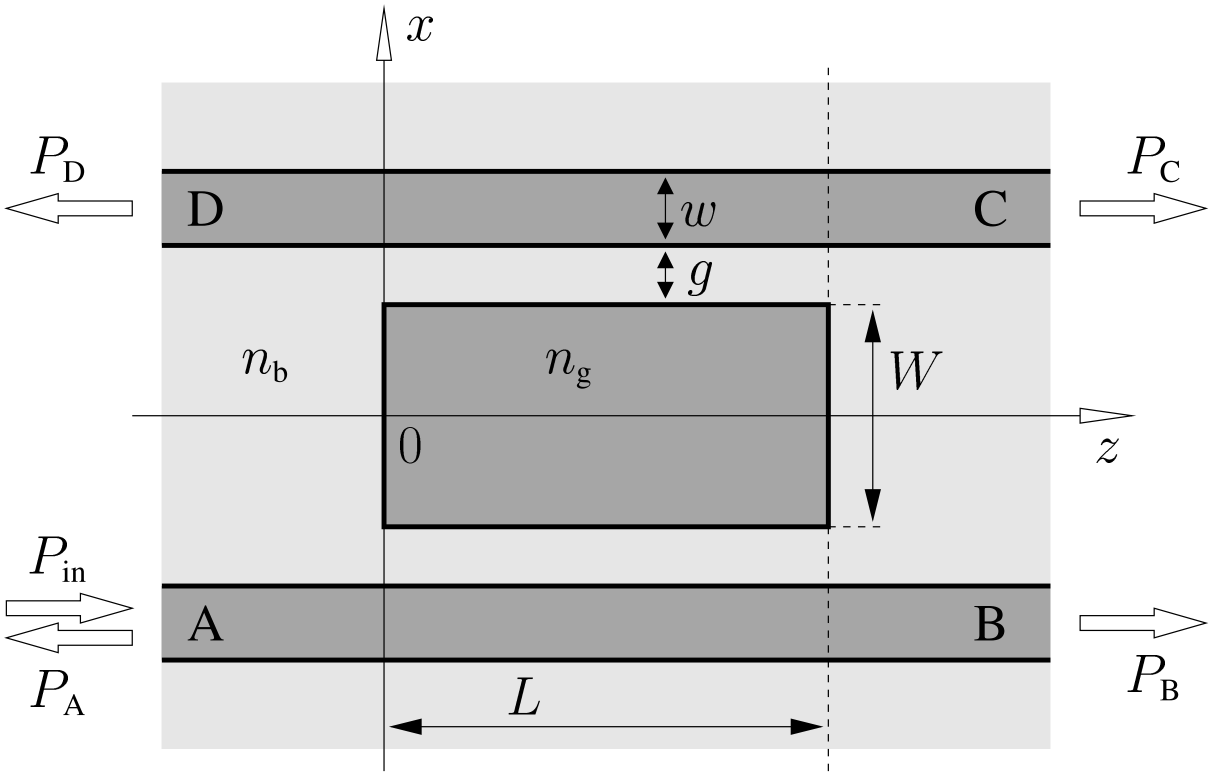

$$P_{\mathrm{in}}(\lambda) = 1. \quad P_{\mathrm{A}}, P_{\mathrm{B}}, P_{\mathrm{C}}, P_{\mathrm{D}} \; = \; ?$$

Analysis of resonant coupling between dielectric waveguides via rectangular microcavities

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- Rectangular microresonators
- Mode expansion simulations
- Resonances I
- Coupled mode modeling
- Detaching the cavity
- Multimode facet reflectivity
- Slab mode characterization
- Resonances II

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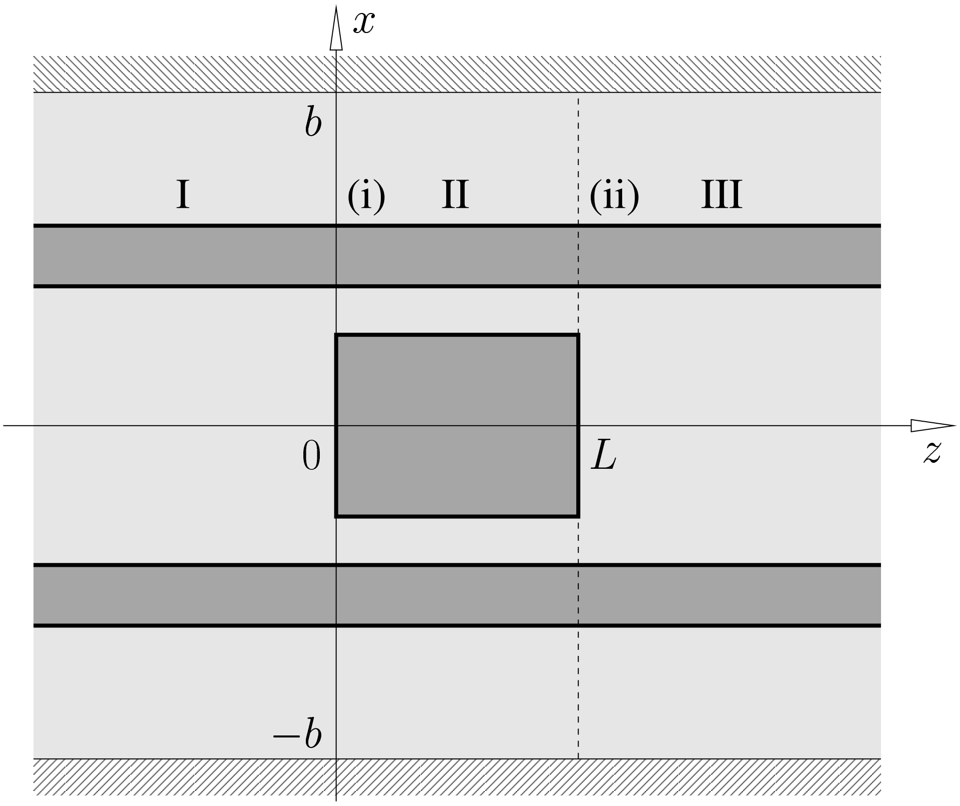


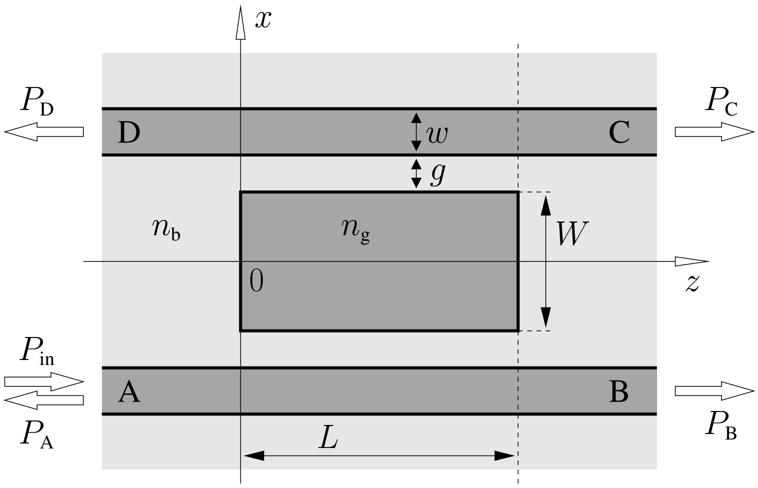
- Spatial 2D setting, fixed angular frequency ω / vacuum wavelength λ.
- Division into z-homogeneous segments I, II, III.
- Artificial boundaries $\rightarrow$ discretization of the mode spectrum.
- Mode expansion ansatz, $s = \mathrm{I}, \mathrm{II}, \mathrm{III}$:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}^{s}(x,z,t) = \sum_m F_m^s \boldsymbol{\psi}_m^{s,\mathrm{f}}(x)\, \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\beta_m^s z} + \sum_m B_m^s \boldsymbol{\psi}_m^{s,\mathrm{b}}(x)\, \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}\beta_m^s z}.$$

