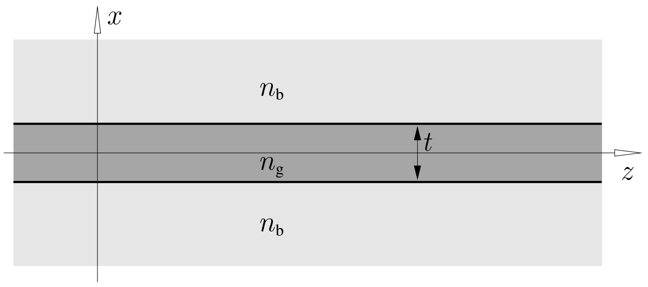
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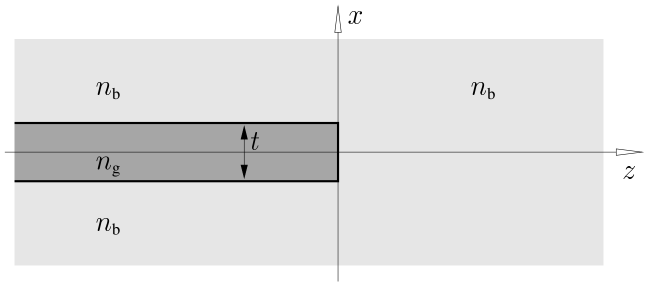
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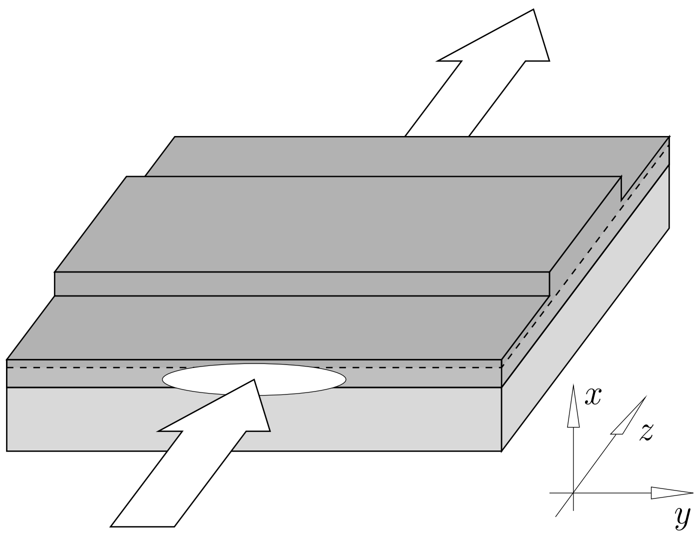
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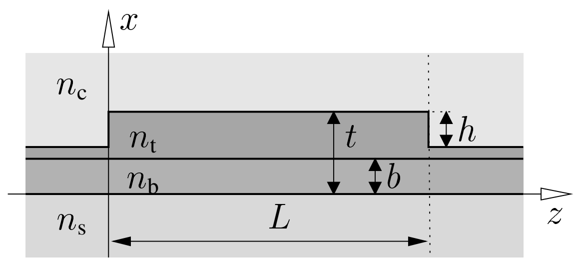
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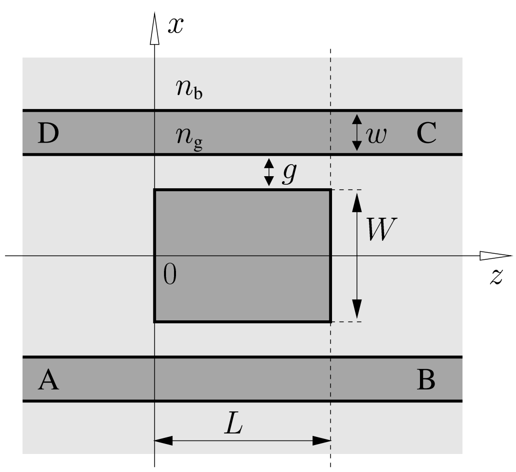
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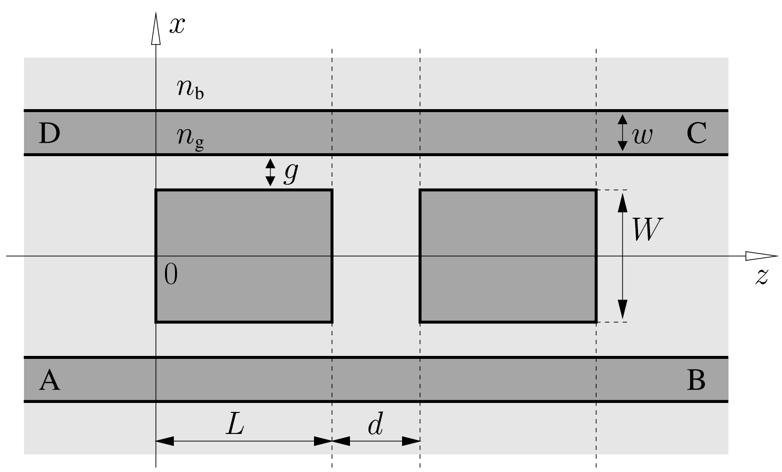
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d

Modeling of planar photonic devices by mode expansion techniques

*Manfred Hammer**
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- Mode expansion simulations
- Cross strip interferometer
  - Operation principle
  - Polarizer device
  - Magnetooptic isolator
- Rectangular microresonators
  - Waveguide end facet
  - Single cavity devices
  - Wavelength filter

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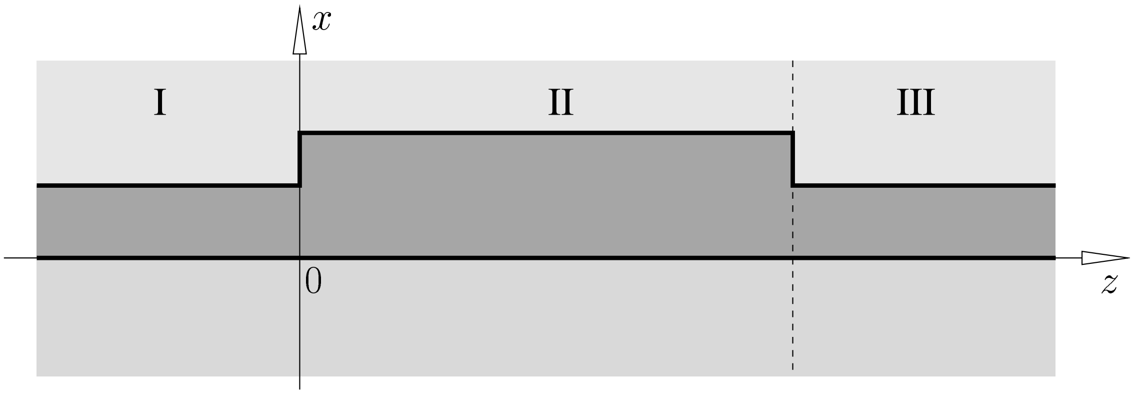
A segmented waveguide structure:



Given: Optical permittivity $\epsilon(x,z) = \epsilon_s(x)$ if $z \in$ segment s,
$+$ (guided wave) input.

Unknown: Optical output.

- Frequency domain: Time dependence $\sim\ \mathrm{e}^{\mathrm{i}\omega t}$.
- Uncharged, linear, isotropic, nonmagnetic media.
- Restriction to two spatial dimensions: $\partial_y = 0$.
- Maxwell equations for the optical field
  curl $\bar{\boldsymbol{E}} = -\mathrm{i}\omega\mu_0\bar{\boldsymbol{H}}$, div $\bar{\boldsymbol{H}} = 0$, curl $\bar{\boldsymbol{H}} = \mathrm{i}\omega\epsilon_0\epsilon\bar{\boldsymbol{E}}$, div $\epsilon\bar{\boldsymbol{E}} = 0$
  split into two decoupled sets ...

$k = \omega\sqrt{\mu_0\epsilon_0} = \omega/c = 2\pi/\lambda,$

TE

$$\bar{\boldsymbol{E}}(x,z) = \begin{pmatrix} 0 \\ \bar{E}_y \\ 0 \end{pmatrix}(x,z),$$

$$\bar{\boldsymbol{H}}(x,z) = \frac{\mathrm{i}}{\omega\mu_0}\begin{pmatrix} -\partial_z\bar{E}_y \\ 0 \\ \partial_x\bar{E}_y \end{pmatrix}(x,z),$$

$$(\partial_x^2 + \partial_z^2 + k^2\epsilon)\bar{E}_y = 0,$$

TM

$$\bar{\boldsymbol{E}}(x,z) = \frac{\mathrm{i}}{\omega\epsilon_0\epsilon}\begin{pmatrix} \partial_z\bar{H}_y \\ 0 \\ -\partial_x\bar{H}_y \end{pmatrix}(x,z),$$

$$\bar{\boldsymbol{H}}(x,z) = \begin{pmatrix} 0 \\ \bar{H}_y \\ 0 \end{pmatrix}(x,z),$$

$$(\epsilon\partial_x\frac{1}{\epsilon}\partial_x + \epsilon\partial_z\frac{1}{\epsilon}\partial_z + k^2\epsilon)\bar{H}_y = 0.$$

- z-invariant segments $\epsilon(x) = n^2(x)$ $\rightarrow$ mode ansatz:

TE:

$$\bar{E}_y(x,z) = E_y(x)\,\mathrm{e}^{-\mathrm{i}\beta z}$$

$$(\partial_x^2 + k^2\epsilon)E_y = \beta^2 E_y$$

TM:

$$\bar{H}_y(x,z) = H_y(x)\,\mathrm{e}^{-\mathrm{i}\beta z}$$

$$(\epsilon\partial_x\frac{1}{\epsilon}\partial_x + k^2\epsilon)H_y = \beta^2 H_y$$

- Eigenvalue problems for β and E_y, H_y
  $\rightarrow$ propagation constants β_m, mode profiles $\boldsymbol{\psi}_m = (\boldsymbol{E}_m, \boldsymbol{H}_m)$:

TE:

$$\boldsymbol{E}(x) = \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix}(x),$$

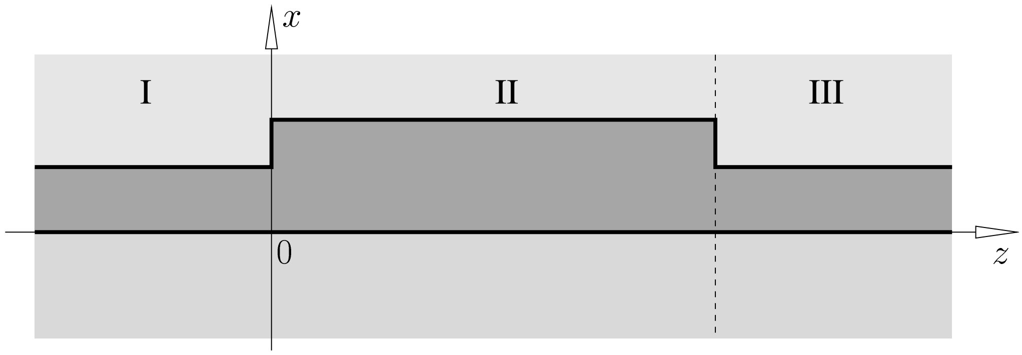
$$\boldsymbol{H}(x) = \frac{1}{\omega\mu_0}\begin{pmatrix} -\beta E_y \\ 0 \\ \mathrm{i}\partial_x E_y \end{pmatrix}(x),$$

TM:

$$\boldsymbol{E}(x) = \frac{1}{\omega\epsilon_0\epsilon}\begin{pmatrix} \beta H_y \\ 0 \\ -\mathrm{i}\partial_x H_y \end{pmatrix}(x),$$

$$\boldsymbol{H}(x) = \begin{pmatrix} 0 \\ H_y \\ 0 \end{pmatrix}(x).$$

- Vertically unbounded setting:

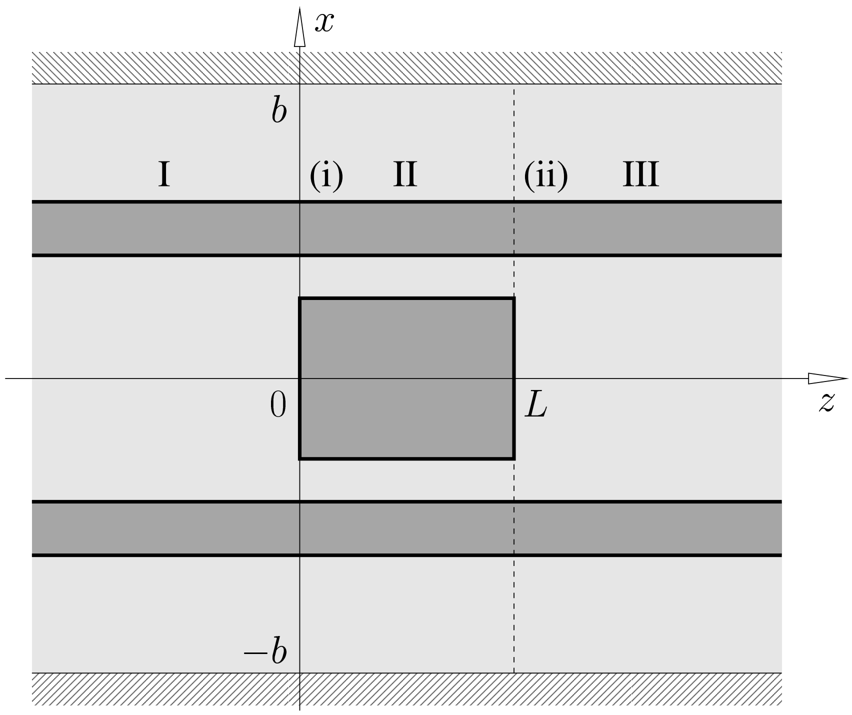


Condition: $\int |E_y(x)|^2 \,\mathrm{d}x < \infty, \quad \int \frac{1}{\epsilon} |H_y(x)|^2 \,\mathrm{d}x < \infty,$

a few confined solutions on each segment.

$\leftrightarrow$ Modeling of guided wave propagation.

- Vertically bounded setting:

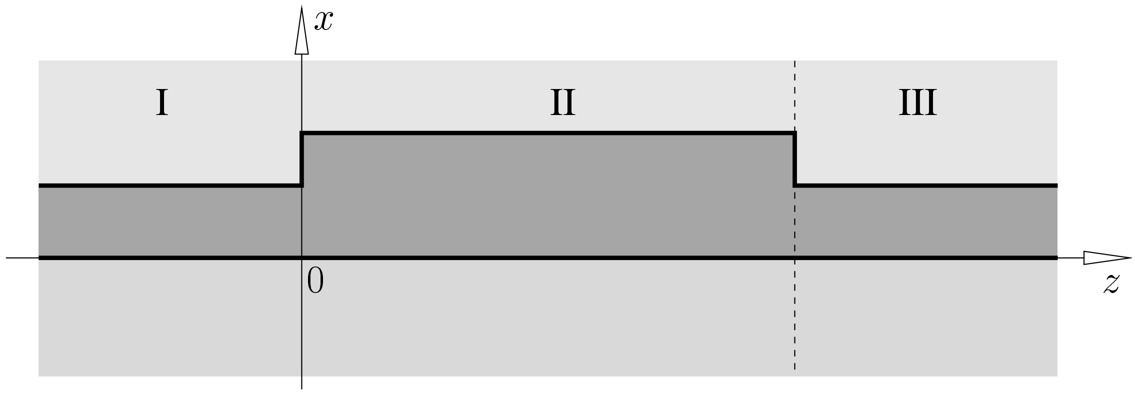


Artificial boundaries at $x = \pm b$: $E_y(\pm b) = 0$ or $H_y(\pm b) = 0$,

a large number of terms per segment,

approximations for guided and nonguided parts of the optical field.

$\leftrightarrow$ Rigorous mode expansion simulations.



