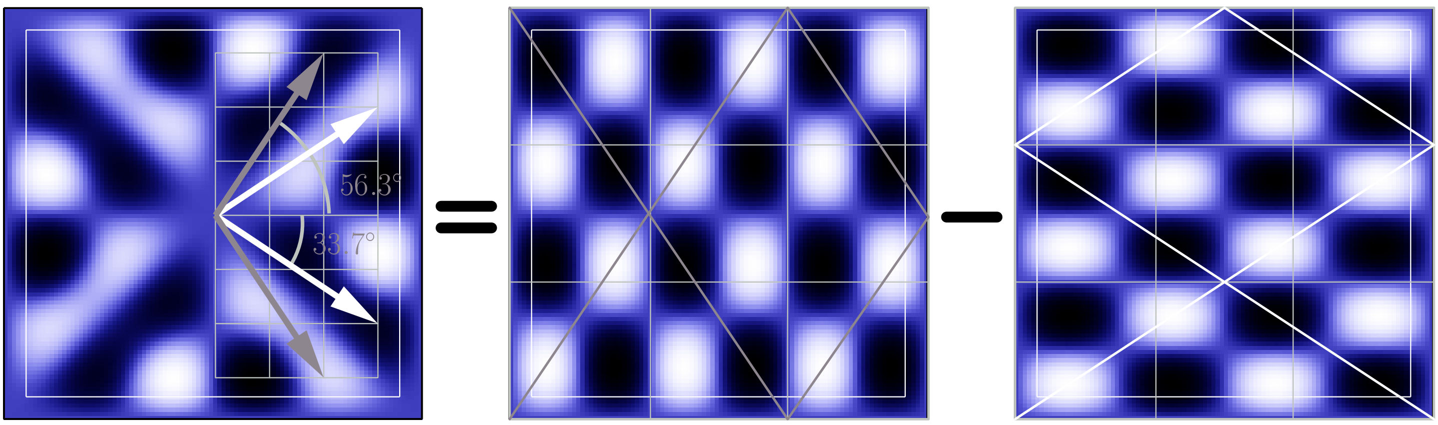
Compare with an eigenmode on a Dirichlet square:



Internal structure: $2:3$ - resonance.

The problem $\left\{ \begin{array}{l} \partial_x^2 E + \partial_z^2 E + k^2 n_g^2 E = 0, \quad k = 2\pi/\lambda, \\ E = 0 \;\text{ for }\; x = 0, S \;\text{ and }\; z = 0, S, \end{array} \right\}$ permits the solution

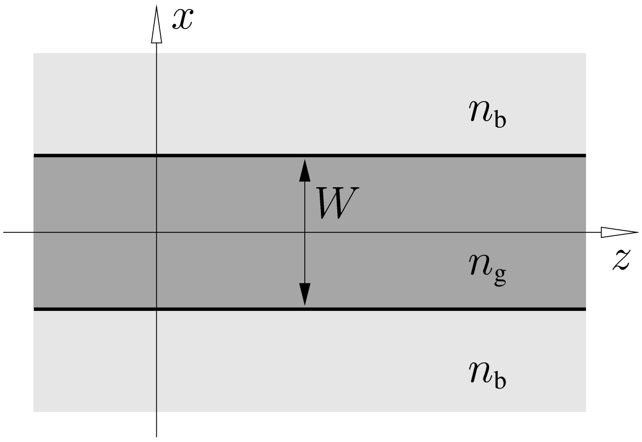
$$E(x,z) = E_0\,\{\sin(\beta_2 x)\sin(\beta_3 z) - \sin(\beta_3 x)\sin(\beta_2 z)\}, \quad \beta_m = m\frac{2\pi}{S}, \quad m = 2,3:$$

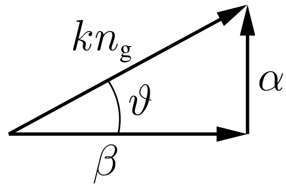


The resonance at $\lambda = 1.532\,\mu$m requires $S = (4^2+6^2)^{1/2}\lambda/2n_g = 1.726\,\mu$m.
Physical parameters: $L = W = 1.54\,\mu$m, $n_g = 3.2$.