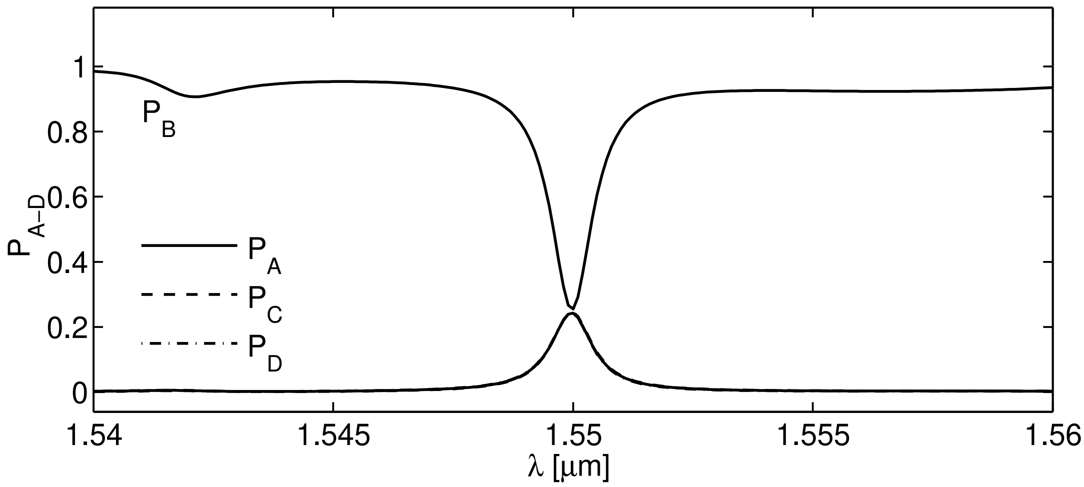
  $\{\boldsymbol{\psi}_m^{s,\mathrm{f}}, \boldsymbol{\psi}_m^{s,\mathrm{b}}\}$: forward + backward, propagating + evanescent terms.

- Inner product for overlap computations + orthogonality relations
  $\rightarrow$ bidirectional field expansions at the junctions (i), (ii),
  $\rightarrow$ linear set of equations for $\{F_m^s, B_m^s\}$.
- $F_0^{\mathrm{I}} = 1, \quad F_j^{\mathrm{I}} = 0, j \neq 0, \quad B_m^{\mathrm{III}} = 0; \quad B_m^{\mathrm{I}}, F_m^{\mathrm{II}}, B_m^{\mathrm{II}}, F_m^{\mathrm{III}} = \ ?$

Rigorous numerical basis.

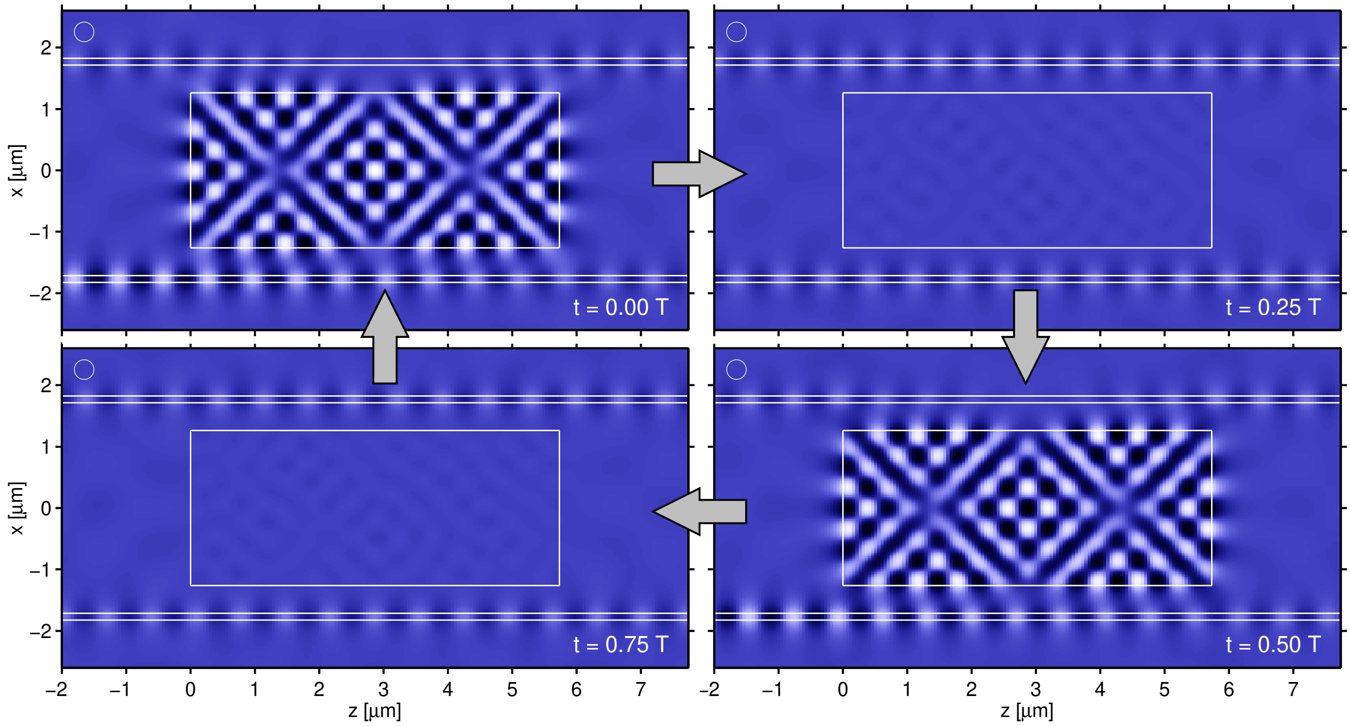


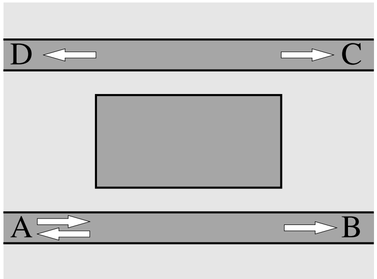
$n_{\rm g} = 3.40,\ n_{\rm b} = 1.45,$
$W = 2.524\,\mu\text{m},\ L = 5.738\,\mu\text{m},$
$w = 0.112\,\mu\text{m},\ g = 0.450\,\mu\text{m},$
TE polarization,
$\boldsymbol{\mathcal{E}}(x, z, t) = \text{Re}\,(0, E_y, 0)(x, z)\ \text{e}^{\text{i}\omega t},$
$\omega = 2\pi/T = 2\pi\text{c}/\lambda.$