- Total electromagnetic field on each segment $s = \mathrm{I}, \mathrm{II}, \mathrm{III}$:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}^{s}(x, z, t) = \sum_m F_m^s \boldsymbol{\psi}_m^{s,\mathrm{f}}(x)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta_m^s z} + \sum_m B_m^s \boldsymbol{\psi}_m^{s,\mathrm{b}}(x)\, \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}\beta_m^s z}.$$

$\boldsymbol{\psi}_m^{s,\mathrm{f}}, \boldsymbol{\psi}_m^{s,\mathrm{b}}$: Forward + backward, propagating + evanescent terms.

field type	forward	backward
propagating, $\beta^2 > 0$	$\sim\ \mathrm{e}^{-\mathrm{i}\lvert\beta\rvert z}$	$\sim\ \mathrm{e}^{+\mathrm{i}\lvert\beta\rvert z}$
evanescent, $\beta^2 < 0$	$\sim\ \mathrm{e}^{-\lvert\beta\rvert z}$	$\sim\ \mathrm{e}^{+\lvert\beta\rvert z}$

- Inner product for two fields $\boldsymbol{\psi} = (\boldsymbol{E}_1, \boldsymbol{H}_1)$ and $\boldsymbol{\phi} = (\boldsymbol{E}_2, \boldsymbol{H}_2)$:

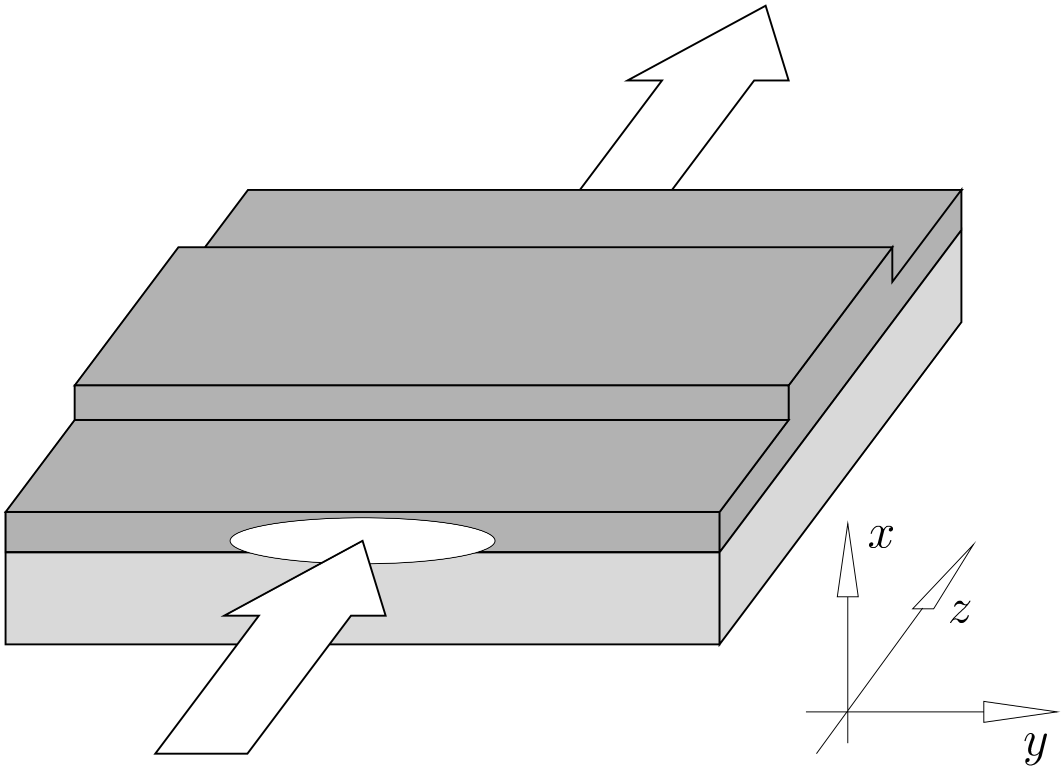
$$(\boldsymbol{\psi}; \boldsymbol{\phi}) = \frac{1}{4} \int (E_{1x}^* H_{2y} - E_{1y}^* H_{2x} + E_{2x} H_{1y}^* - E_{2y} H_{1x}^*)\, \mathrm{d}x\,.$$

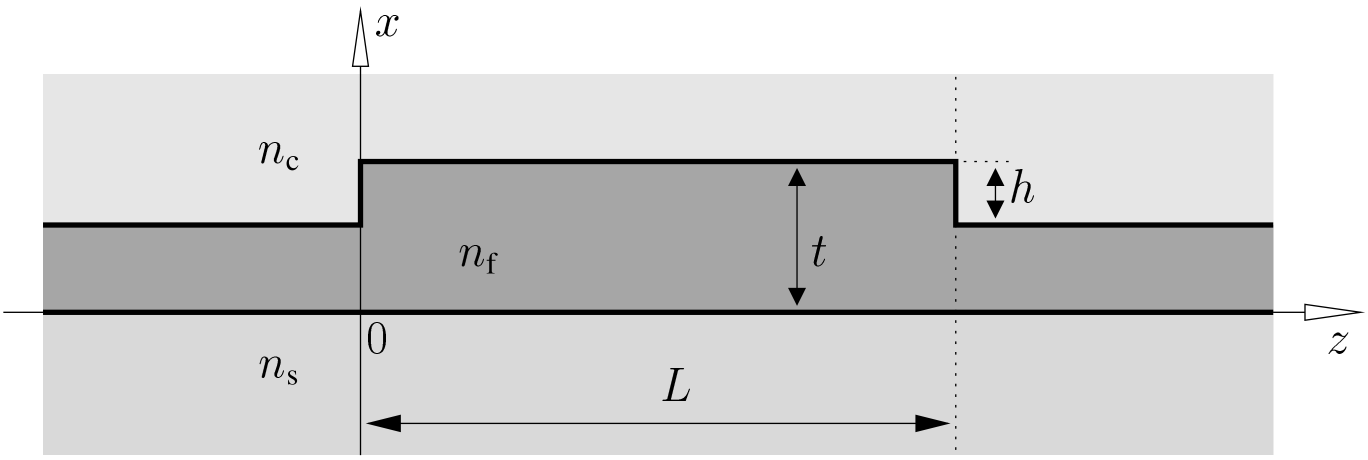
$\leftrightarrow$ Orthogonality relations for the basis modes.

$\rightarrow$ Bidirectional field expansions at the junctions I/II, II/III.

$\rightarrow$ Linear set of equations in $\{F_m^s, B_m^s\}$.

F_m^{I} given, $B_m^{\mathrm{III}} = 0$: $B_m^{\mathrm{I}}, F_m^{\mathrm{II}}, B_m^{\mathrm{II}}, F_m^{\mathrm{III}} = ?$


x
z
y


x
n_c
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L
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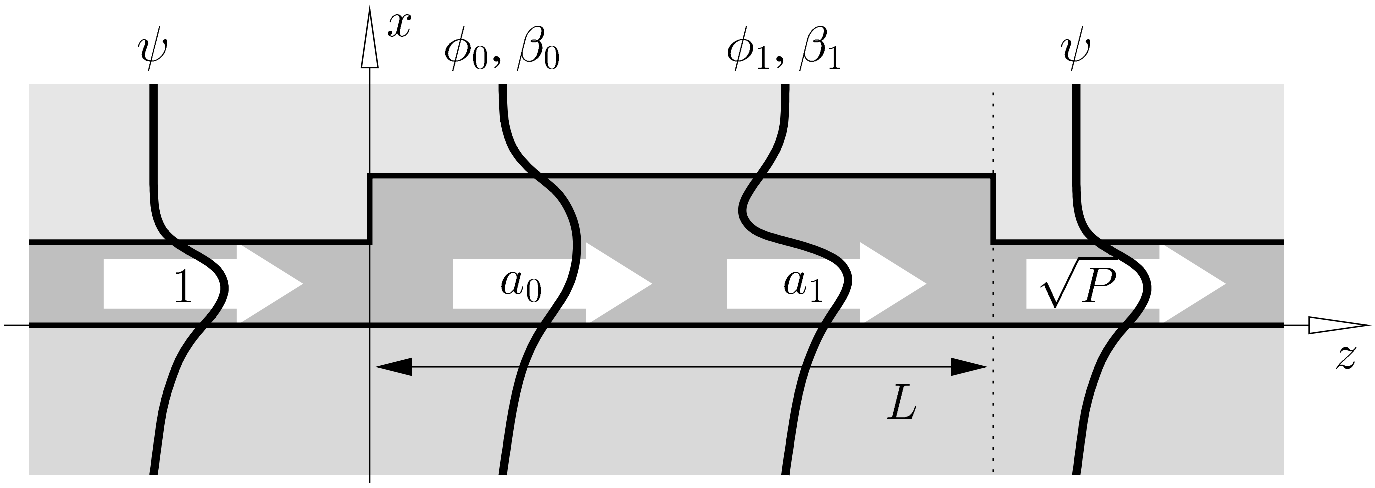


Ingredients:

ψ:	input / output mode field,
ϕ_0, ϕ_1:	fundamental and first order mode of the strip segment,
β_0, β_1:	the corresponding propagation constants,
$(\,.\, , \,.\,)$:	a suitable bilinear product.

Relative power transfer:

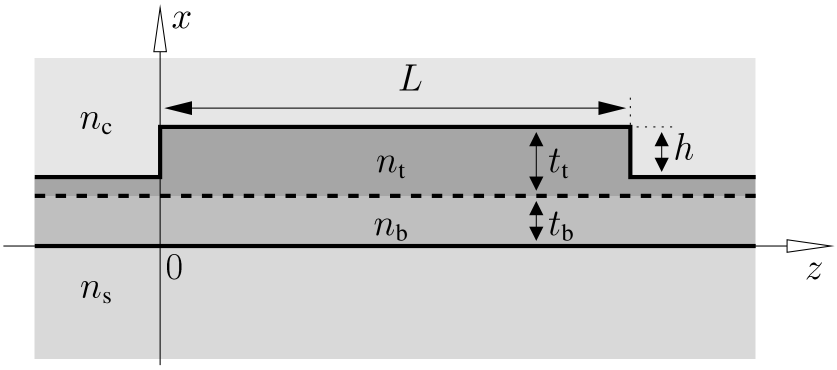
$$P(L) = w_0^2 + w_1^2 + 2w_0w_1 \cos(\pi L/L_\mathrm{c})\,,$$

with $\quad a_j = (\psi, \phi_j)\,, \quad w_j = a_j^2\,, \quad L_\mathrm{c} = \dfrac{\pi}{\beta_0 - \beta_1}\,.$

L: the length of the thick section.

Modulation range of the transmission:

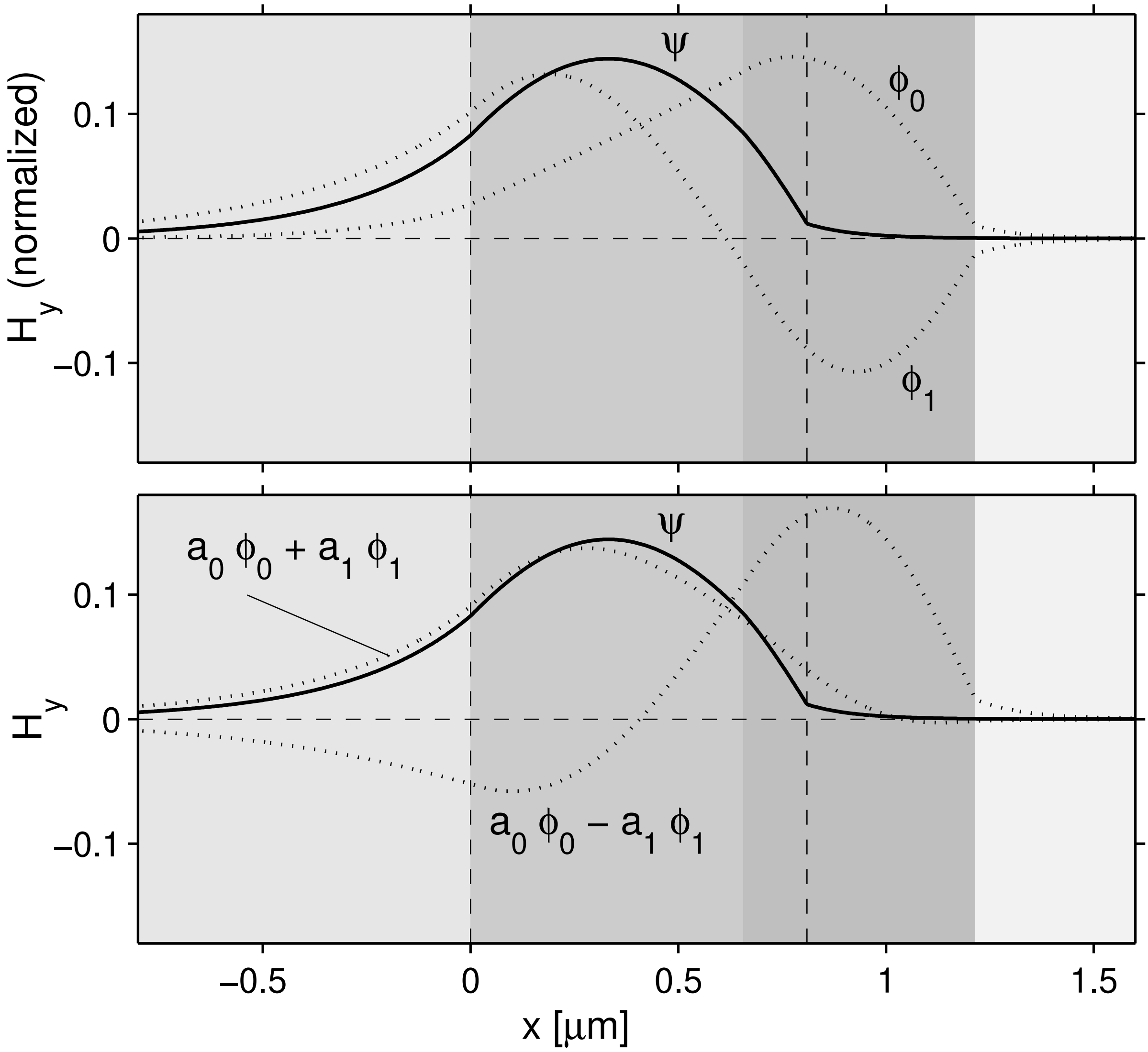
$$(w_0 - w_1)^2 \leq P \leq (w_0 + w_1)^2\,.$$



$t_b = 0.656\,\mu$m, $t_t = 0.559\,\mu$m,
$h = 0.405\,\mu$m, $\lambda = 1.3\,\mu$m,
$n_s = 1.950$, $n_b = 2.157$,
$n_t = 2.275$, $n_c = 1.0$.

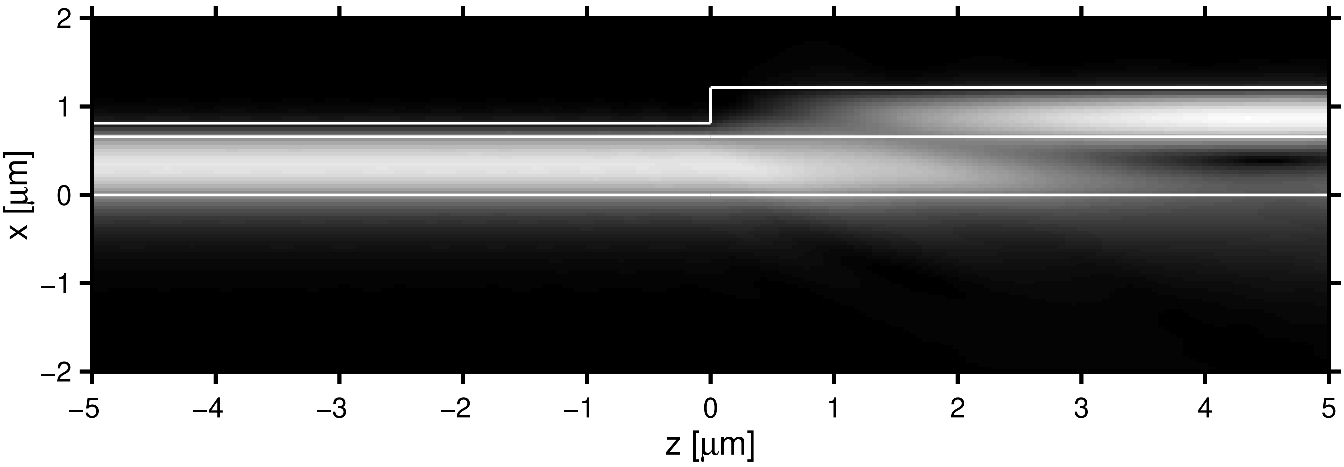
$$(w_0 - w_1)^2 \leq P \leq (w_0 + w_1)^2, \quad w_0 = (\psi, \phi_0)^2, \quad w_1 = (\psi, \phi_1)^2.$$

Adjust $t_b + t_t$ and h for $w_0 = w_1$ and maximum $(w_0 + w_1)^2$:

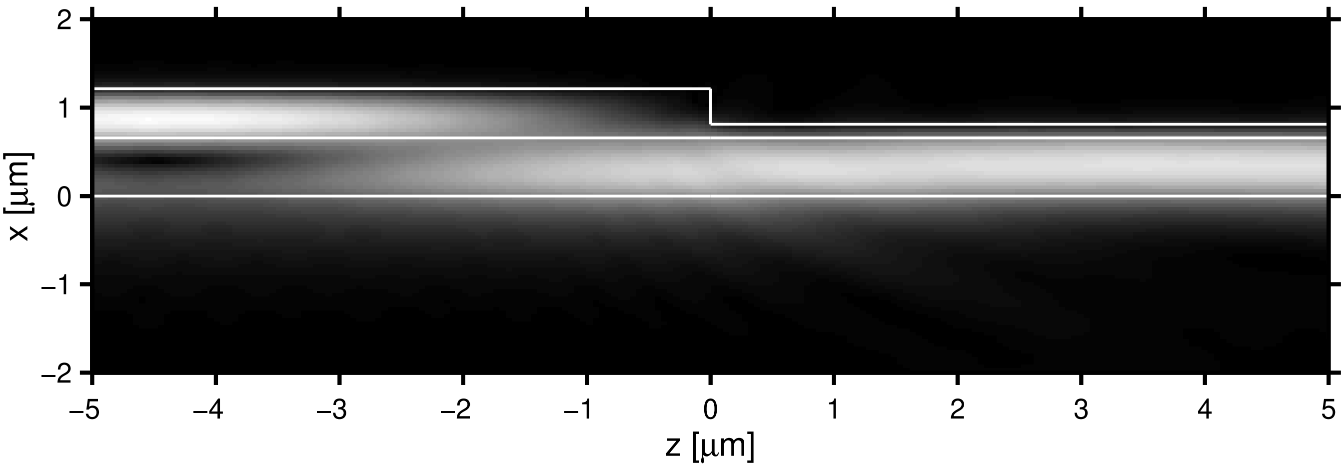


$(\phi_0 + \phi_1)/\sqrt{2} \approx \psi$, $\qquad$ $(\phi_0 - \phi_1)/\sqrt{2} \perp \psi$.