6 A guided wave approach to the resonance

Consider a waveguide with the parameters of the resonant cavity:



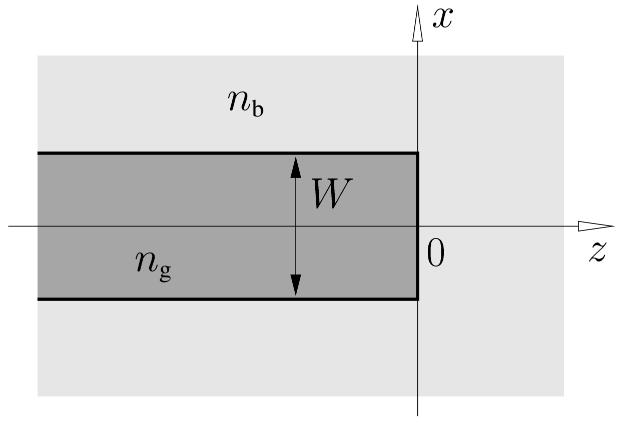


$\lambda = 1.532\,\mu$m, $W = 1.54\,\mu$m,
$n_g = 3.2$, $n_b = 1.0$.

Wavevector components of the guided modes:

	TE_0	TE_1	TE_2	TE_3	TE_4	TE_5	TE_6
$\beta/\,\mu m^{-1}$	13.0	12.6	12.0	10.9	9.4	7.3	4.3
$\alpha/\,\mu m^{-1}$	1.8	3.7	5.5	7.3	9.1	10.9	12.4
$\vartheta/°$	8.1	16.3	24.9	34.0	44.0	55.9	70.7

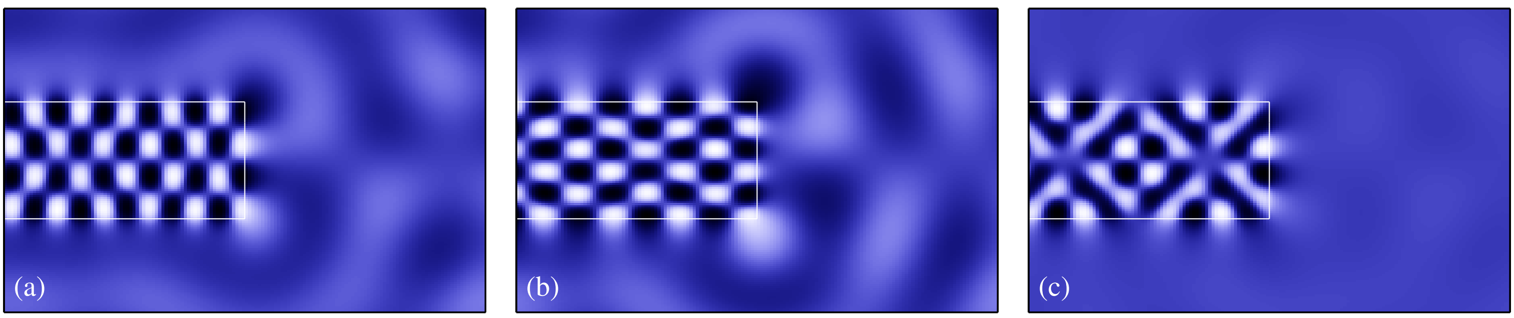
Resonator cavity:
A finite segment of this waveguide.