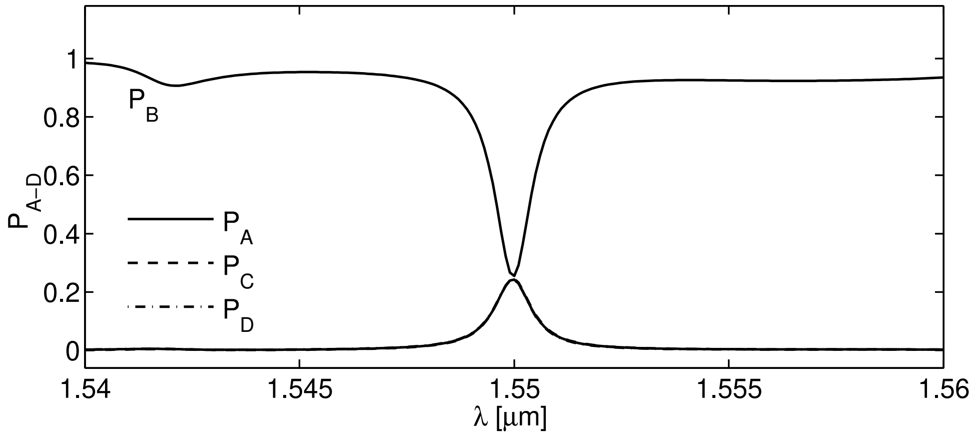


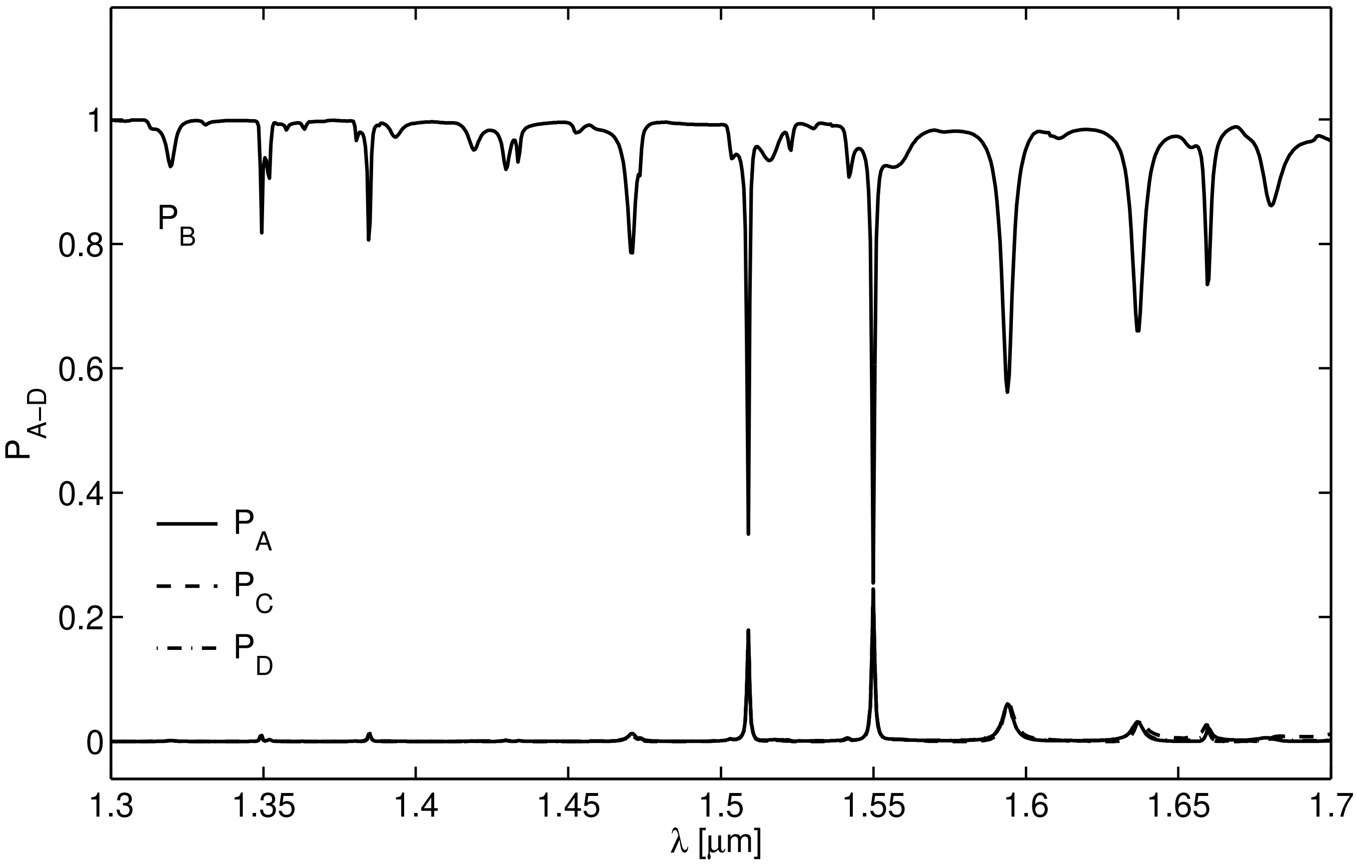
$\lambda = 1.55\,\mu\text{m}$:
$P_{\rm D} = 0.24,$
$P_{\rm C} = 0.24,$
$P_{\rm A} = 0.24,$
$P_{\rm B} = 0.25.$

$\lambda = 1.55\,\mu\text{m},\ T = 5.17\,\text{fs},\ \mathcal{E}_y(x, z, t)$:

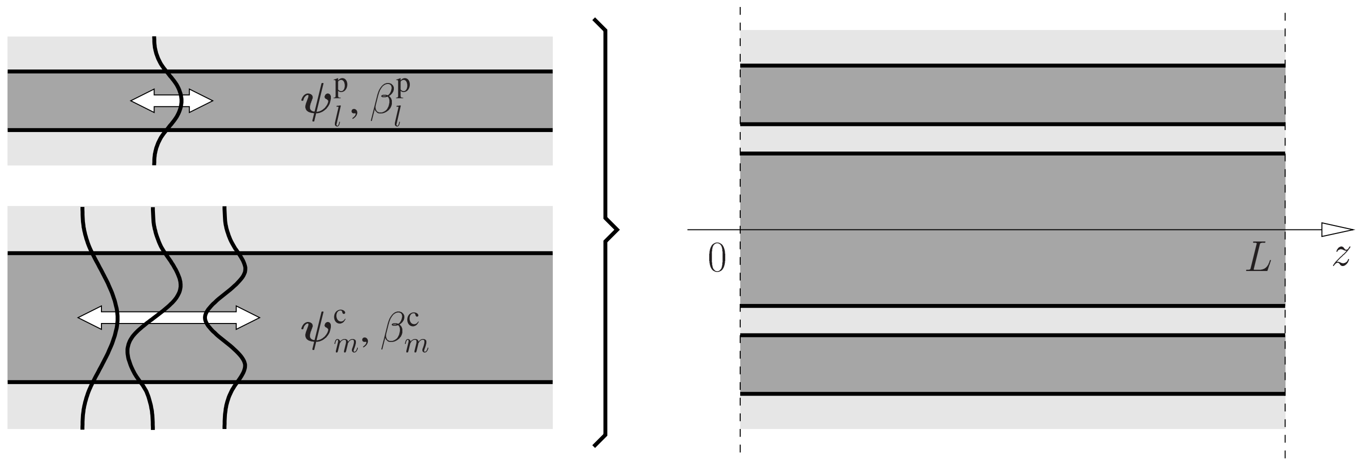








Classification / characterization of resonances ?