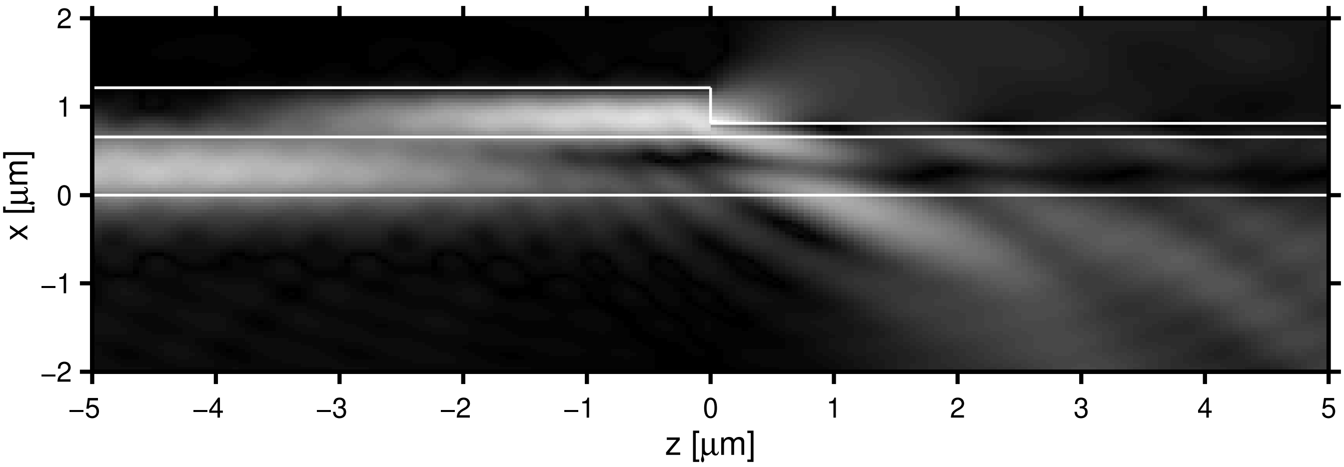
Low losses / reflections at the input junction:

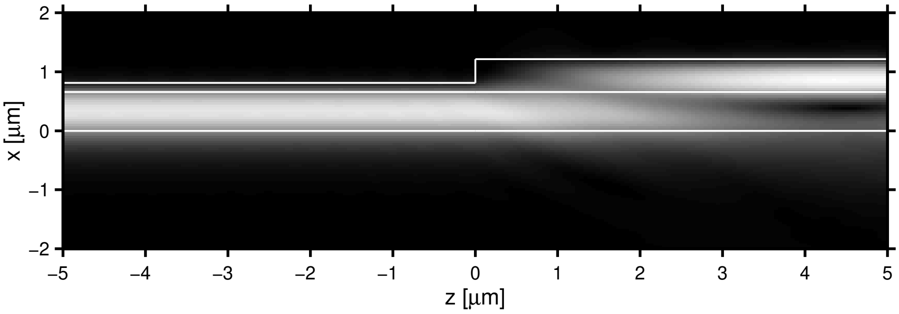


Complete constructive . . .



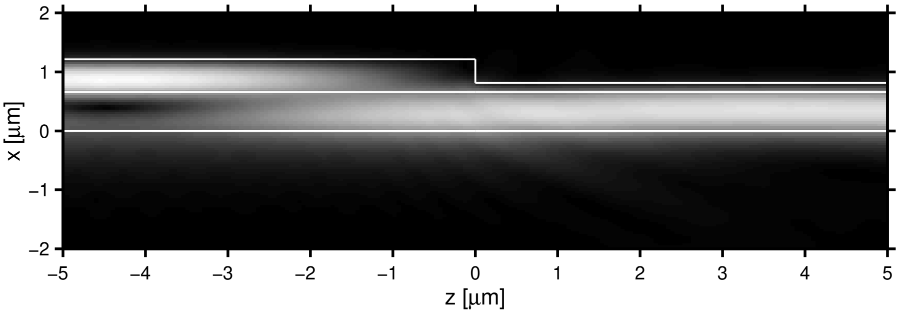
. . . or destructive interference at the output junction:





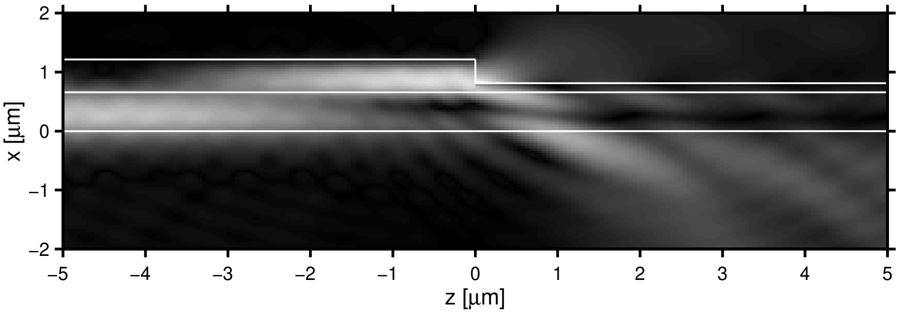
Input: ψ

	$\psi, \rightarrow$	$\psi, \leftarrow$	$\phi_0, \rightarrow$	$\phi_0, \leftarrow$	$\phi_1, \rightarrow$	$\phi_1, \leftarrow$
ME	1	$1.1 \cdot 10^{-6}$	0.49	0	0.50	0
FDTD	1	$3.0 \cdot 10^{-5}$	0.48	0	0.51	0



Input: $(\phi_0 + \phi_1)/\sqrt{2}$

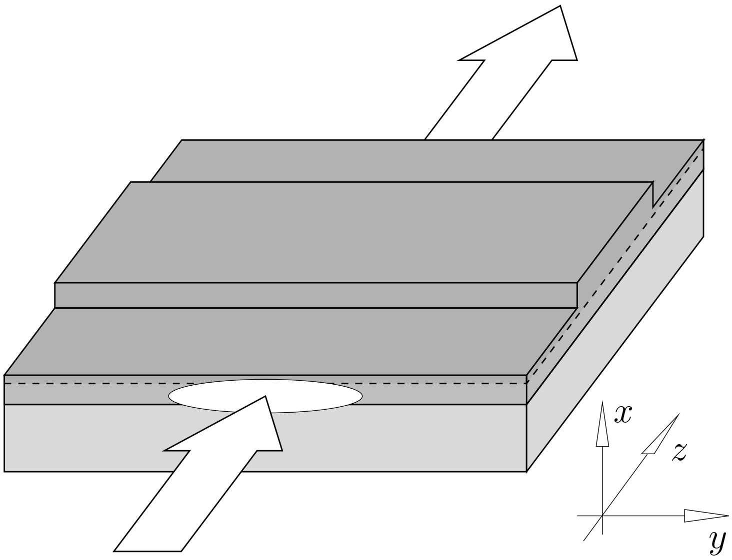
	$\phi_0, \rightarrow$	$\phi_0, \leftarrow$	$\phi_1, \rightarrow$	$\phi_1, \leftarrow$	$\psi, \rightarrow$	$\psi, \leftarrow$
ME	$1/2$	$2.1 \cdot 10^{-4}$	$1/2$	$1.1 \cdot 10^{-4}$	0.99	0
FDTD	$1/2$	$5.0 \cdot 10^{-4}$	$1/2$	$3.0 \cdot 10^{-4}$	0.99	0

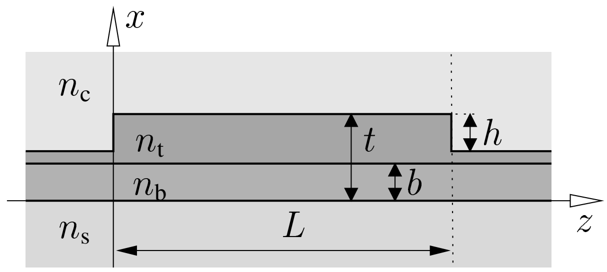


Input: $(\phi_0 - \phi_1)/\sqrt{2}$

	$\phi_0, \rightarrow$	$\phi_0, \leftarrow$	$\phi_1, \rightarrow$	$\phi_1, \leftarrow$	$\psi, \rightarrow$	$\psi, \leftarrow$
ME	$1/2$	$2.0 \cdot 10^{-2}$	$1/2$	$1.3 \cdot 10^{-2}$	$6.5 \cdot 10^{-5}$	0
FDTD	$1/2$	$3.6 \cdot 10^{-2}$	$1/2$	$2.8 \cdot 10^{-2}$	$9.0 \cdot 10^{-4}$	0

$L_\mathrm{c}^\mathrm{TE} \neq L_\mathrm{c}^\mathrm{TM}$: Adjust $L = 2m\, L_\mathrm{c}^\mathrm{TE} = (2m+1) L_\mathrm{c}^\mathrm{TM}$ with natural m.

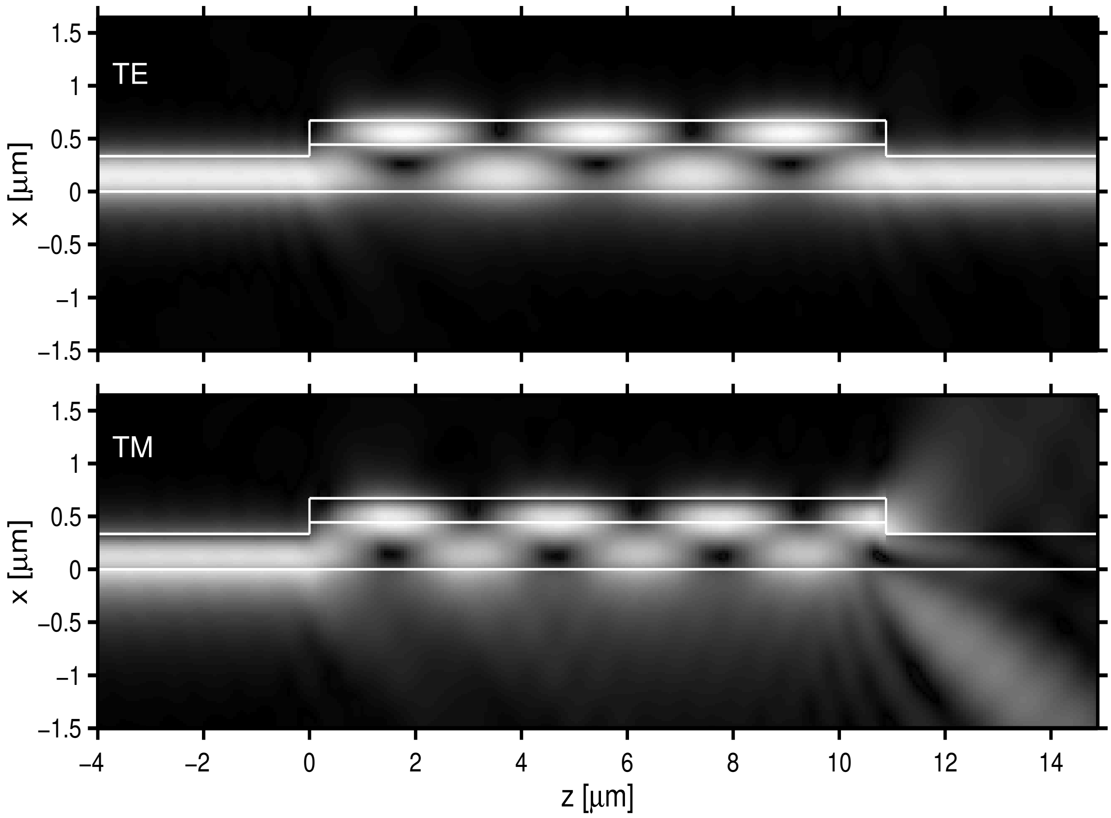




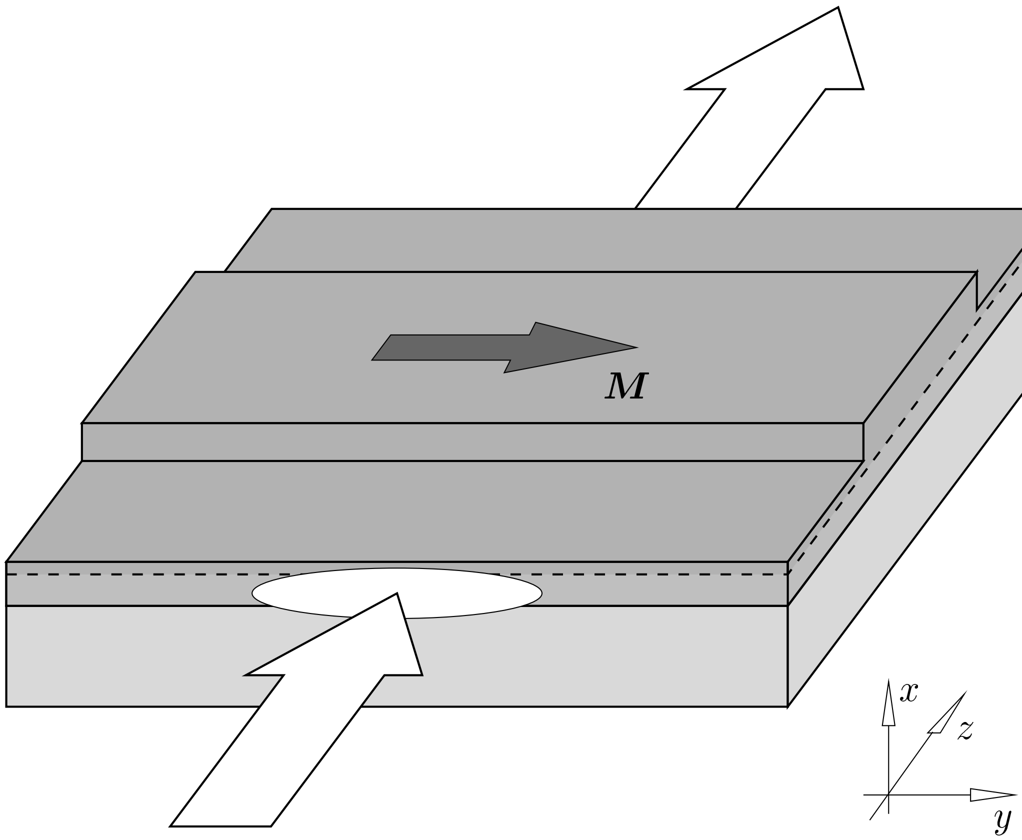
ER $= 10\log_{10} P^\mathrm{TE}/P^\mathrm{TM}$, LO $= -10\log_{10} P^\mathrm{TE}$, $q_0 - \Delta q \leq q \leq q_0 + \Delta q$: ER $\geq 20\,$dB, LO $\leq 1\,$dB.

	t	b	h	L	λ	n_s	n_b	n_t	n_c
q_0	$0.673\,\mu$m	$0.444\,\mu$m	$0.339\,\mu$m	$10.882\,\mu$m	$1.3\,\mu$m	1.465	2.01	2.3	1.0
Δq	26 nm	9 nm	18 nm	100 nm	20 nm	0.007	0.009	0.024	0.08

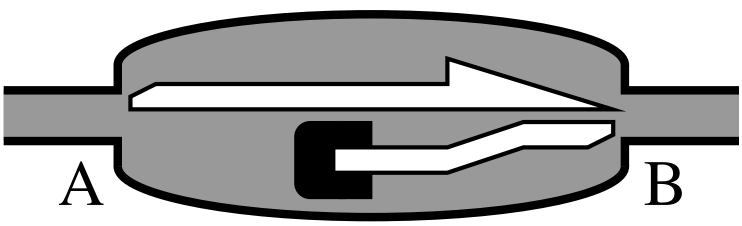
$P^\mathrm{TE} = 97\%$, $P^\mathrm{TM} = 0.09\%$, ER $= 30\,$dB, LO $= 0.1\,$dB, $L = 6L_\mathrm{c}^\mathrm{TE} = 7L_\mathrm{c}^\mathrm{TM}$.

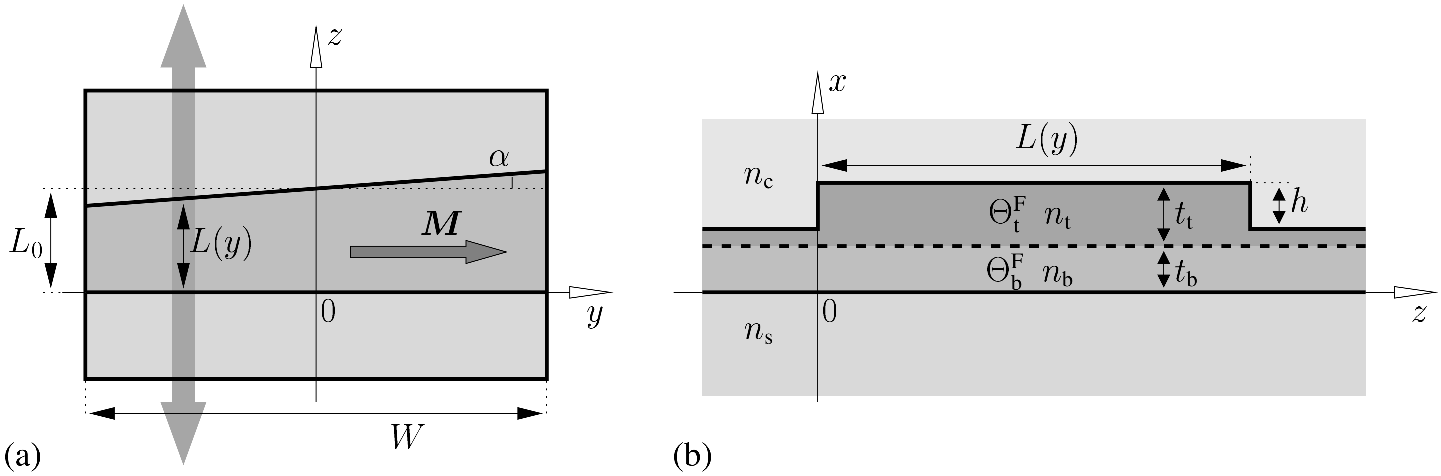


A strip made of magnetooptic garnet films . . .