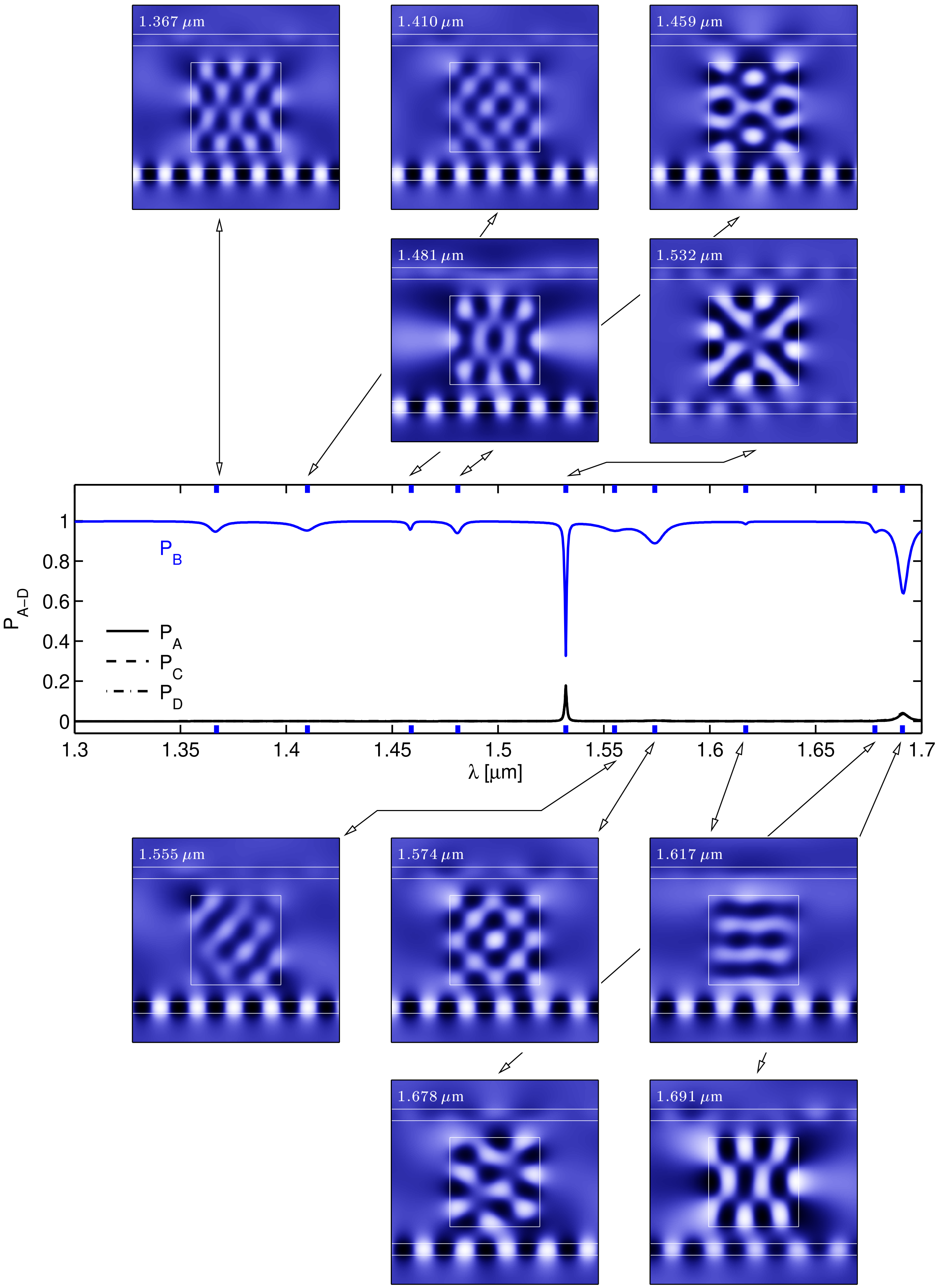
Guided wave reflection at the facet:

	Input	Relative power reflected into mode							$\sum$
		TE_0	TE_1	TE_2	TE_3	TE_4	TE_5	TE_6	
(a)	TE_3	0	1 %	0	80 %	0	3 %	0	84 %
(b)	TE_5	0	1 %	0	3 %	0	71 %	0	75 %
(c)	$a\,TE_3 + b\,TE_5$	0	0.1 %	0	61 %	0	38 %	0	99 %

$$\frac{|a|^2}{|b|^2} \approx \frac{61}{38}$$

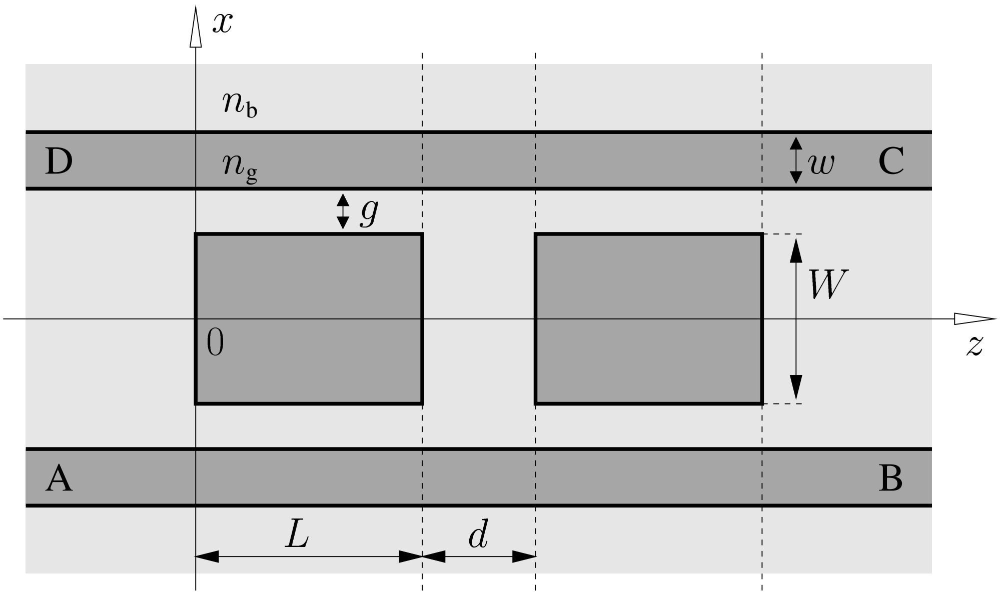


7 *Single cavity — More Resonances*



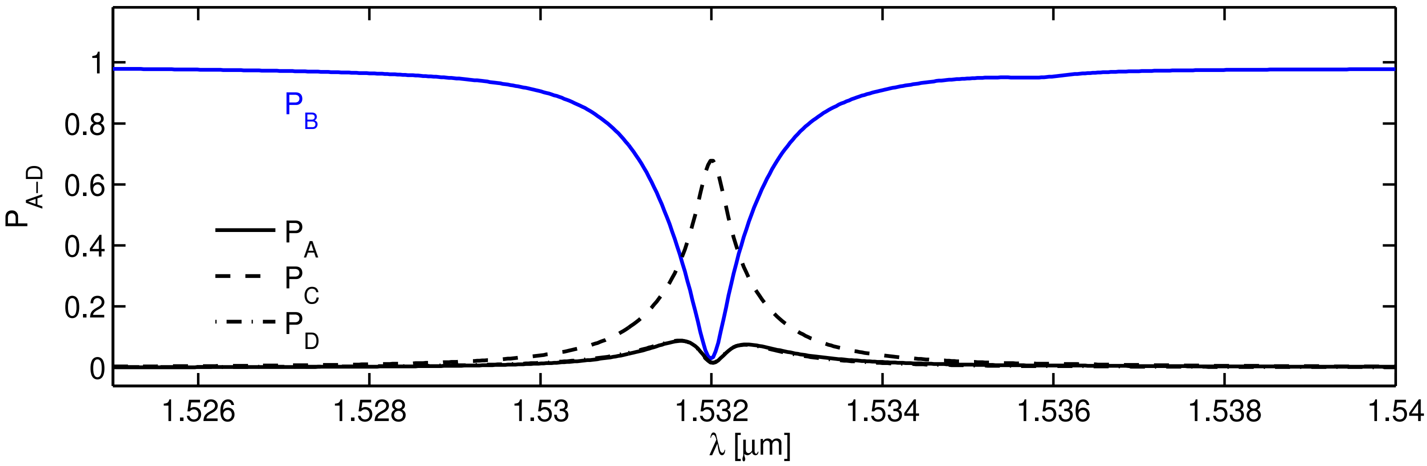