Basis: Guided fields $\boldsymbol{\psi}_l^{\mathrm{p}}$, $\boldsymbol{\psi}_m^{\mathrm{c}}$, and propagation constants β_l^{p}, β_m^{c}, of the port and cavity waveguides.

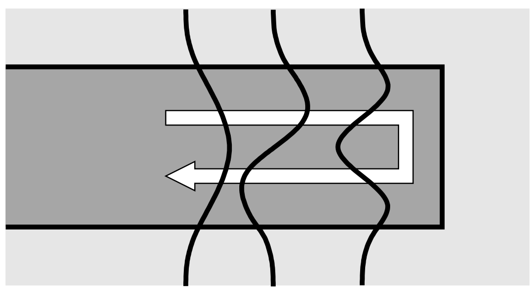
Amplitudes of forward and backward mode copies:

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_{\mathrm{p}} \\ \boldsymbol{F}_{\mathrm{c}} \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} \boldsymbol{B}_{\mathrm{p}} \\ \boldsymbol{B}_{\mathrm{c}} \end{pmatrix}.$$

CMT model for the propagation along the cavity segment:

$$\boldsymbol{F}(L) = \mathsf{T}(L)\,\boldsymbol{F}(0),$$
$$\boldsymbol{B}(0) = \mathsf{T}(L)\,\boldsymbol{B}(L),$$

$$\mathsf{T} = \begin{pmatrix} \mathsf{T}_{\mathrm{pp}} & \mathsf{T}_{\mathrm{pc}} \\ \mathsf{T}_{\mathrm{cp}} & \mathsf{T}_{\mathrm{cc}} \end{pmatrix}.$$



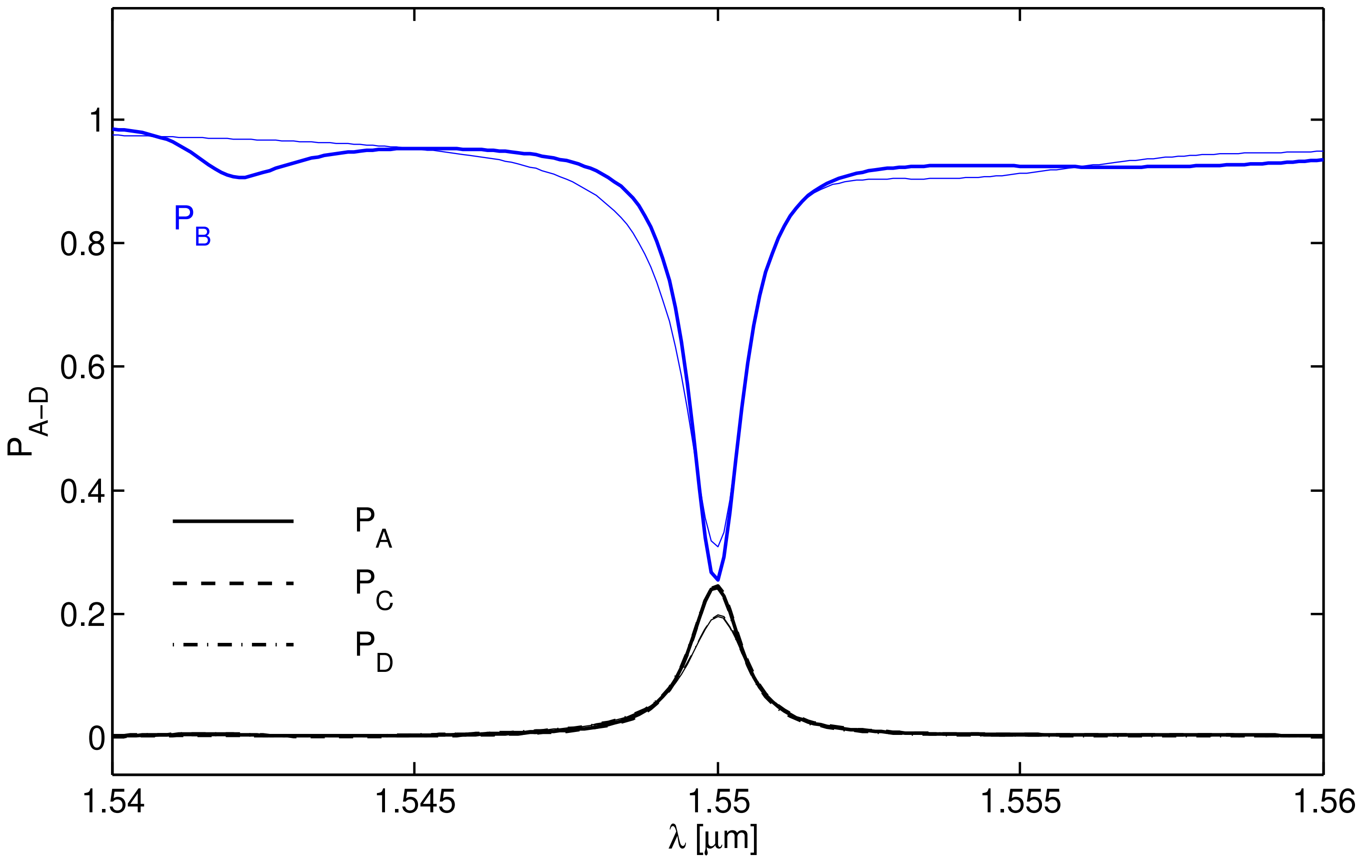
Rigorous model for the cavity facet, guided field reflectivity R:

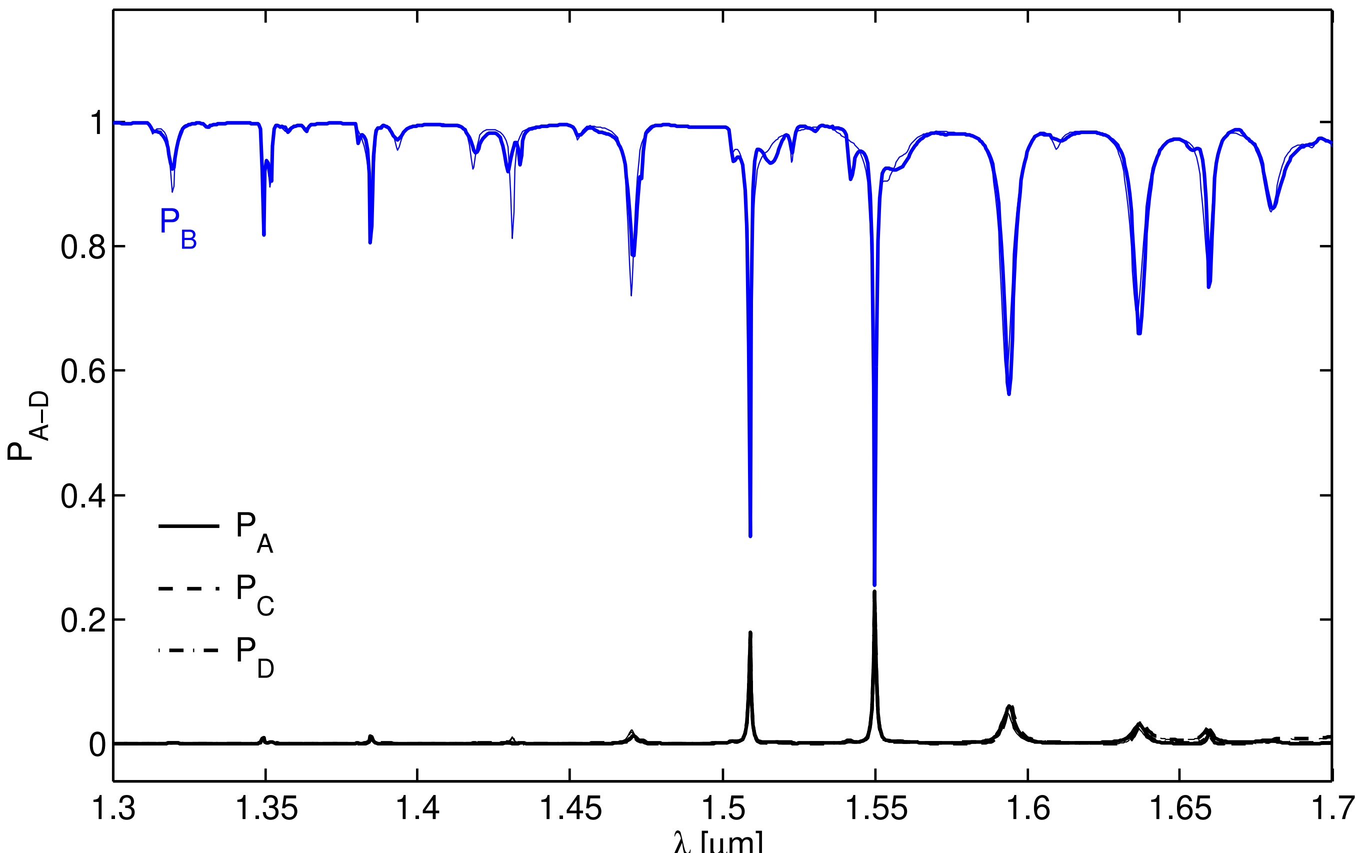
$$\boldsymbol{B}_{\mathrm{c}}(L) = \mathsf{R}\,\boldsymbol{F}_{\mathrm{c}}(L),$$
$$\boldsymbol{F}_{\mathrm{c}}(0) = \mathsf{R}\,\boldsymbol{B}_{\mathrm{c}}(0).$$

$$\boldsymbol{F}_{\mathrm{p}}(0) = (1,0)^{\mathsf{T}}, \qquad \boldsymbol{F}_{\mathrm{p}}(L) = (\mathsf{T}_{\mathrm{pp}} + \mathsf{T}_{\mathrm{pc}}\mathsf{R}\Omega^{-1}\mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cp}})\boldsymbol{F}_{\mathrm{p}}(0),$$
$$\boldsymbol{B}_{\mathrm{p}}(L) = (0,0)^{\mathsf{T}}, \qquad \boldsymbol{B}_{\mathrm{p}}(0) = \mathsf{T}_{\mathrm{pc}}\mathsf{R}\Omega^{-1}\mathsf{T}_{\mathrm{cp}}\,\boldsymbol{F}_{\mathrm{p}}(0),$$

where $\Omega = 1 - \mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$.

$$P_{\mathrm{A}} = |B_{\mathrm{p},1}(0)|^2, \quad P_{\mathrm{B}} = |F_{\mathrm{p},1}(L)|^2, \quad P_{\mathrm{C}} = |F_{\mathrm{p},2}(L)|^2, \quad P_{\mathrm{D}} = |B_{\mathrm{p},2}(0)|^2.$$





Rigorous simulations, CMT model.

- *Resonances in the coupled mode model*
  $\leftrightarrow$ Singularities in the resonance denominator.

- *The cavity amplifies*
  a field $\boldsymbol{F}_{\mathrm{c}}(L) = \boldsymbol{v}$ that corresponds to a large eigenvalue a of Ω^{-1}:
  $\Omega^{-1}\boldsymbol{v} = a\,\boldsymbol{v}$ or $\Omega\,\boldsymbol{v} = (1/a)\,\boldsymbol{v}$, amplification $A = |a|^2$.

  $\rightarrow$ Find a configuration, where approximately

  - $\Omega = 1 - \mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has a zero eigenvalue.
  - $\mathsf{T}_{\mathrm{cc}}\mathsf{R}\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has an eigenvalue 1:

$$\boldsymbol{F}_{\mathrm{c}}(L) \xrightarrow{\mathsf{R}} \boldsymbol{B}_{\mathrm{c}}(L) \xrightarrow{\mathsf{T}_{\mathrm{cc}}} \boldsymbol{B}_{\mathrm{c}}(0) \xrightarrow{\mathsf{R}} \boldsymbol{F}_{\mathrm{c}}(0) \xrightarrow{\mathsf{T}_{\mathrm{cc}}} \boldsymbol{F}_{\mathrm{c}}(L)\,.$$

  - $\mathsf{T}_{\mathrm{cc}}\mathsf{R}$ has an eigenvalue $\mu = +1$ or $\mu = -1$.

  Amplification for a field $\boldsymbol{v}$ with $\mathsf{T}_{\mathrm{cc}}\mathsf{R}\,\boldsymbol{v} = \mu\,\boldsymbol{v}$, $\mu = r\,\mathrm{e}^{\mathrm{i}\chi}$:

$$A = \frac{1}{|\mu^2 - 1|^2} = \frac{1}{1 + r^4 - 2r^2\cos(2\chi)}\,.$$

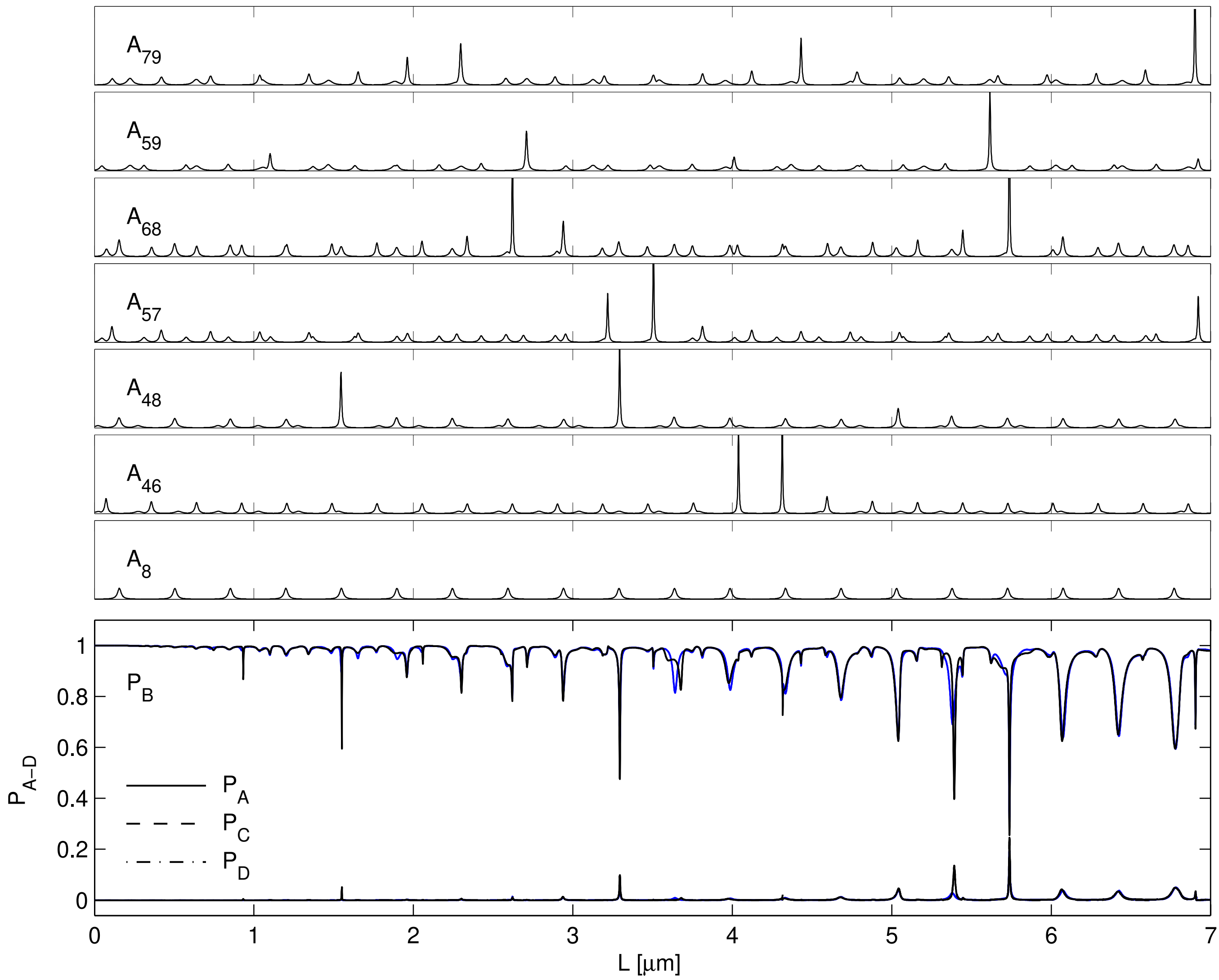
- *Isolated cavity:*

$$\mathsf{T}_{\mathrm{cc}} \xrightarrow{g \rightarrow \infty} \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\beta_1^{\mathrm{c}}L}, \ldots, \mathrm{e}^{-\mathrm{i}\beta_N^{\mathrm{c}}L}\right)\,.$$

- *Requirements for resonant configurations:*
  - High facet reflectivity for a specific mode or mode set, and
  - Correct phase relations for all involved basis fields.

  $\rightarrow$ Consider only single mode and bimodal resonances.

Amplification factors and power transmission versus the cavity length L:

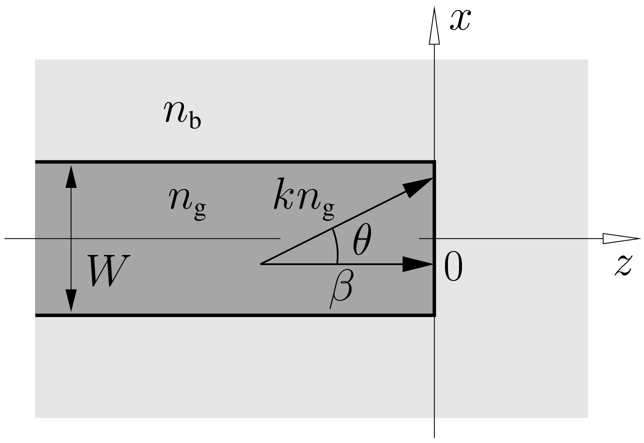


A_m, A_{lm}: Amplification with the cavity field restricted to modes m or l, m.

- *Single mode resonances:*
  $\mathsf{T}_{cc} = \exp(-\mathrm{i}\beta^c L)$, $\mathsf{R} = r\ \exp(\mathrm{i}\varphi)$, $\rightarrow$ $\mu = r\exp(\mathrm{i}\varphi - \mathrm{i}\beta^c L)$.
  Lengths of resonant cavities: $L_j = (\varphi + j\pi)/\beta^c$, integer j.

- *Bimodal resonances:*
  Large reflectivity ($|\mu| \approx 1$) and proper phase relations required.
  Observation: A few pronounced, narrow resonances occur.
  $\rightarrow$ Take a look at facets of multimode slab waveguides.

The guided mode representation is adequate for the field inside the cavity.

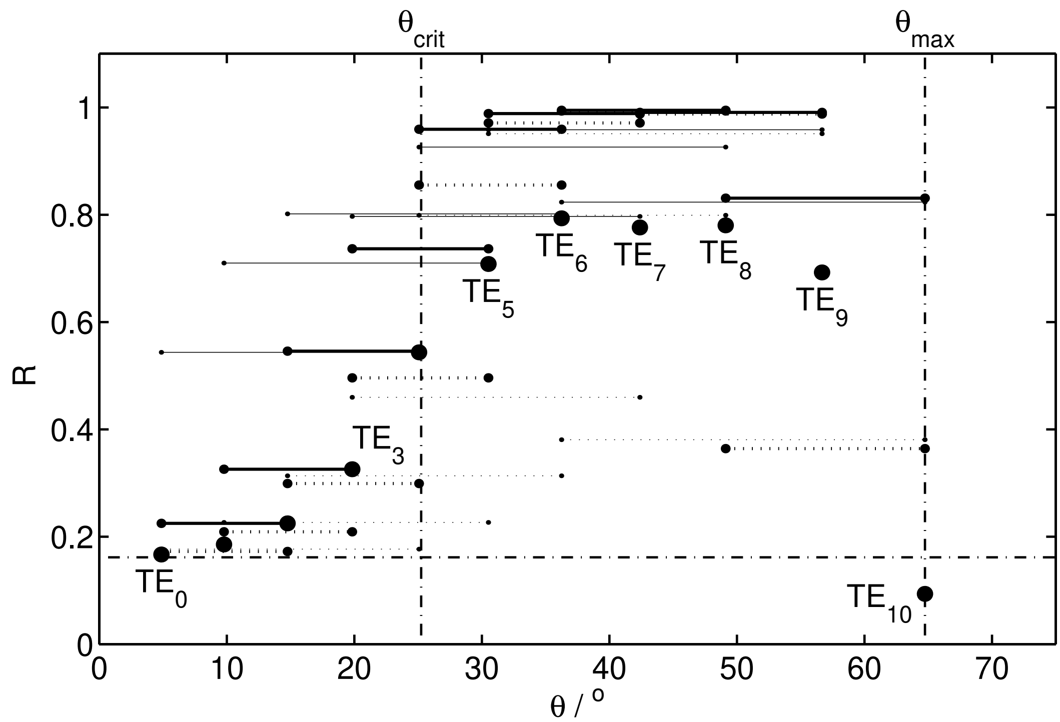


Critical angle for total reflection:
$\sin\theta_{crit} = n_b/n_g$.