. . . can behave like an optical isolator:





Magnetooptic garnet materials, permittivity:

$$\hat{\epsilon} = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -\mathrm{i}\xi \\ 0 & 0 & 0 \\ \mathrm{i}\xi & 0 & 0 \end{pmatrix}, \qquad \xi = n\lambda\Theta^{\mathrm{F}}/\pi\,,$$

$W \approx 2\,\mathrm{cm}$, $\alpha \approx 0.05°$ Θ^{F}: specific Faraday rotation.

$$\boxed{\text{TM}} \qquad \left.\overrightarrow{\beta_0^{\mathrm{f}}, \beta_1^{\mathrm{f}}, L_{\mathrm{c}}^{\mathrm{f}}, P^{\mathrm{f}}}\right\} \neq \left\{\overleftarrow{\beta_0^{\mathrm{b}}, \beta_1^{\mathrm{b}}, L_{\mathrm{c}}^{\mathrm{b}}, P^{\mathrm{b}}}\right.$$

Isolator performance for $L = 2mL_{\mathrm{c}}^{\mathrm{f}} = (2m+1)L_{\mathrm{c}}^{\mathrm{b}}$ with natural m.

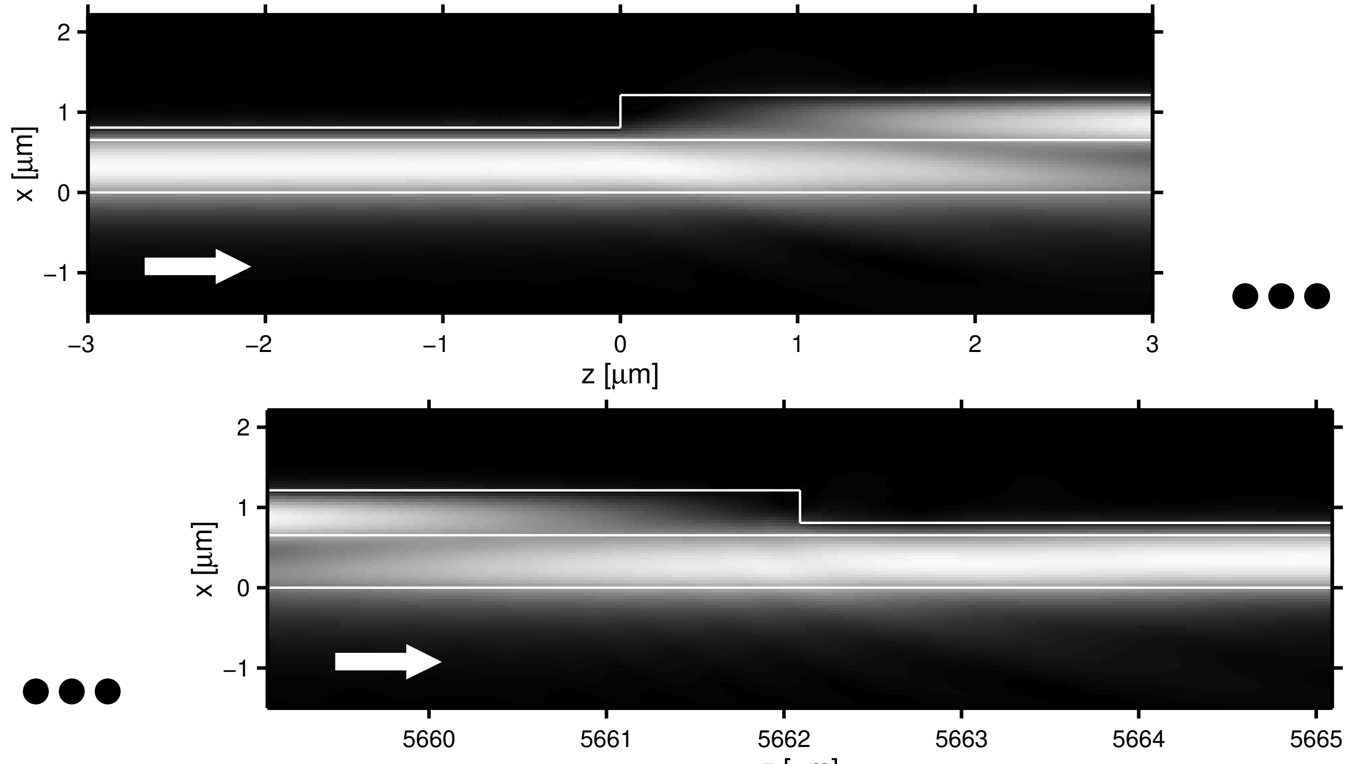
	q_0	Δq
t_{b}	$0.656\,\mu$m	$\geq 0.2\,\mu$m
t_{t}	$0.559\,\mu$m	$0.16\,\mu$m
h	$0.405\,\mu$m	$30\,$nm
L_0	$5.663\,$mm	$1.6\,$mm
λ	$1.3\,\mu$m	$\geq 90\,$nm

	q_0	Δq
n_{s}	1.950	0.1
n_{b}	2.157	0.02
n_{t}	2.275	0.02
n_{c}	1.0	≥ 0.4
$\Theta_{\mathrm{b}}^{\mathrm{F}}$	$150°/$ cm	$202°/$ cm
$\Theta_{\mathrm{t}}^{\mathrm{F}}$	$-1000°/$ cm	$360°/$ cm

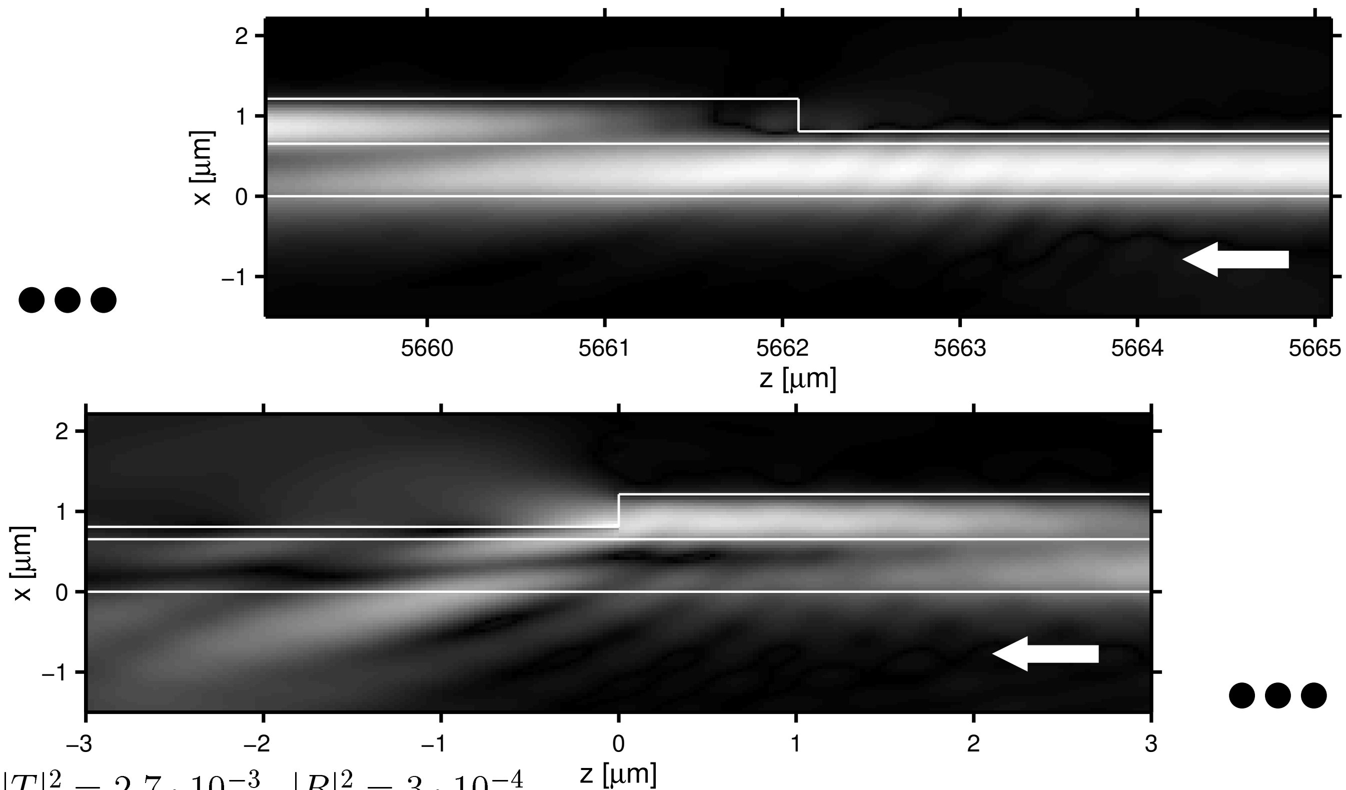
LO $= -10\log_{10} P^{\mathrm{f}}$, IS $= 10\log_{10} P^{\mathrm{f}}/P^{\mathrm{b}}$.

$q_0 - \Delta q \leq q \leq q_0 + \Delta q$: The experiment can be carried out with
IS $\geq 20\,$dB, LO $\leq 1\,$dB $\quad (P^{\mathrm{f}}/P^{\mathrm{b}} \geq 100$, $P^{\mathrm{f}} \geq 0.8)$.

$L_{\mathrm{c}}^{\mathrm{f}} = 4.5008\,\mu$m, $L_{\mathrm{c}}^{\mathrm{b}} = 4.4973\,\mu$m, $L_0/L_{\mathrm{c}}^{\mathrm{f}} \approx 1258$, $L_0/L_{\mathrm{c}}^{\mathrm{b}} \approx 1259$.

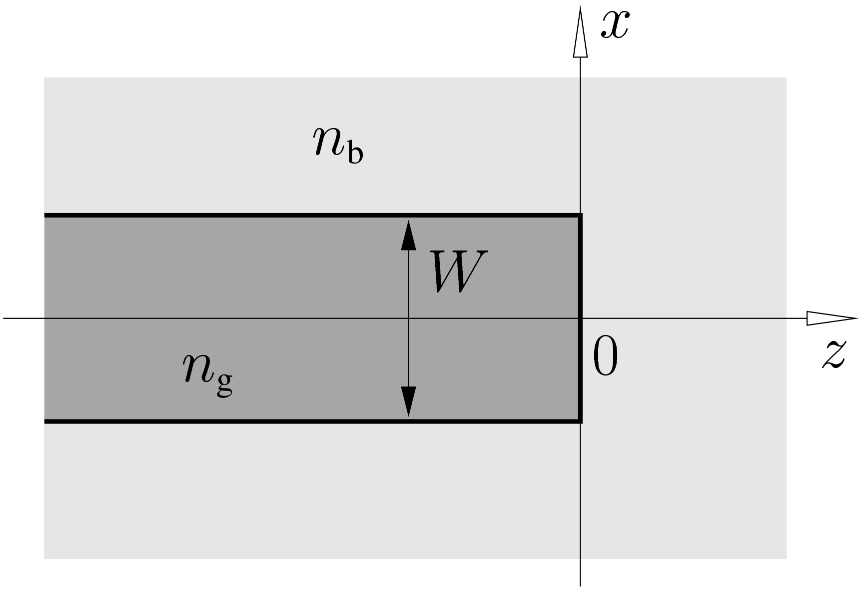


$|T|^2 = 0.98,\ |R|^2 = 3 \cdot 10^{-4}$



$|T|^2 = 2.7 \cdot 10^{-3},\ |R|^2 = 3 \cdot 10^{-4}$

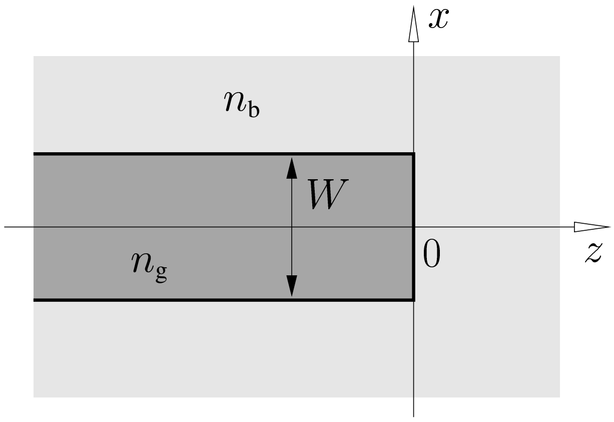
$P^{\mathrm{f}} = 98\%$, $P^{\mathrm{b}} = 0.3\%$, IS $= 26$ dB, LO $= 0.1$ dB.

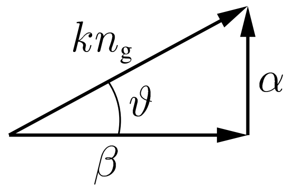


Given: Incident guided waves. $\Longrightarrow$

$\Longrightarrow$ Transmission?

Guided and nonguided reflection? $\Longleftarrow$





$\lambda = 1.532\,\mu$m, $W = 1.54\,\mu$m,
$n_g = 3.2$, $n_b = 1.0$.
$\sin\vartheta_{crit} = n_b/n_g$, $\vartheta_{crit} = 18.2°$.

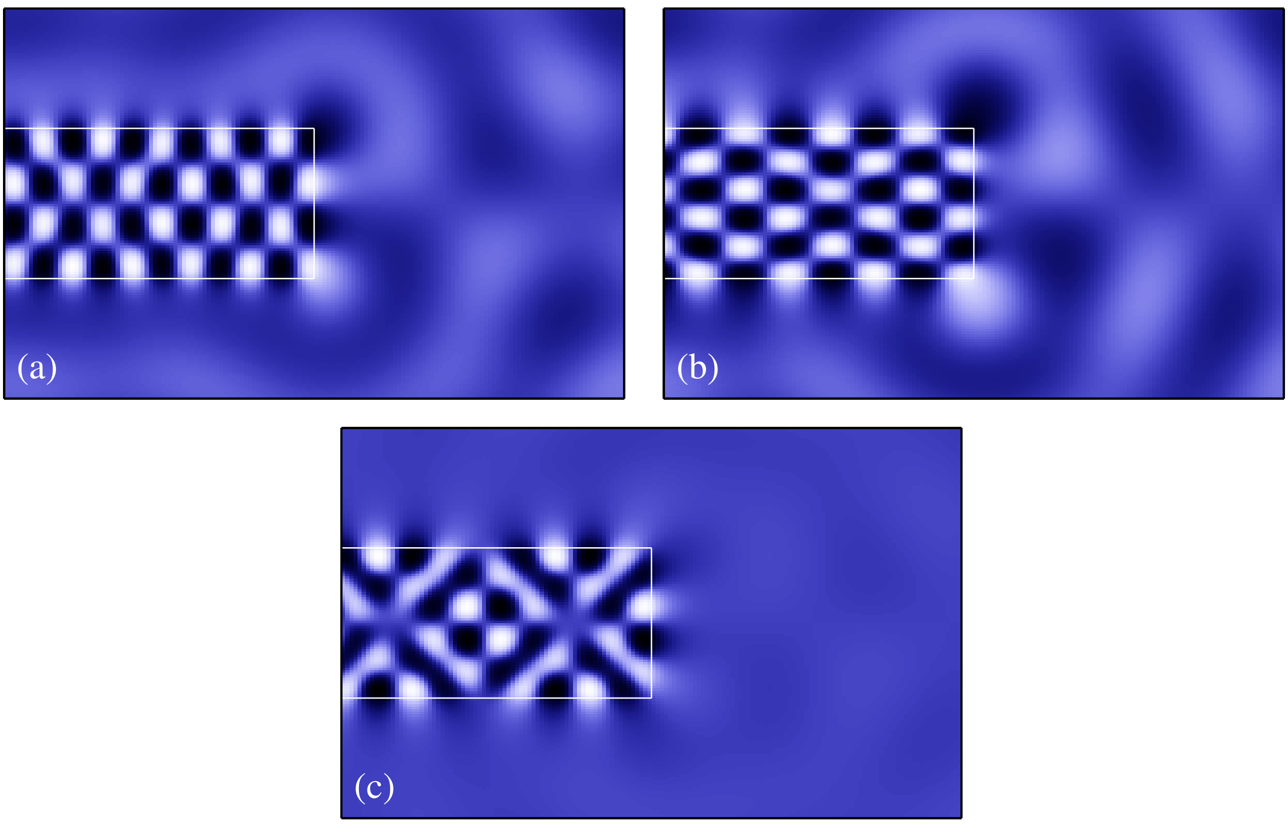
Wavevector components of the guided fields:

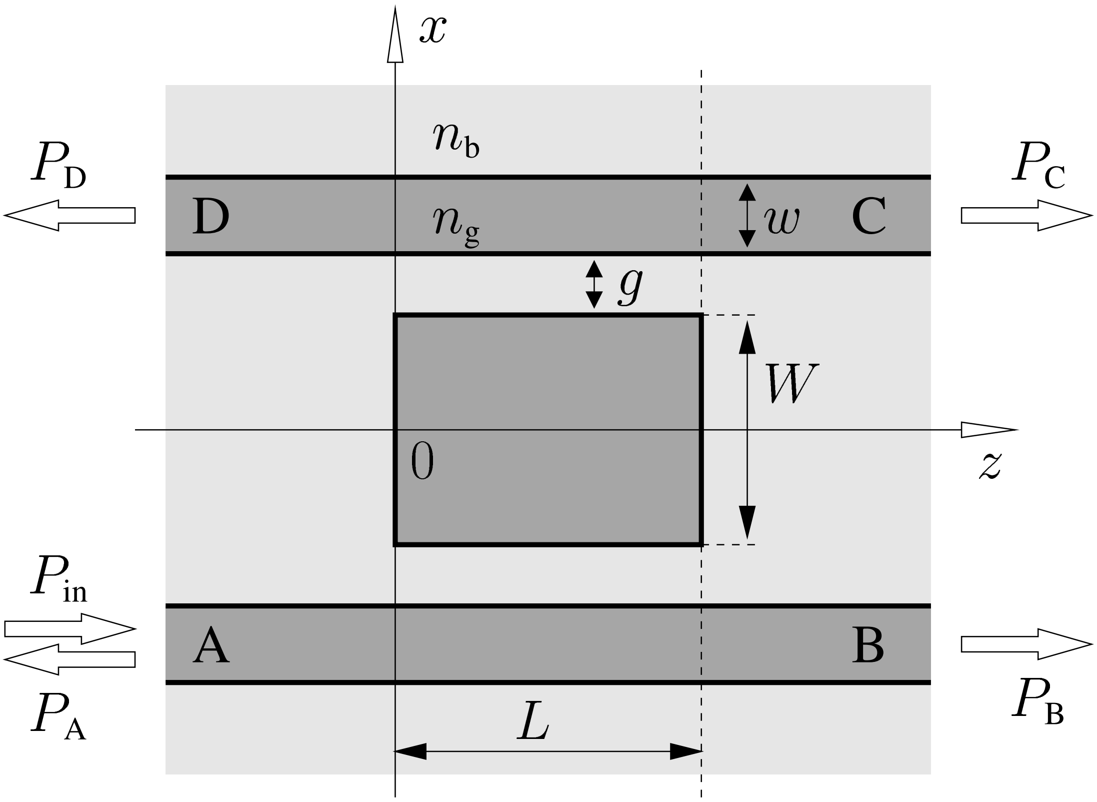
	TE_0	TE_1	TE_2	TE_3	TE_4	TE_5	TE_6
$\beta/\,\mu m^{-1}$	13.0	12.6	12.0	10.9	9.4	7.3	4.3
$\alpha/\,\mu m^{-1}$	1.8	3.7	5.5	7.3	9.1	10.9	12.4
$\vartheta/°$	8.1	16.3	24.9	34.0	44.0	55.9	70.7

Guided wave reflection:

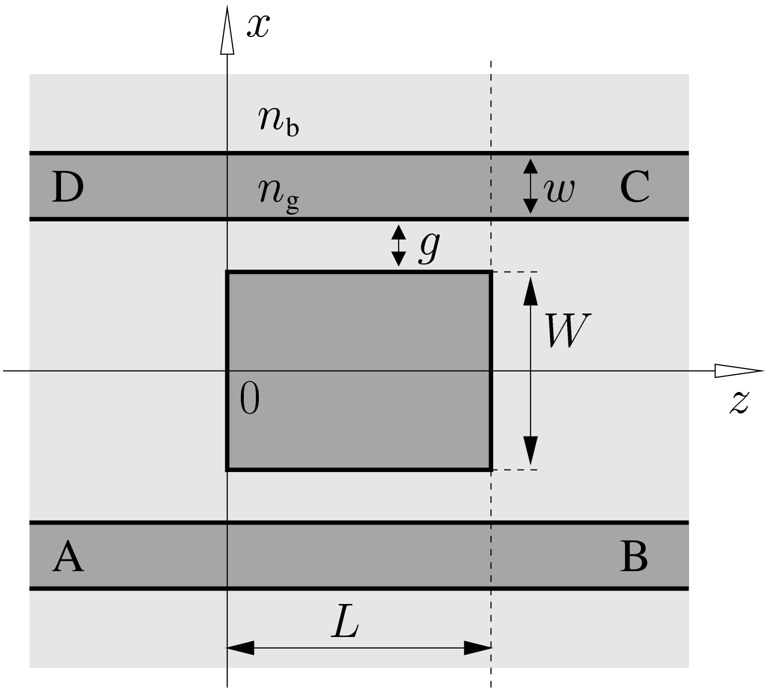
	Input	Relative power reflected into mode							$\sum$
		TE_0	TE_1	TE_2	TE_3	TE_4	TE_5	TE_6	
(a)	TE_3	0	1 %	0	80 %	0	3 %	0	84 %
(b)	TE_5	0	1 %	0	3 %	0	71 %	0	75 %
(c)	$a\,TE_3 + b\,TE_5$	0	0.1 %	0	61 %	0	38 %	0	99 %

$$\frac{|a|^2}{|b|^2} \approx \frac{61}{38}$$

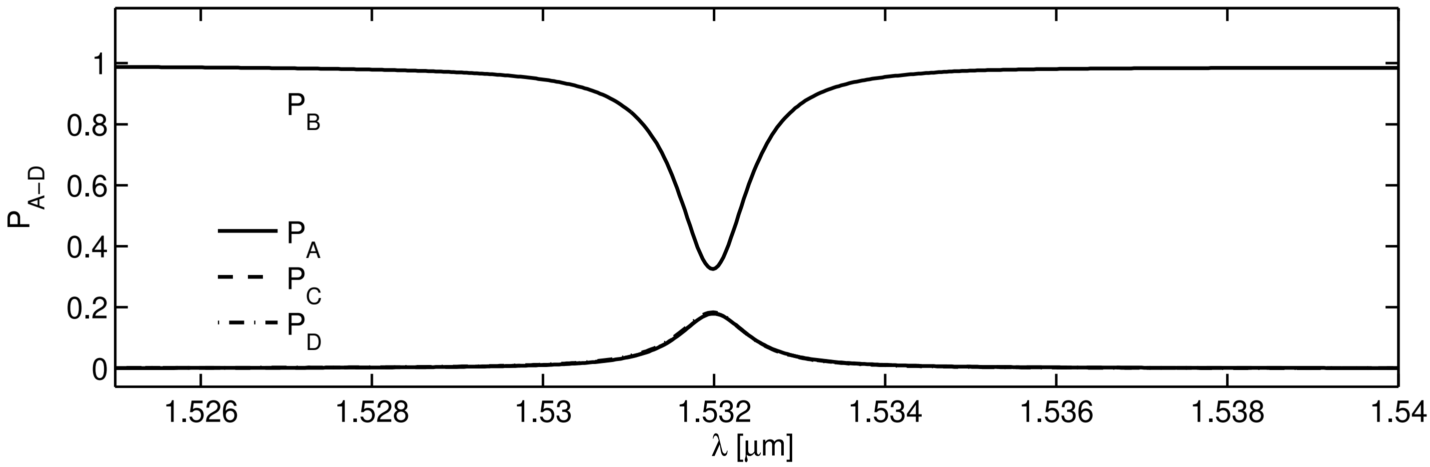




$$P_{\mathrm{in}}(\lambda) = 1. \quad P_{\mathrm{A}}, P_{\mathrm{B}}, P_{\mathrm{C}}, P_{\mathrm{D}} \ = \ ?$$

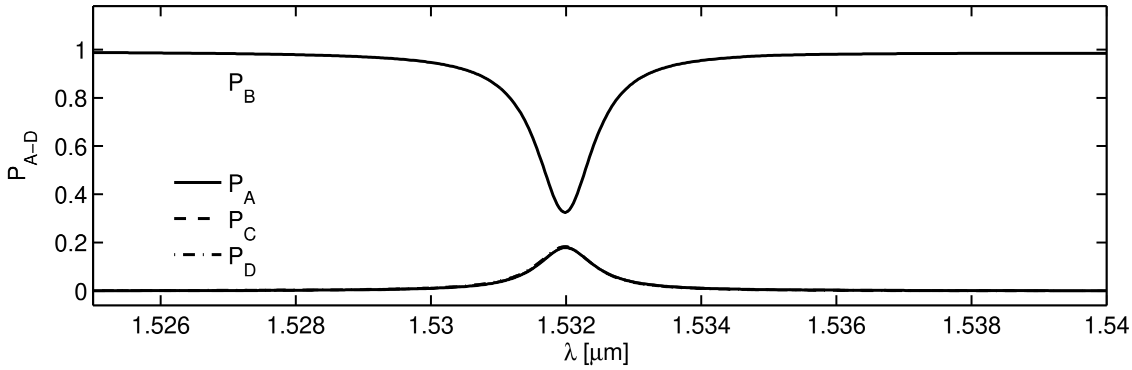


* $n_g = 3.2$, $n_b = 1.0$,
$w = 0.20\,\mu$m, $g = 0.29\,\mu$m,
$W = 1.54\,\mu$m, $L = 1.54\,\mu$m,
$\lambda \in [1.3\,\mu\text{m}, 1.7\,\mu\text{m}]$,
TE fields, $\boldsymbol{\mathcal{E}} = (0, E_y, 0)$.

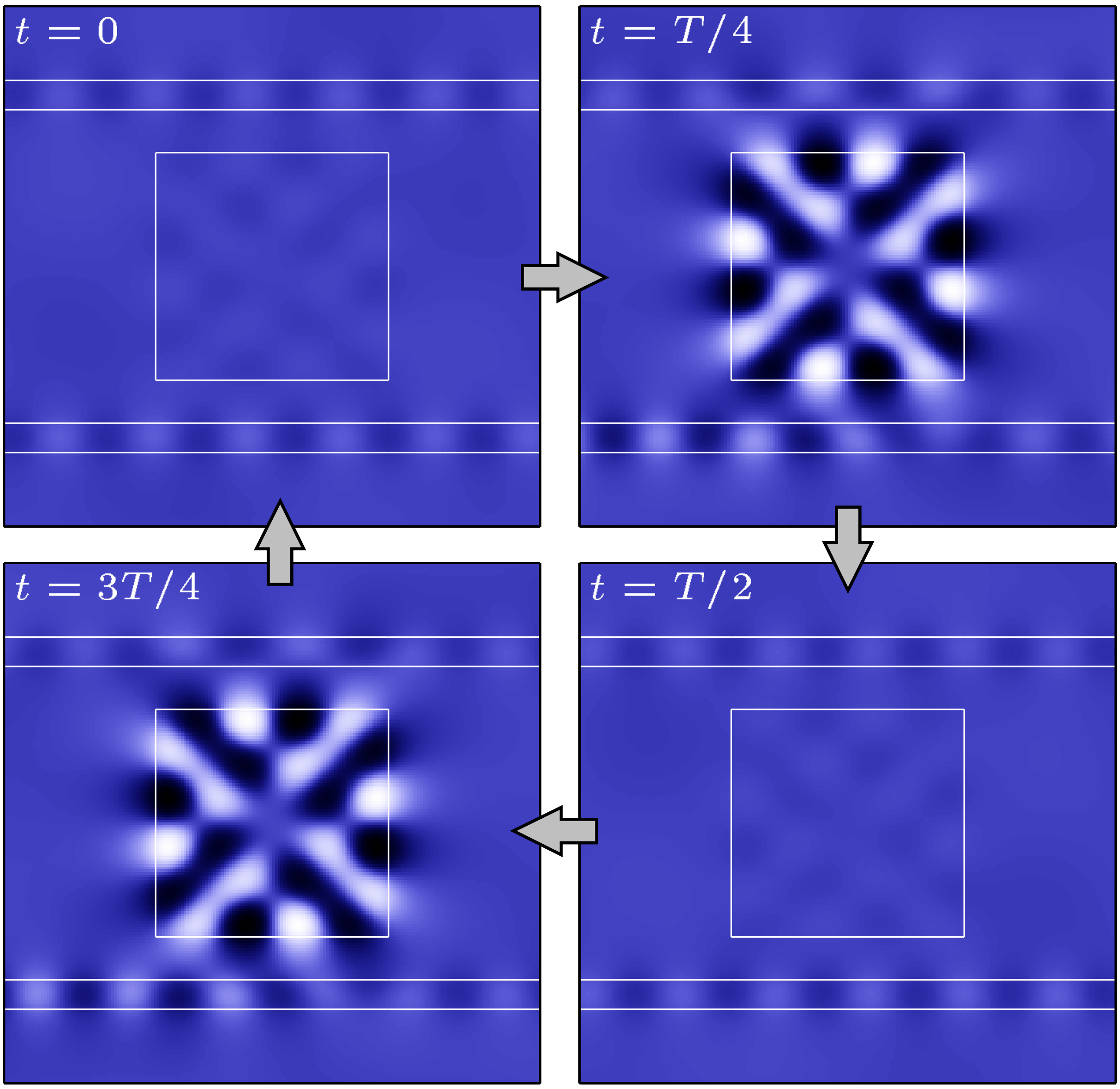


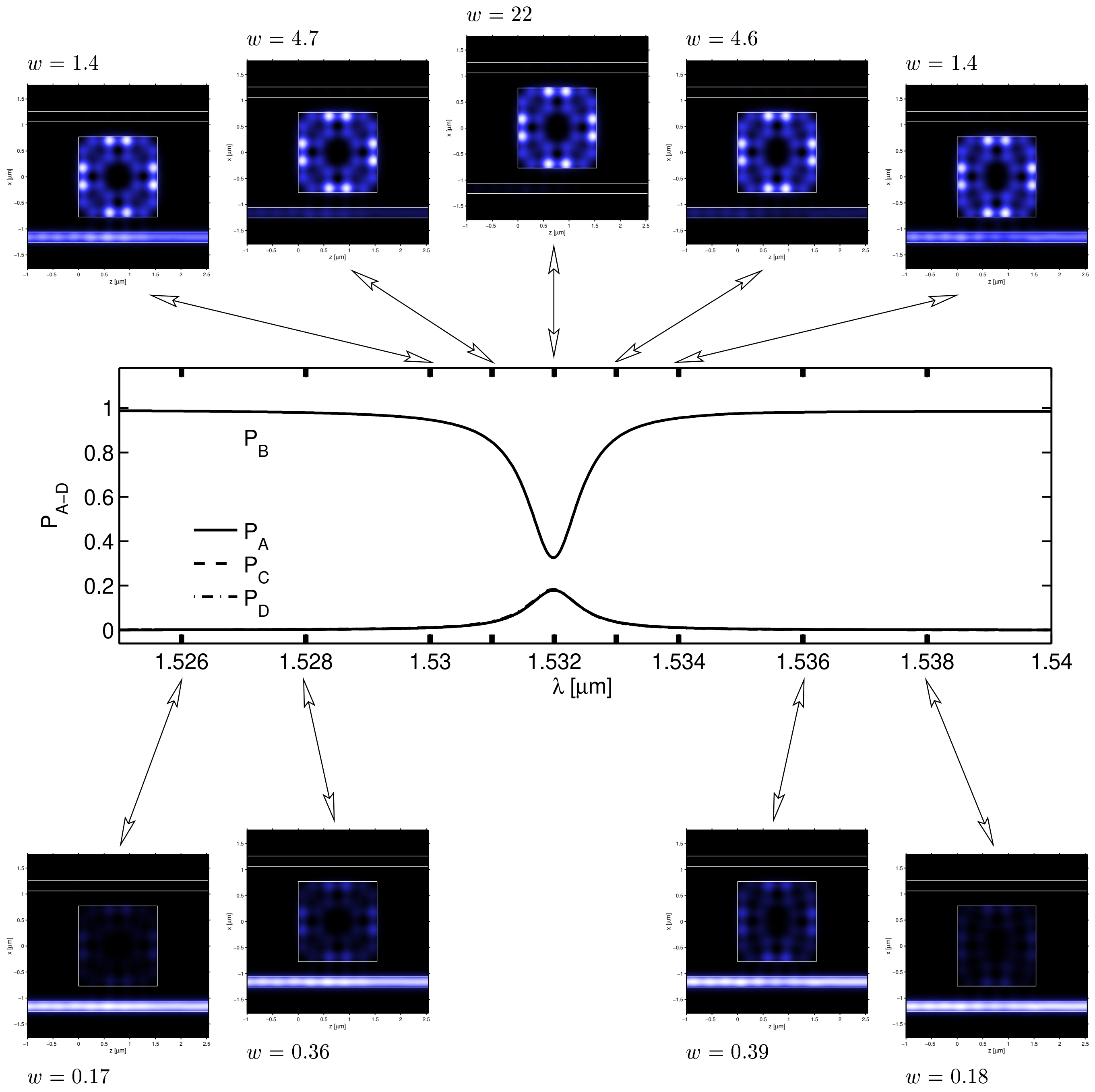
$\lambda = 1.532\,\mu$m: $P_A = 0.18$, $P_B = 0.33$, $P_C = 0.18$, $P_D = 0.18$.