8 Filter — Spectral response

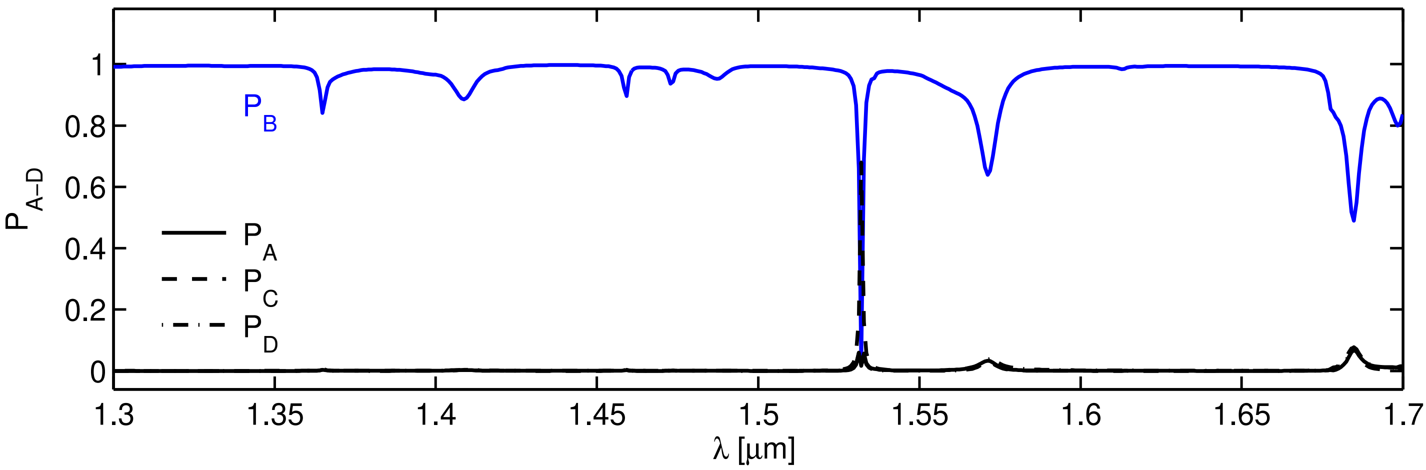
Two rectangular cavities in line — a wavelength filter:



$n_g = 3.2$,
$n_b = 1.0$,
$w = 0.20\,\mu$m,
$g = 0.29\,\mu$m,
$W = 1.54\,\mu$m,
$L = 1.54\,\mu$m,
$d = 0.72\,\mu$m,
$\boldsymbol{\mathcal{E}} = (0, E_y, 0)$.

Excitation in port A, relative power transmission to ports A to D:

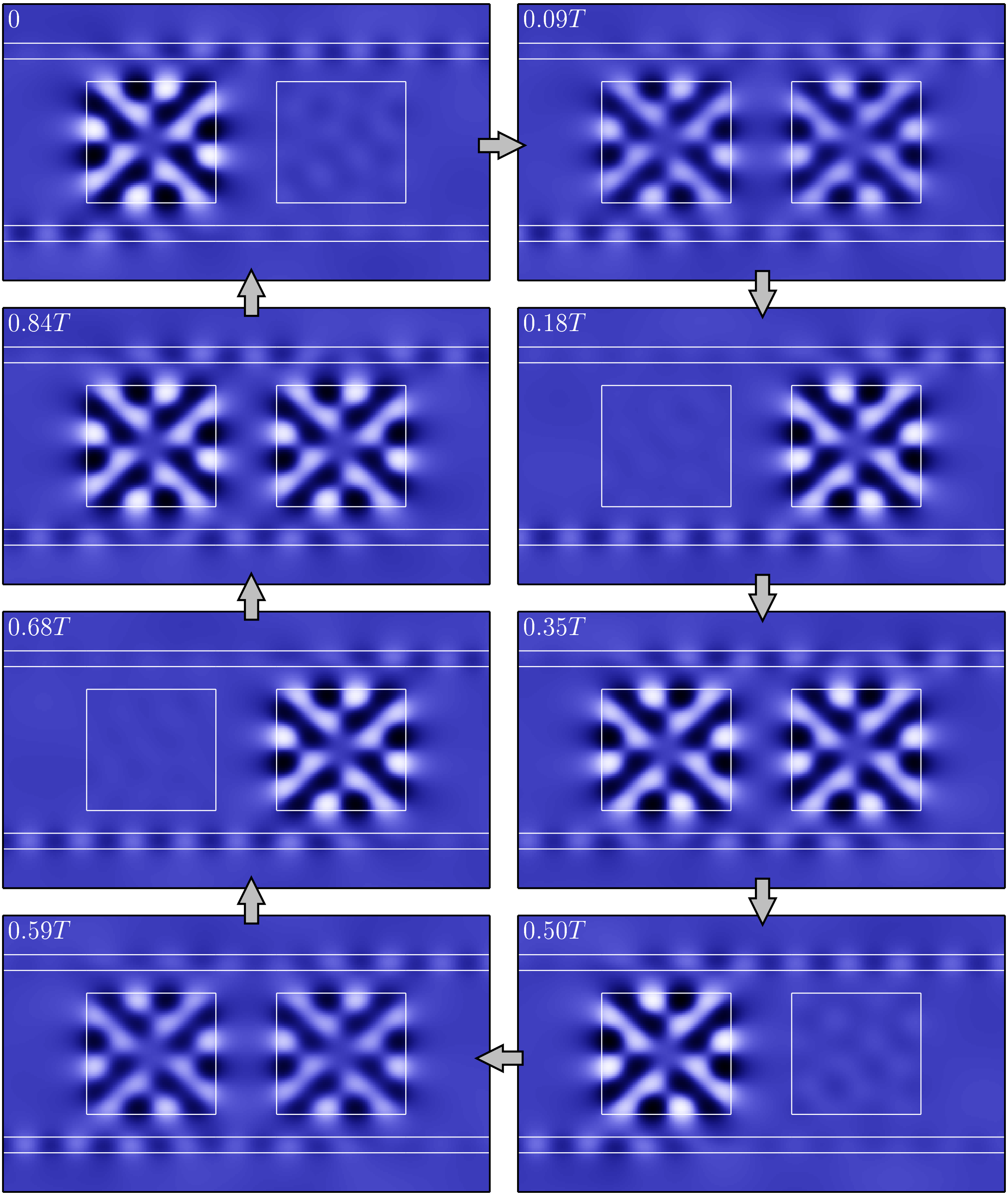




$\lambda = 1.532\,\mu$m: $P_A = 0.02$, $P_B = 0.03$, $P_C = 0.68$, $P_D = 0.02$.
FWHM $\approx 1\,$nm, FSR $\approx 40\,$nm $(150\,$nm$)$.

9 Filter — Resonant field patterns

Time evolution of the electric field $E_y(x, z, t)$ in the filter device:



Snapshots over one time period $T = 5.11\,\mathrm{fs}$, $\lambda = 1.532\,\mu\mathrm{m}$, $f = 196\,\mathrm{THz}$.