Maximum mode angle:
$\sin\theta_{max} = \sqrt{1 - n_b^2/n_g^2}$,

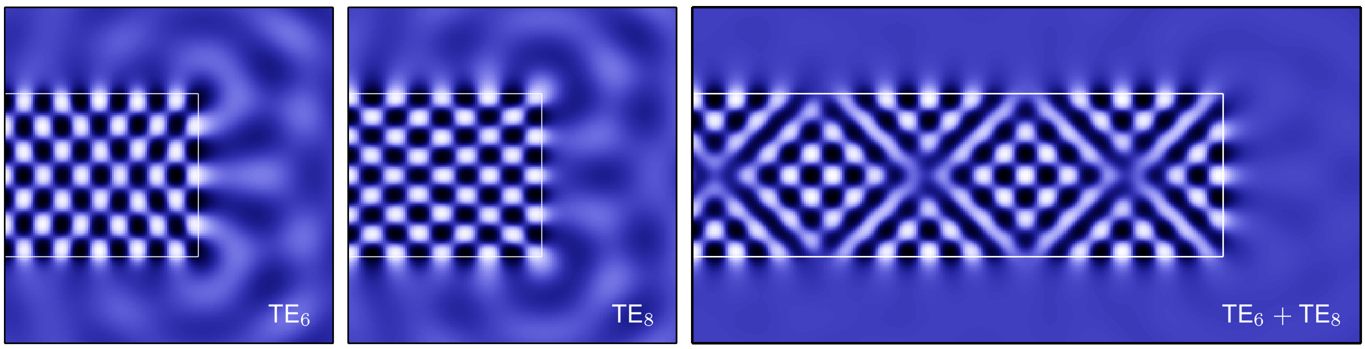
High single mode reflectivity:
$\theta > \theta_{crit} \;\leftrightarrow\; \beta/k < \sqrt{n_g^2 - n_b^2}$,
with $\beta/k > n_b$: $n_g > \sqrt{2}\, n_b$.

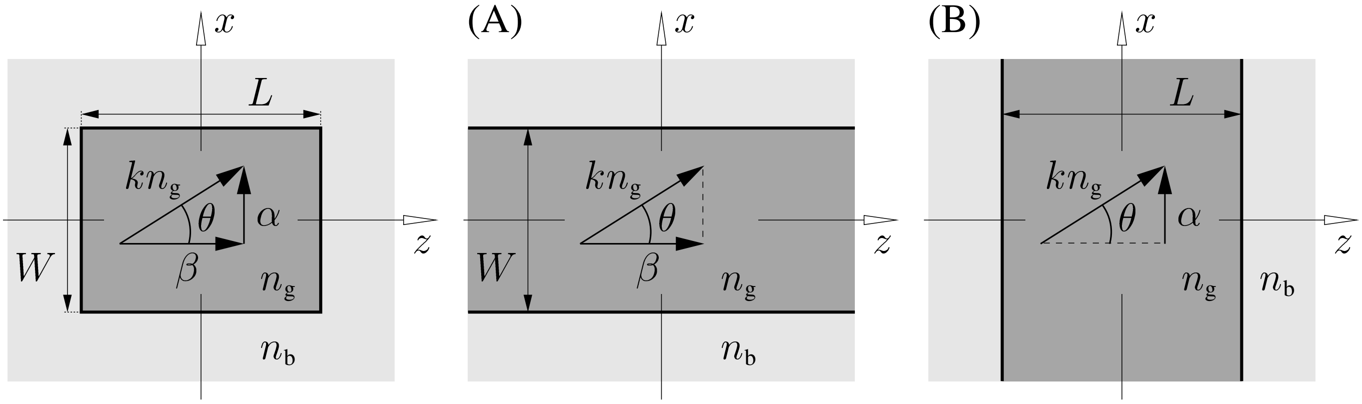
Relative power reflected into the input field versus the mode angle θ:



$n_b = 1.45$,
$n_g = 3.40$,
$W = 2.524\,\mu$m,
$\lambda = 1.55\,\mu$m.

• Single mode reflectivity, ●——● Maximum reflectivity for a two mode superposition,
●·······● Reflectivity for a two mode superposition where $\mathcal{E}_y$ vanishes at the facet edges:





Single mode resonance, field in $-W/2 < x < W/2,\ -L/2 < z < L/2$:

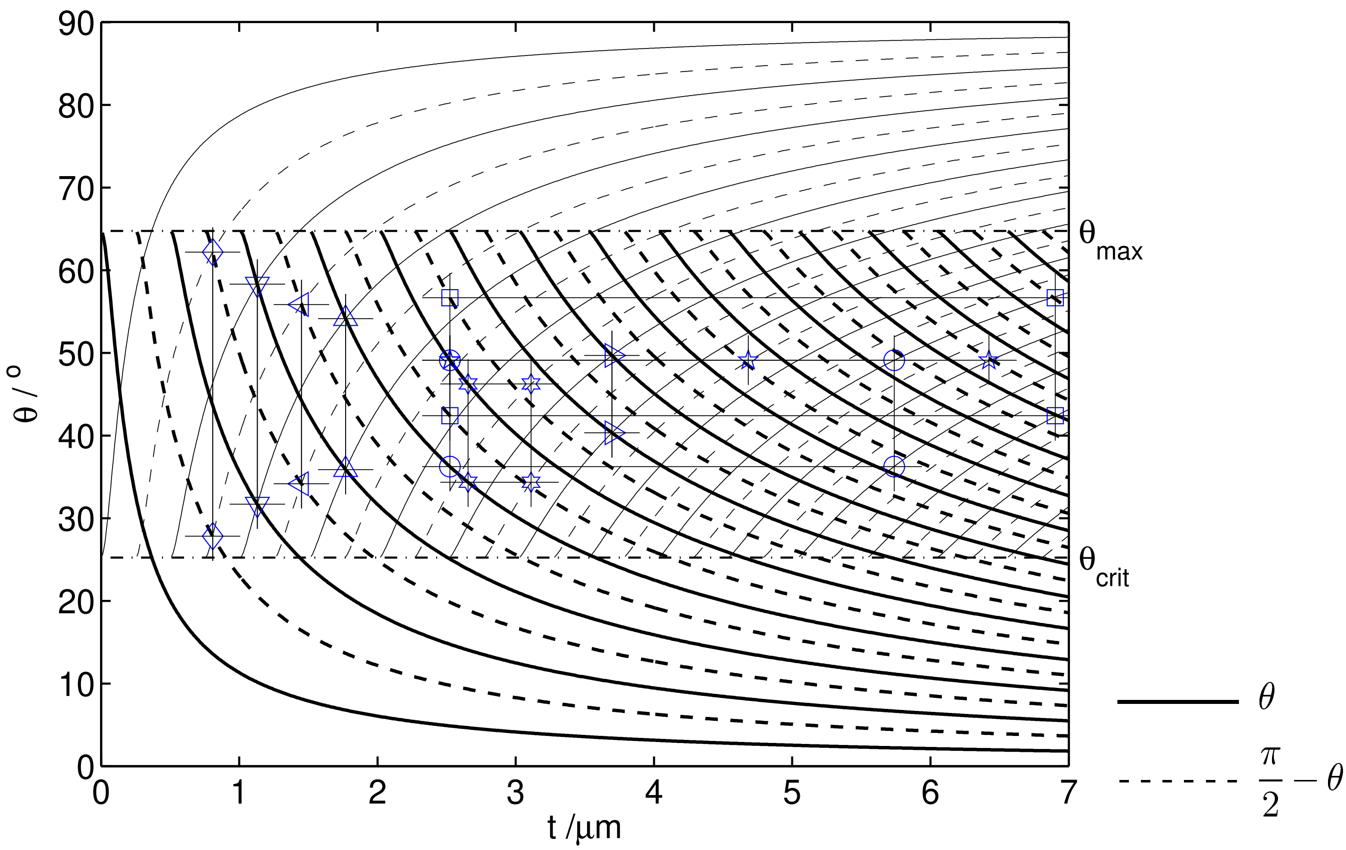
(A) $\quad E_y(x,z) = E_0\,\phi(x)(\,\mathrm{e}^{-\mathrm{i}\beta z} + b\,\mathrm{e}^{\mathrm{i}\beta z}),\ \ \phi(x) = \mathrm{e}^{-\mathrm{i}\alpha x} \pm \mathrm{e}^{\mathrm{i}\alpha x},$