* *C. Manolatou, M. J. Khan, S. Fan, P. R. Villeneuve, H. A. Haus, J. D. Joannopoulos, Coupling of Modes Analysis of Resonant Channel Add-Drop Filters, IEEE Journal of Quantum Electronics 35(9), 1322–1331, 1999*

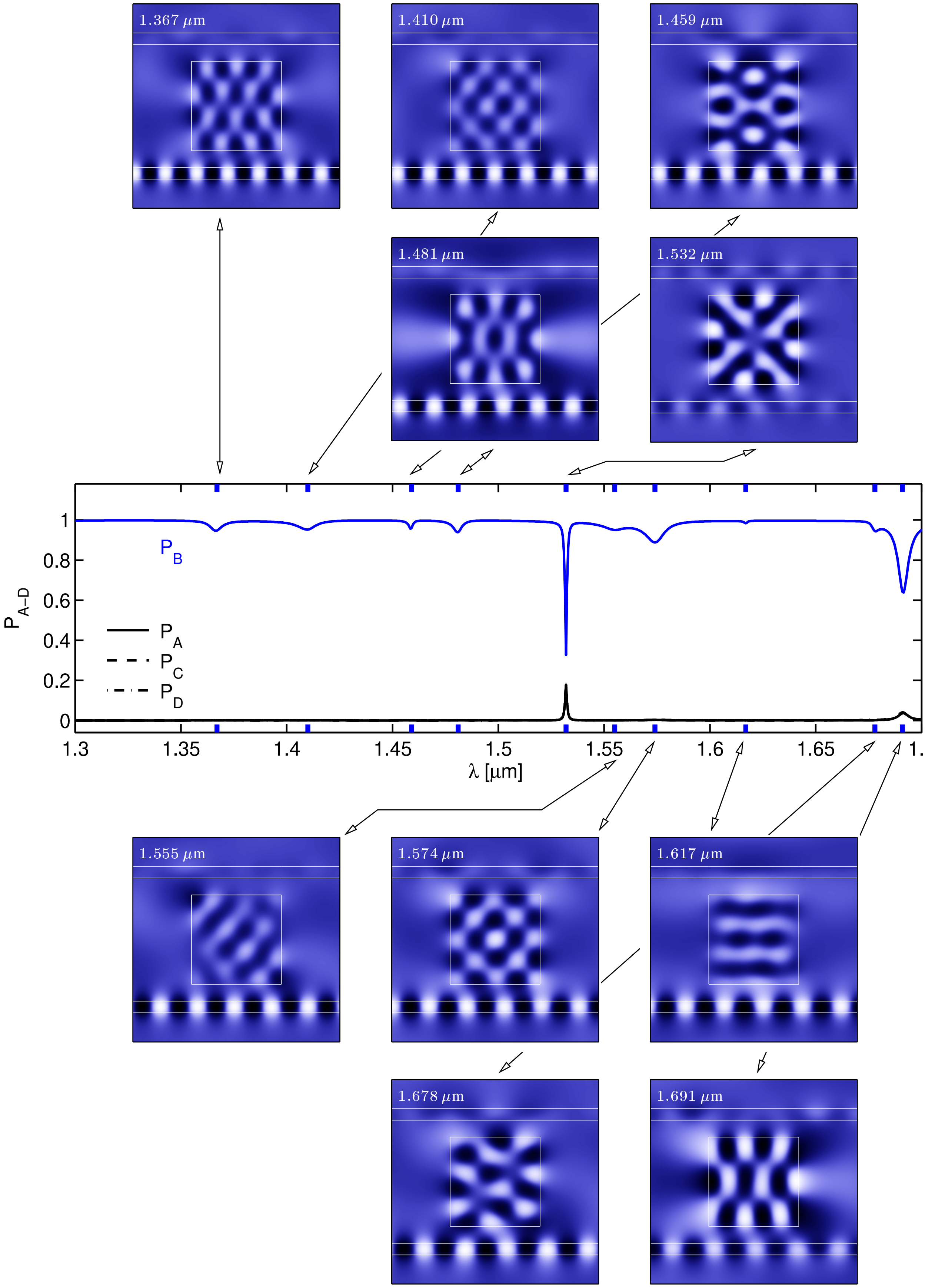


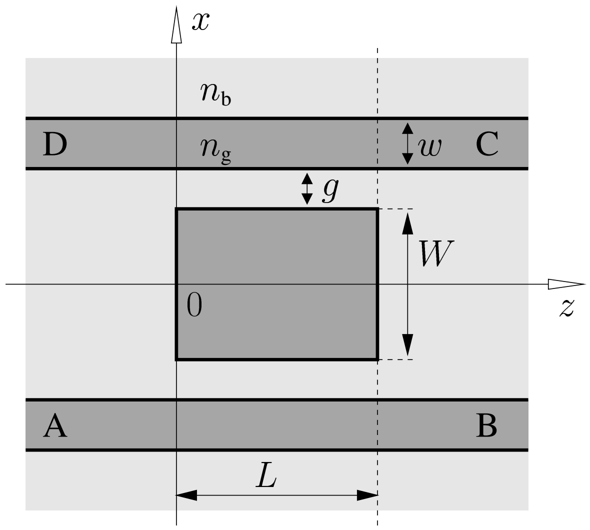
$\lambda = 1.532\,\mu\mathrm{m}, \;\; f = 196\,\mathrm{THz}, \;\; T = 5.11\,\mathrm{fs}, \;\; E_y(x, z, t)$:

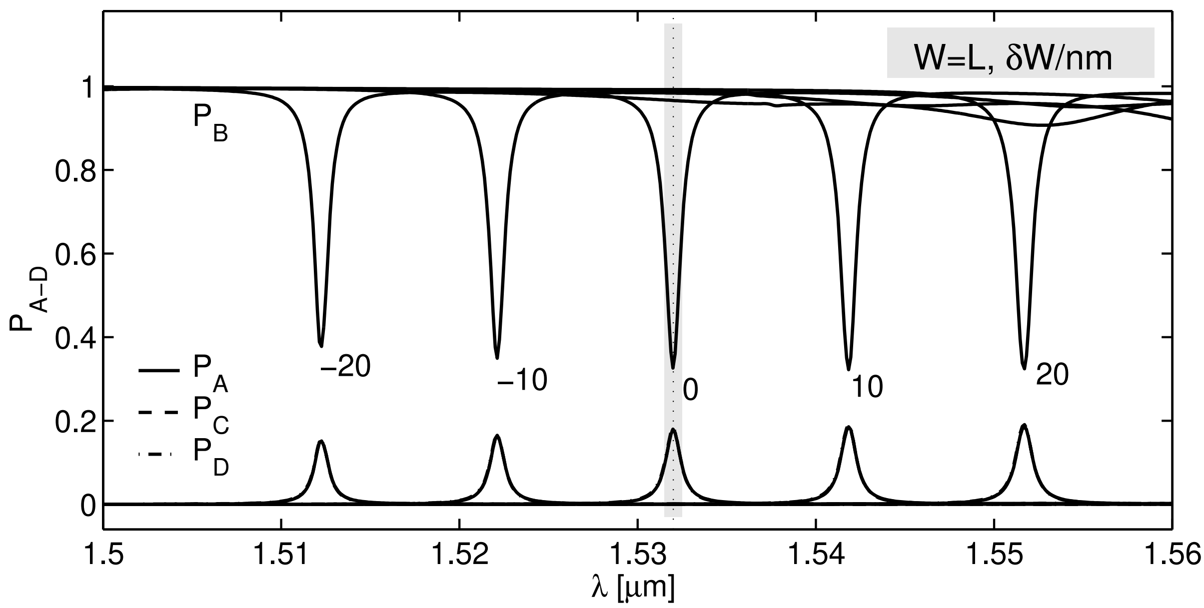


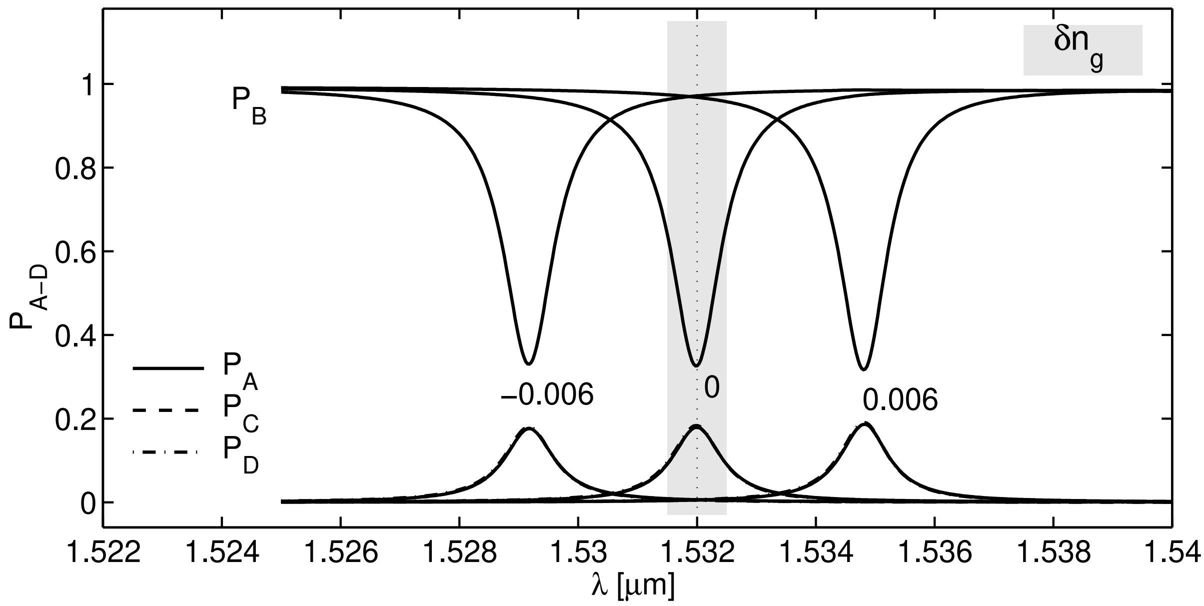


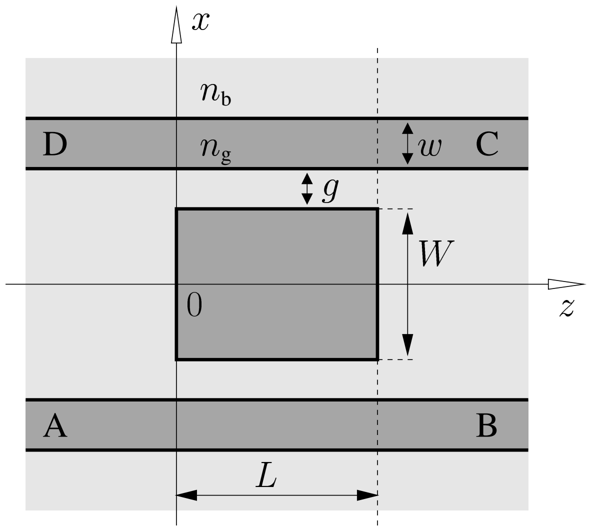
w: maximum electromagnetic energy density in the cavity region, relative to an isolated, equally excited waveguide.

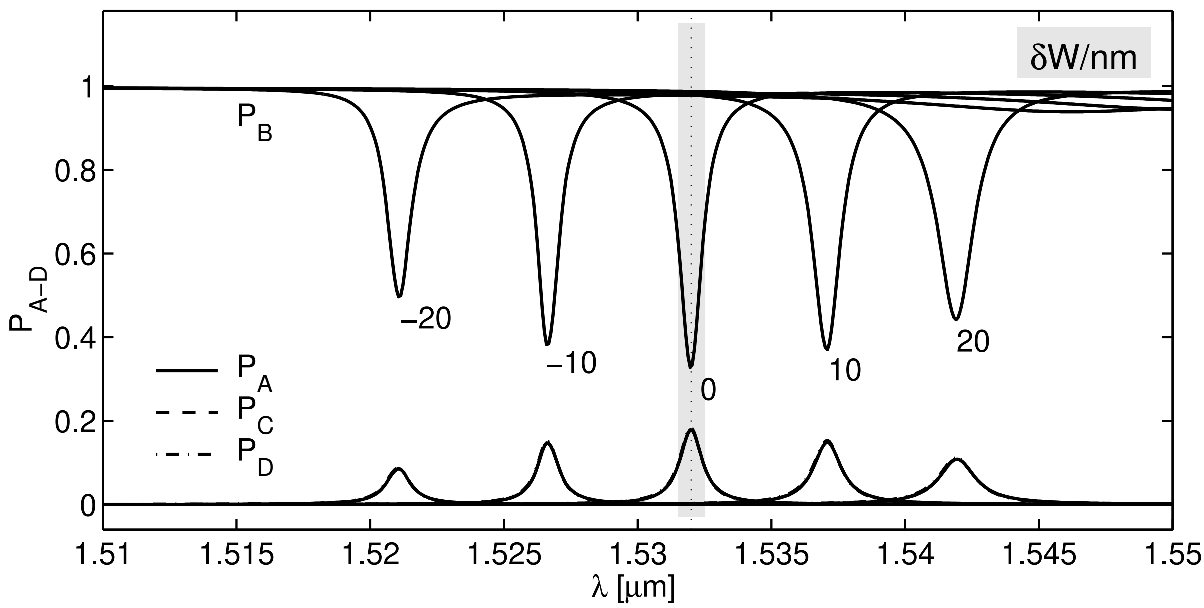

1.367 μm
1.410 μm
1.459 μm
1.481 μm
1.532 μm
1.555 μm
1.574 μm
1.617 μm
1.678 μm
1.691 μm
P_B
P_A
P_C
P_D
P_A–D
λ [μm]
1.3
1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7
0
0.2
0.4
0.6
0.8
1

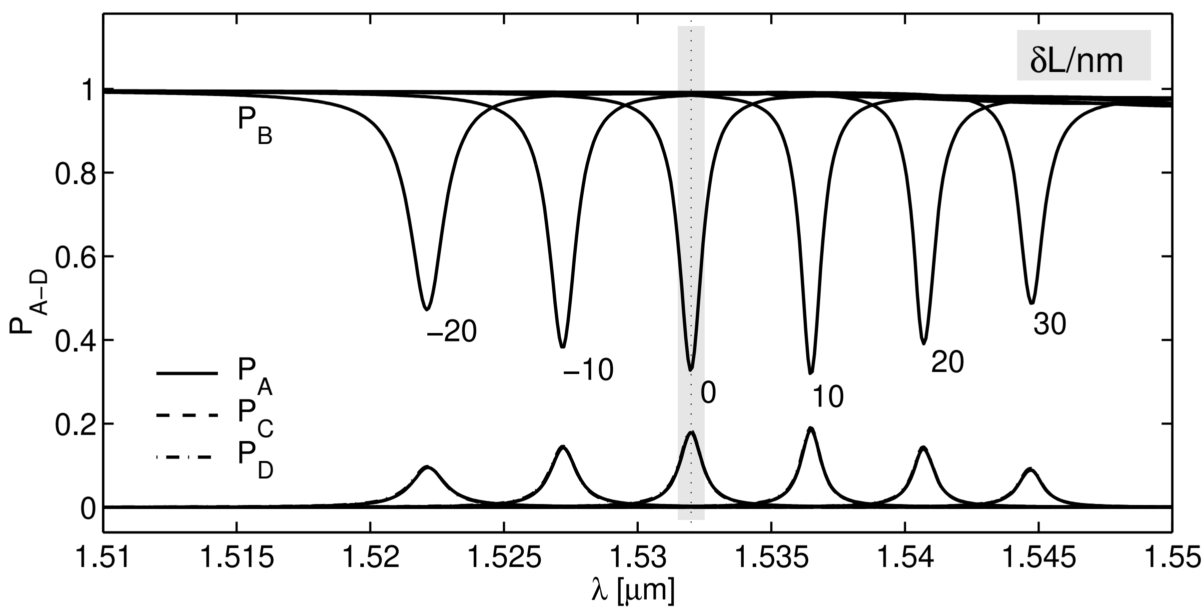

x
n_b
D
n_g
w
C
g
W
0
z
A
B
L

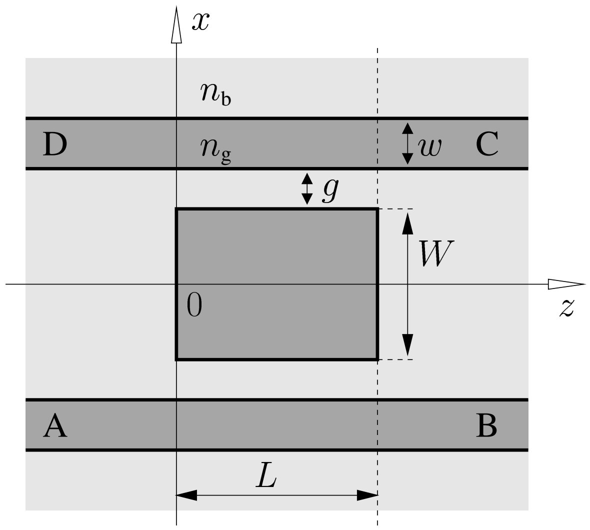

W=L, δW/nm
P_B
P_{A-D}
−20
−10
0
10
20
P_A
P_C
P_D
λ [μm]

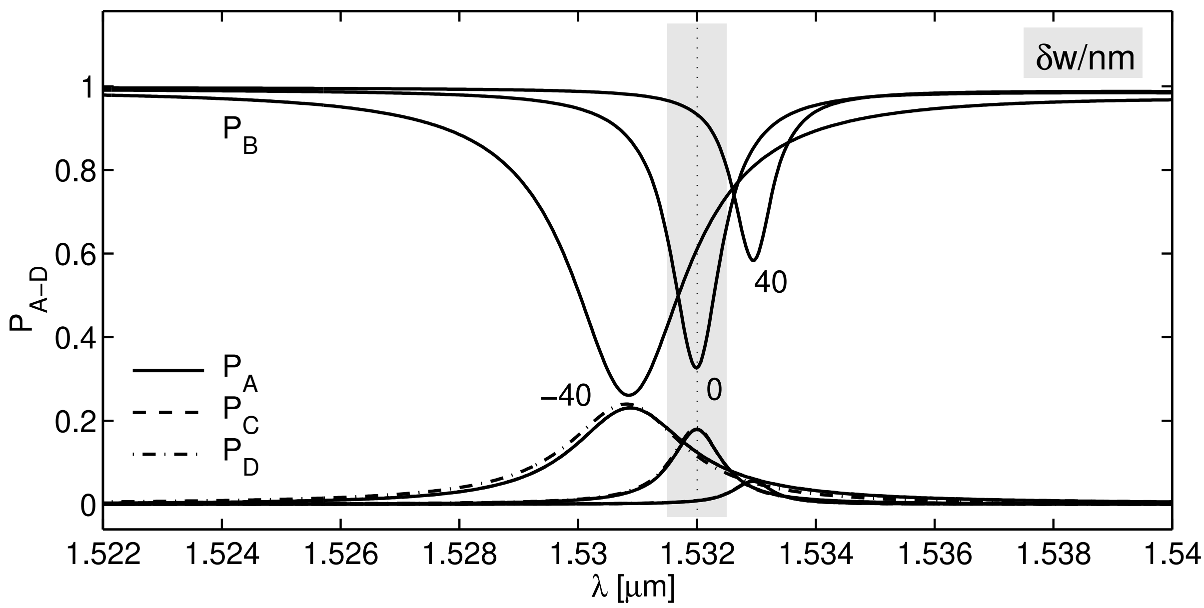

δn_g
P_B
P_{A-D}
−0.006
0
0.006
P_A
P_C
P_D
λ [μm]

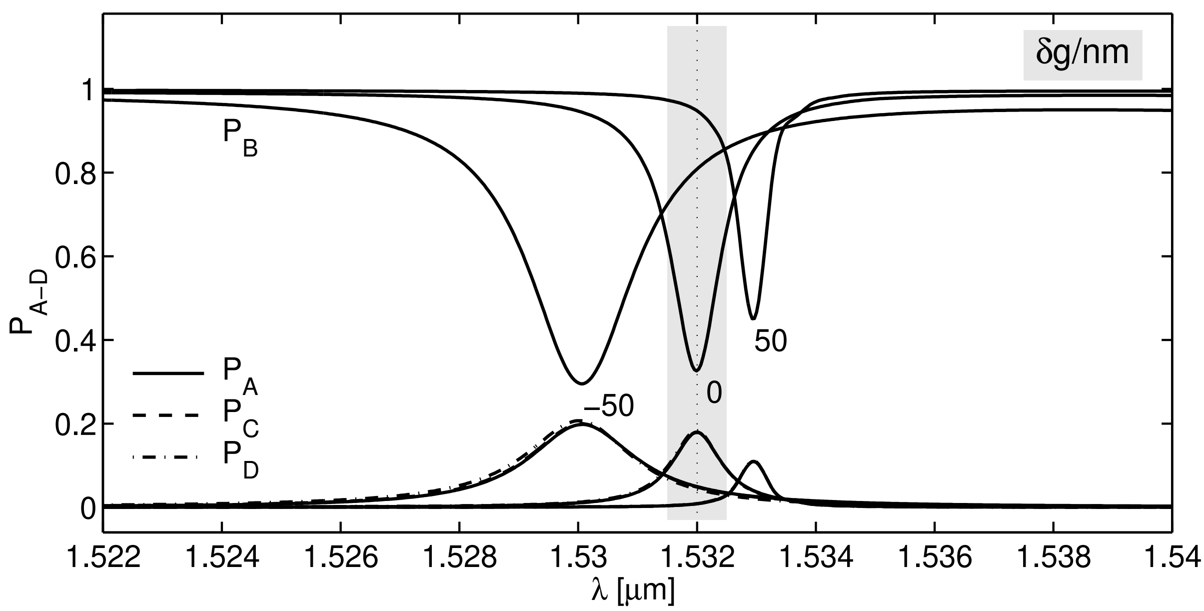

x
n_b
D
n_g
w
C
g
W
0
z
A
B
L

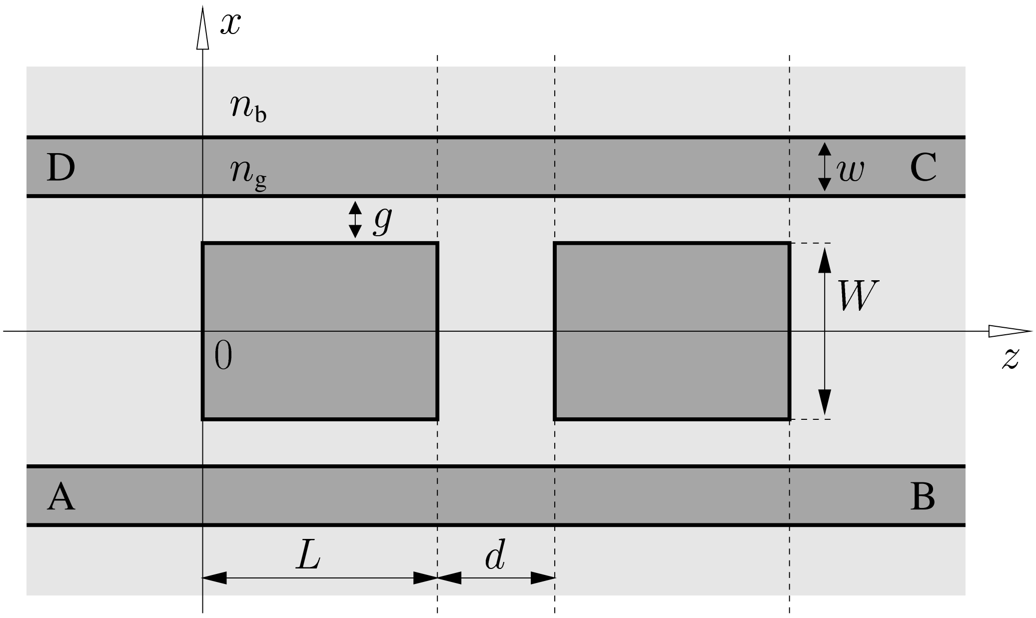

δW/nm
P_B
P_{A-D}
−20
−10
0
10
20
P_A
P_C
P_D
λ [µm]


δL/nm
P_B
P_{A-D}
−20
−10
0
10
20
30
P_A
P_C
P_D
λ [µm]

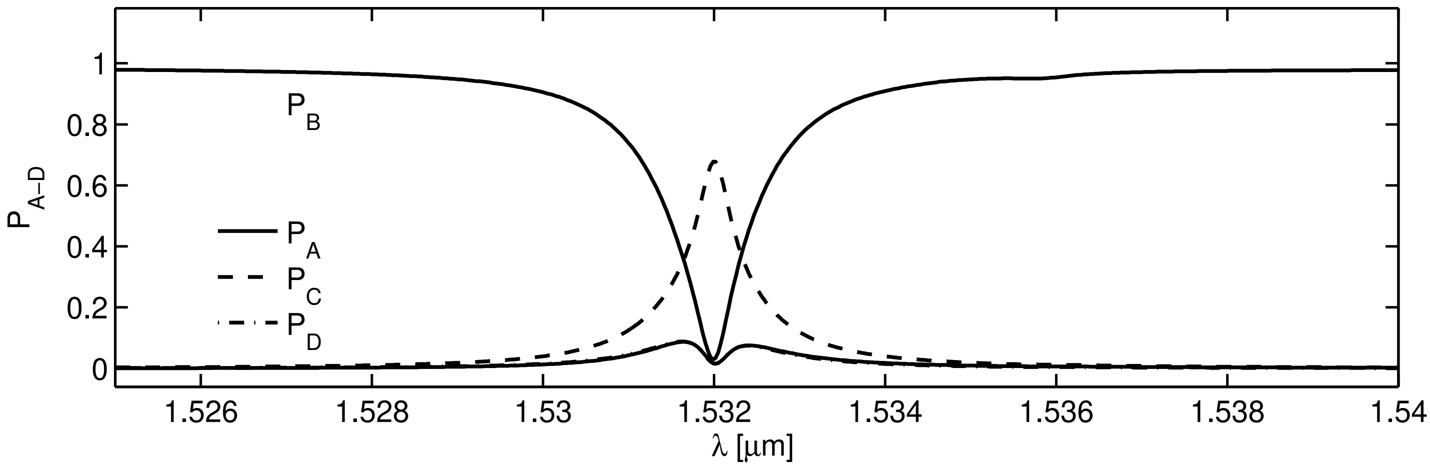

x
n_b
D
n_g
w
C
g
W
0
z
A
B
L


δw/nm
P_B
40
0
−40
P_A
P_C
P_D
P_{A-D}
λ [μm]


δg/nm
P_B
50
0
−50
P_A
P_C
P_D
P_{A-D}
λ [μm]



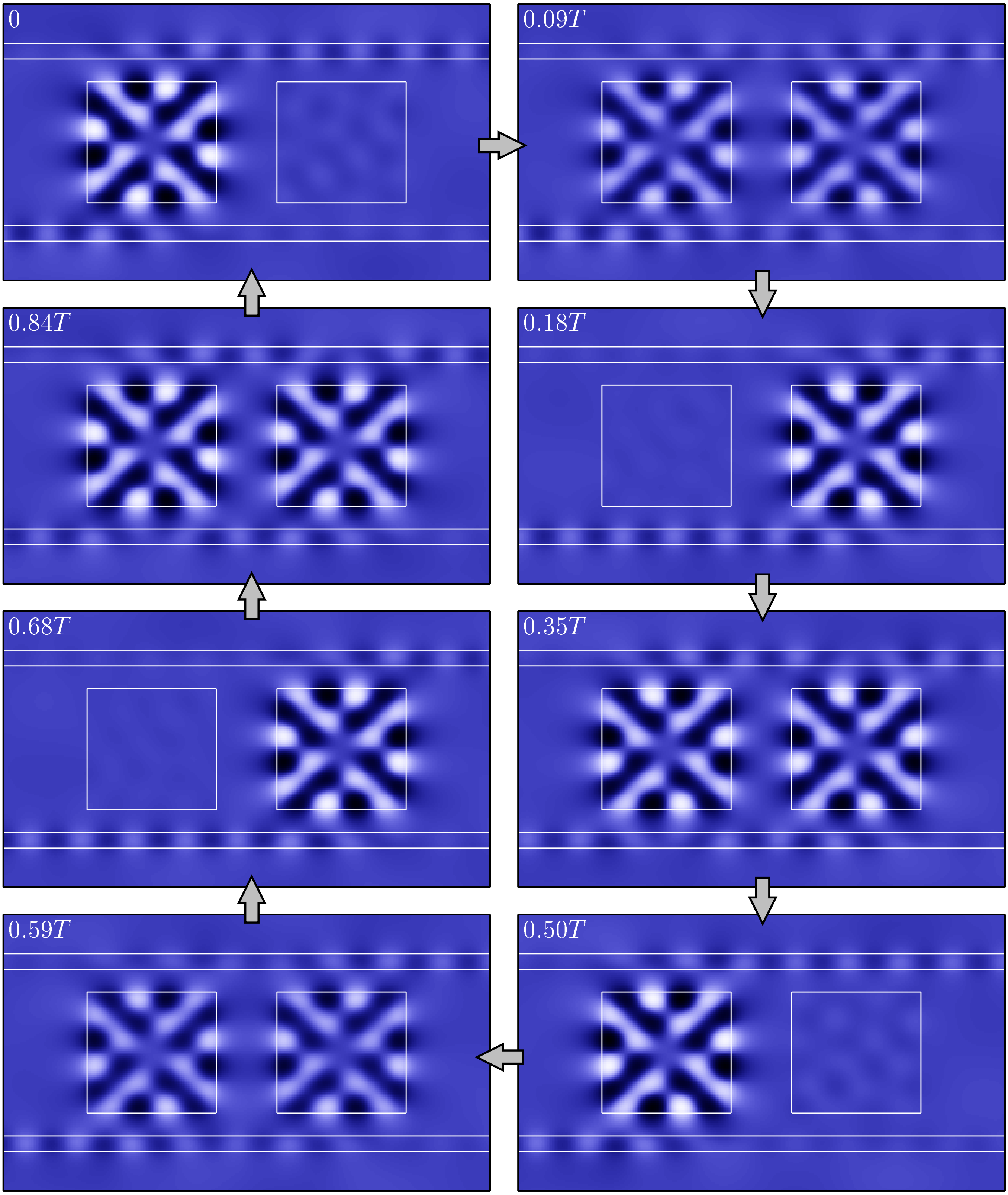
$n_g = 3.2,\ n_b = 1.0,\ w = 0.20\,\mu$m, $g = 0.29\,\mu$m, $d = 0.72\,\mu$m,
$W = 1.54\,\mu$m, $L = 1.54\,\mu$m, $\lambda \in [1.3\,\mu\text{m}, 1.7\,\mu\text{m}]$, TE fields, $\boldsymbol{\mathcal{E}} = (0, E_y, 0)$.

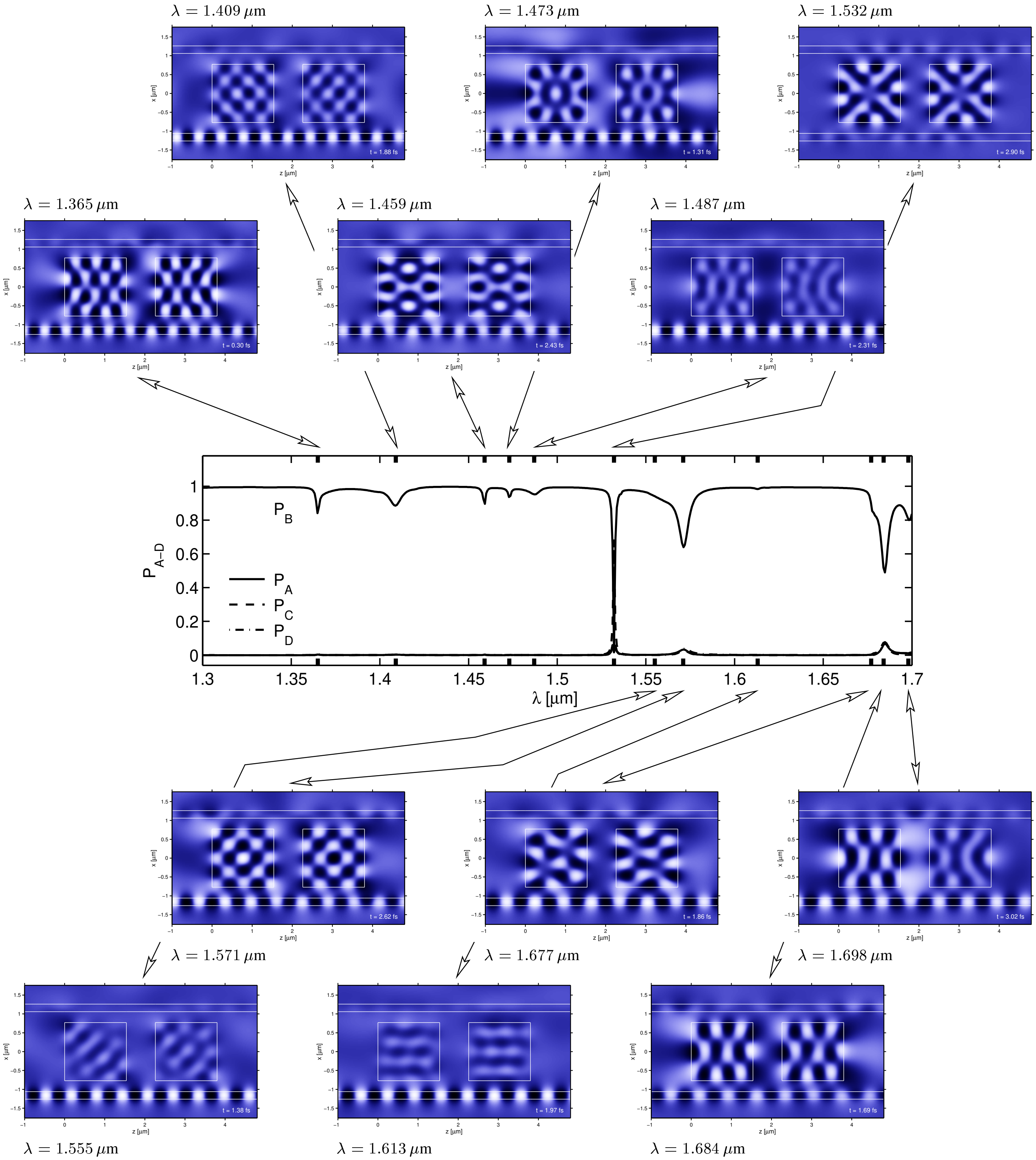


$\lambda = 1.532\,\mu$m: $P_A = 0.02$, $P_B = 0.03$, $P_C = 0.68$, $P_D = 0.02$.

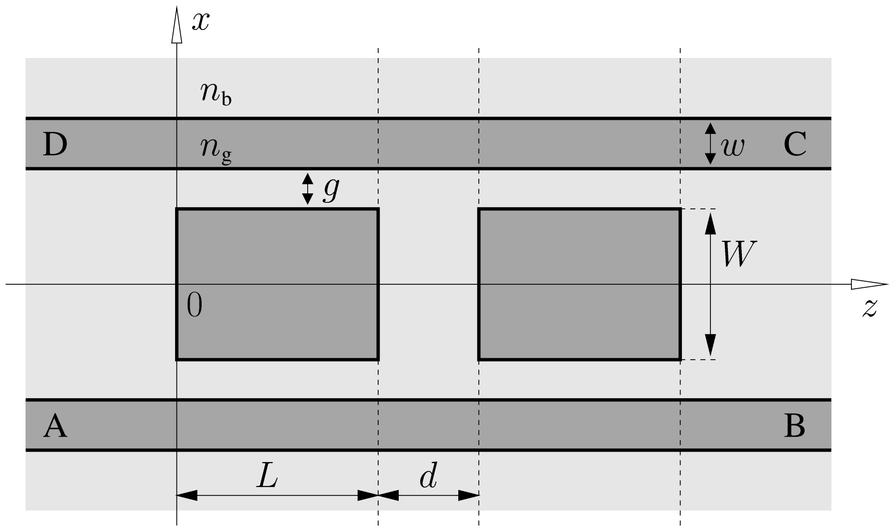
FWHM $\approx 1\,$nm.

$\lambda = 1.532\,\mu\text{m}, \;\; f = 196\,\text{THz}, \;\; T = 5.11\,\text{fs}, \;\; E_y(x, z, t)$:

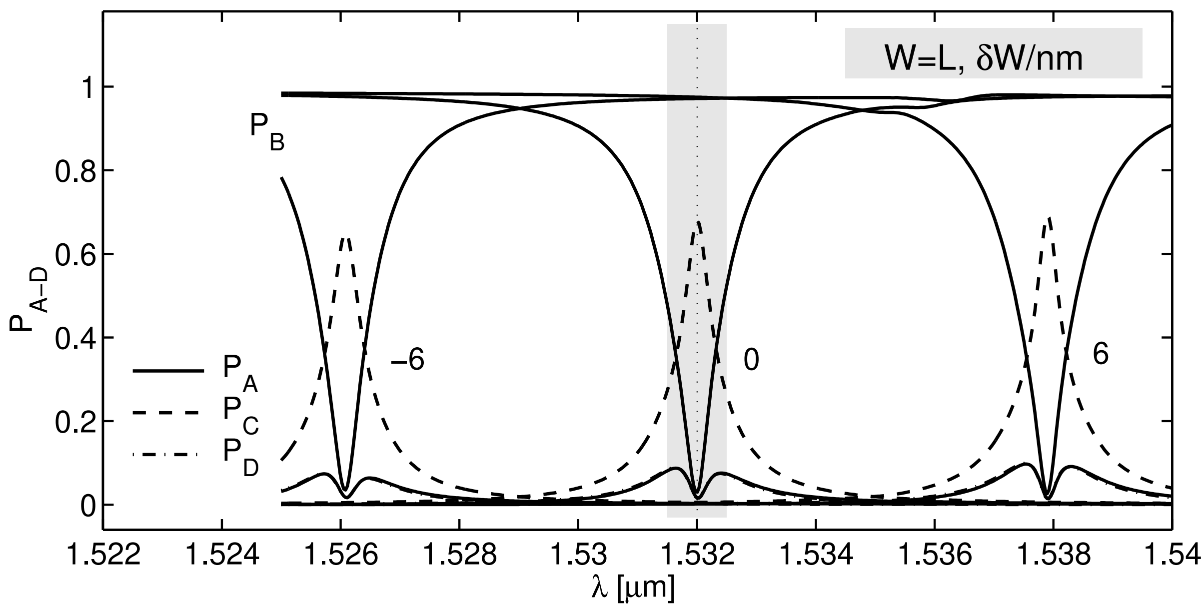




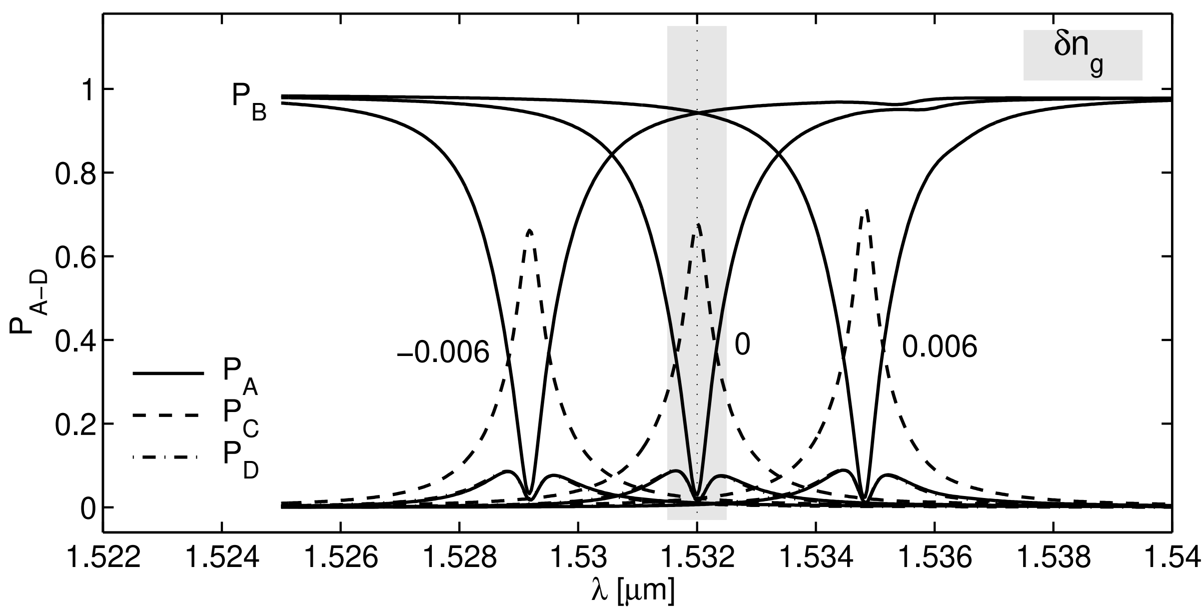
FWHM $\approx 1\,$nm, FSR $\approx 40\,$nm $(150\,$nm$)$.



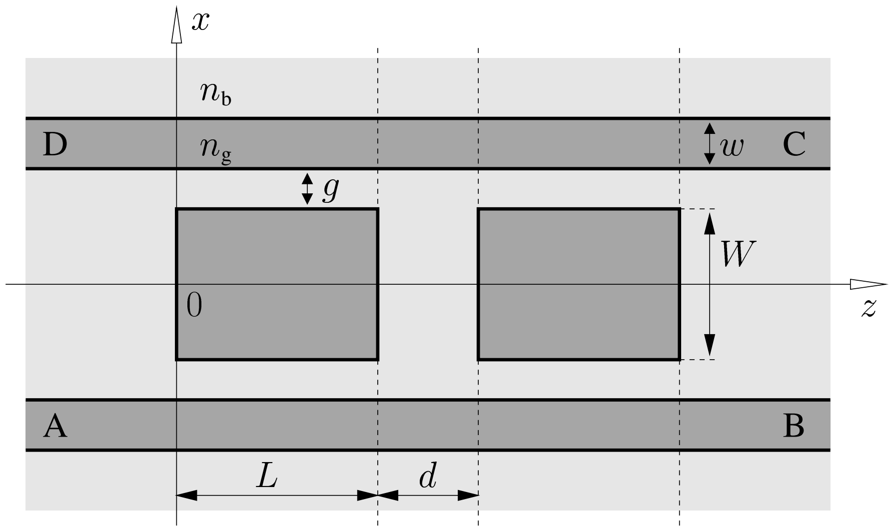
x
n_b
D
n_g
w
C
g
W
0
z
A
B
L
d

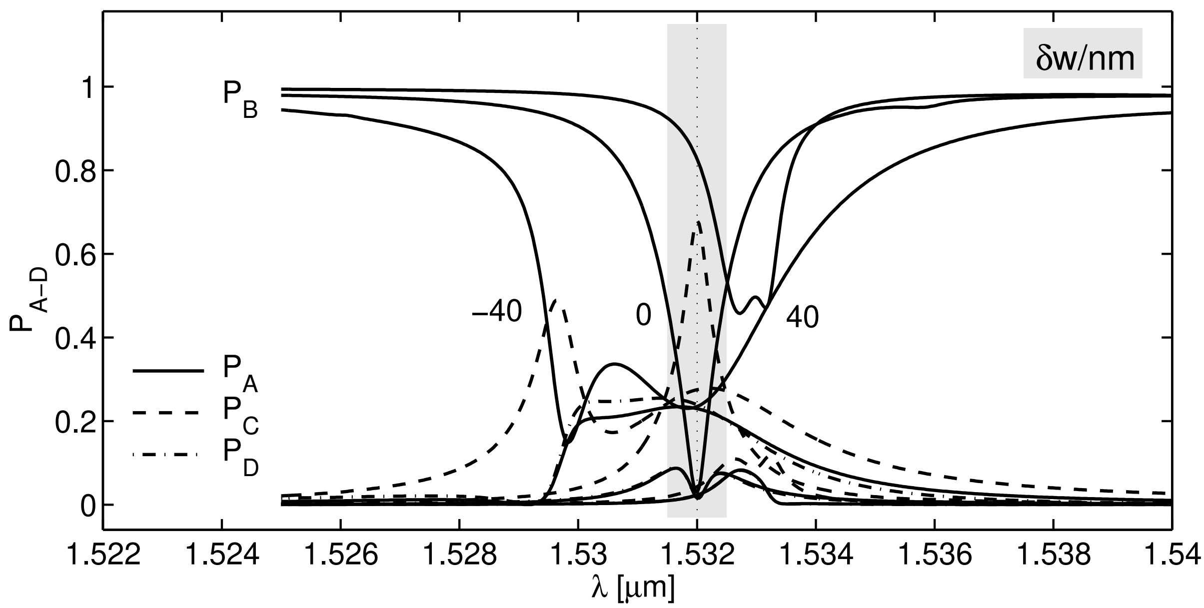


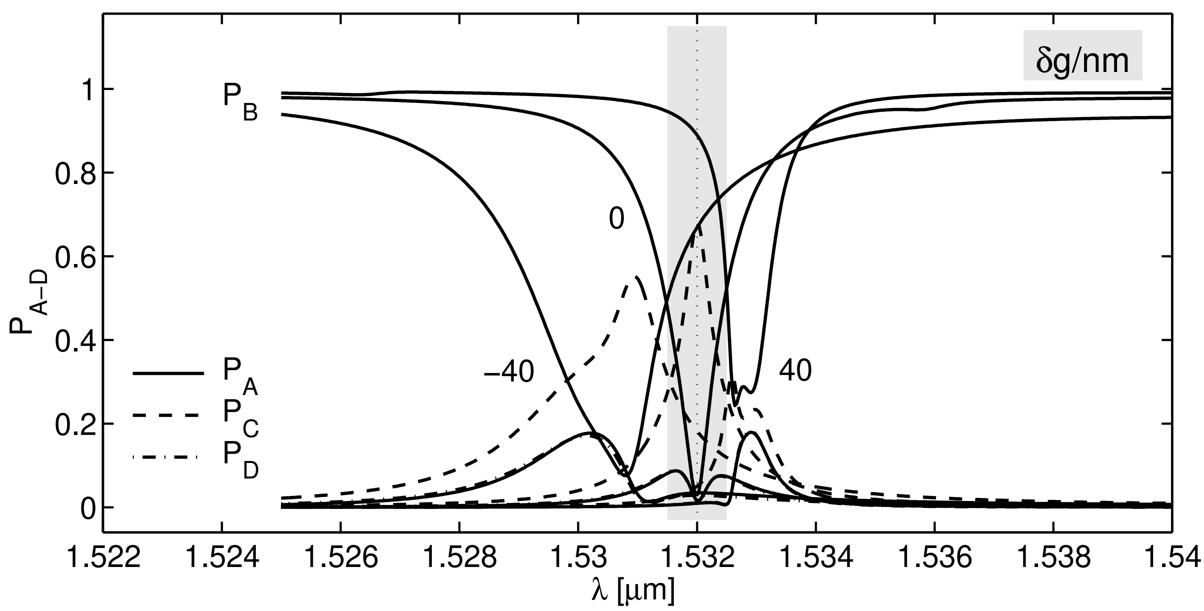
W=L, δW/nm
P_B
P_{A-D}
–6
0
6
P_A
P_C
P_D
λ [μm]

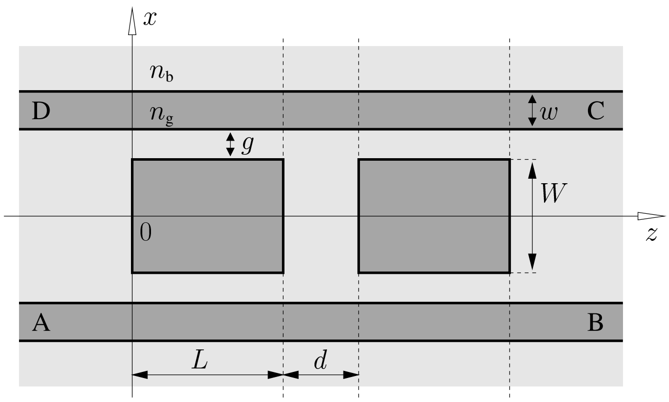


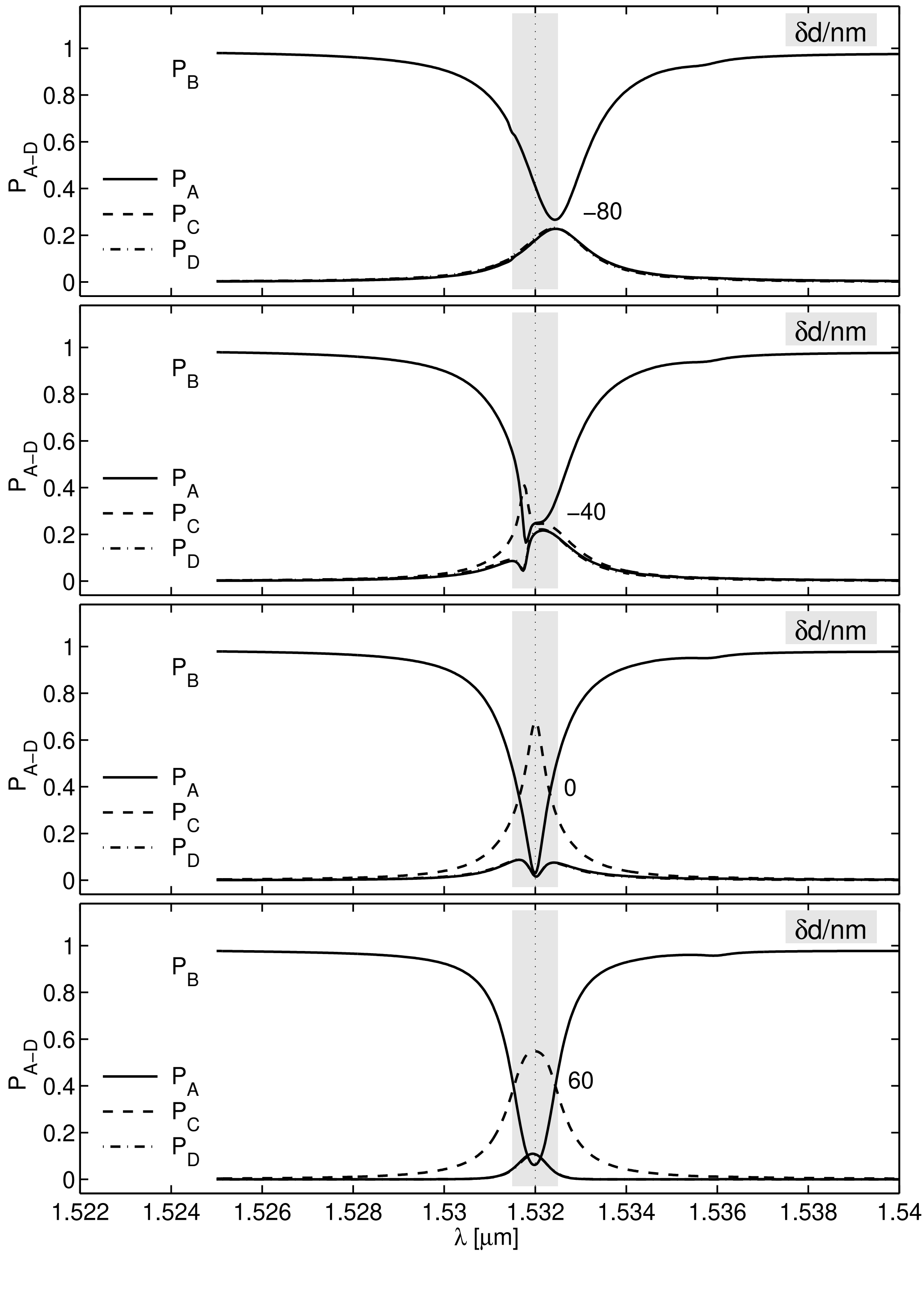
δn_g
P_B
P_{A-D}
–0.006
0
0.006
P_A
P_C
P_D
λ [μm]

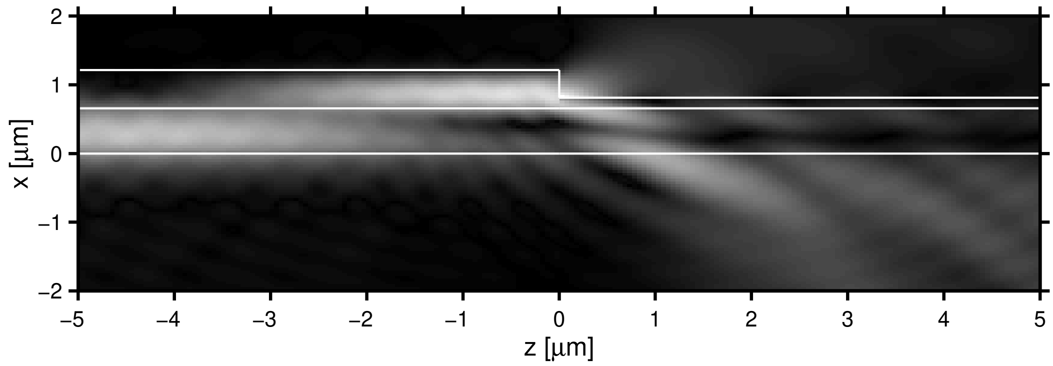

x
n_b
D
n_g
w
C
g
W
0
z
A
B
L
d


δw/nm
P_B
–40
0
40
P_A
P_C
P_D
P_{A-D}
λ [μm]


δg/nm
P_B
0
–40
40
P_A
P_C
P_D
P_{A-D}
λ [μm]


x
n_b
D
n_g
w
C
g
W
0
z
A
B
L
d


δd/nm
P_B
P_{A-D}
P_A
P_C
P_D
–80
–40
0
60
λ [µm]



- Apparently reliable 2-D simulations of segmented planar integrated optical structures, quite efficient in terms of computational resources.
- Examples: Explicitely simple interferometer devices and concepts for rectangular microresonators and wavelength filters.
- Adequate design basis that allows for convenient numerical experiments on these structures.
- Suitable model system for investigations on more fundamental issues in integrated optics . . .