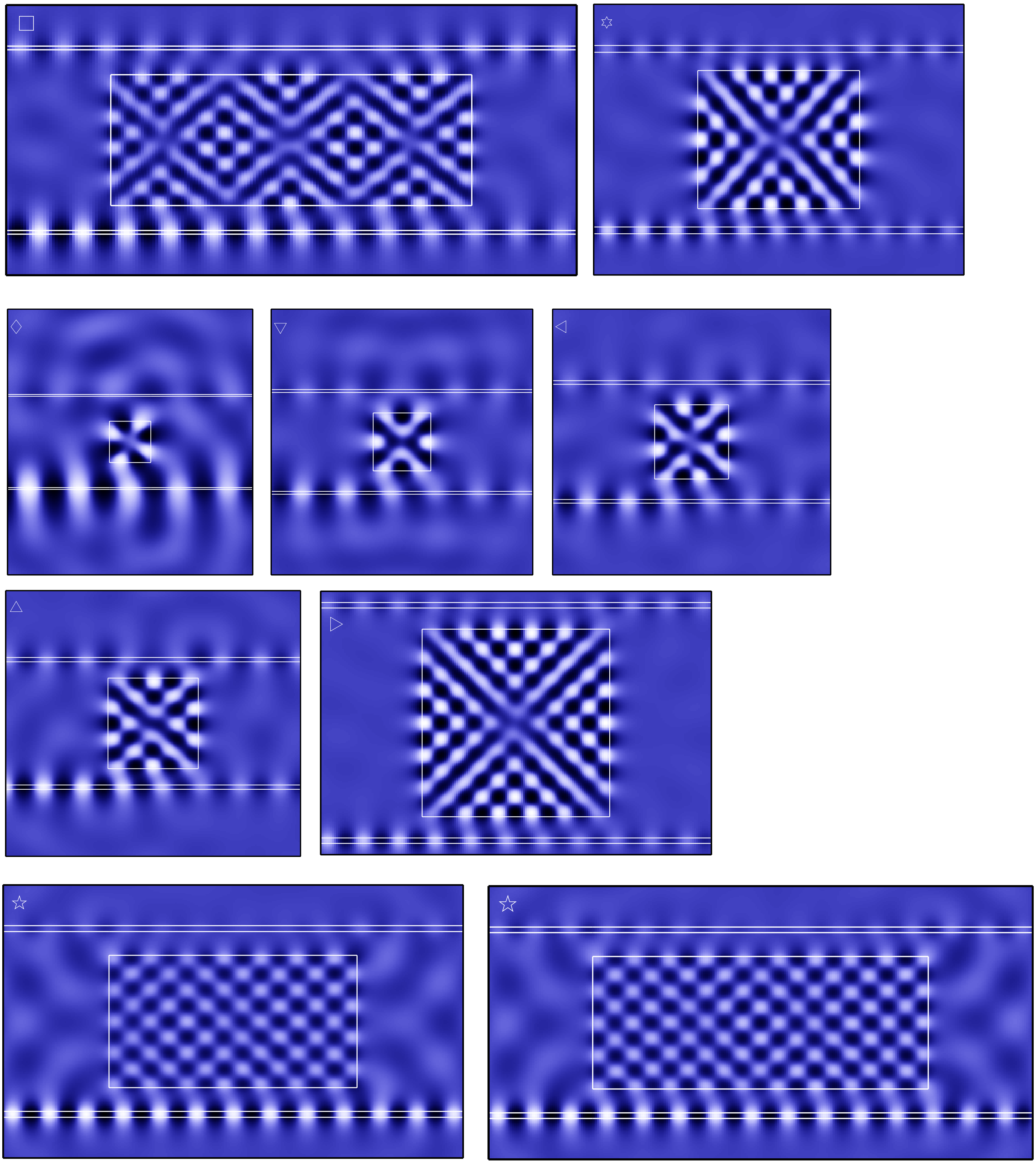
(B) $\quad E_y(x,z) = E_0\,\psi(z)(\,\mathrm{e}^{-\mathrm{i}\alpha x} + d\,\mathrm{e}^{\mathrm{i}\alpha x}),\ \ \psi(z) = \mathrm{e}^{-\mathrm{i}\beta z} \pm \mathrm{e}^{\mathrm{i}\beta z}.$

Viewpoints (A) and (B) are equivalent: $b = \pm 1,\ d = \pm 1$.

$\rightarrow$ *Resonant configurations W, L for given $\lambda = 2\pi/k$, n_g, n_b:*
The slab waveguide of thickness W supports a mode with angle θ, while the slab of thickness L guides a field with angle $\pi/2-\theta$.

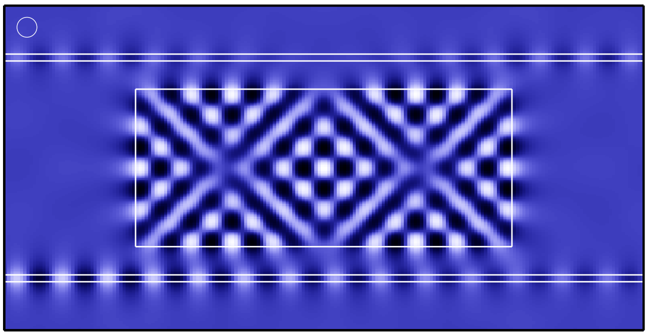
Bimodal resonance: Two pairs of modes with proper symmetry ($\leftrightarrow$ facet edges) satisfy the conditions simultaneously.





Symbol	Mode order	$W/\,\mu$m	$L/\,\mu$m	$w/\,\mu$m	$g/\,\mu$m	Symbol	Mode order	$W/\,\mu$m	$L/\,\mu$m	$w/\,\mu$m	$g/\,\mu$m
○ (0)	$6\times20+8\times16$	2.524	5.738	0.112	0.450	◁ (5)	$3\times5+5\times3$	1.451	1.451	0.073	0.400
□ (1)	$7\times22+9\times16$	2.524	6.902	0.068	0.480	△ (6)	$4\times6+6\times4$	1.772	1.772	0.083	0.320
✡ (2)	$6\times11+8\times9$	2.655	3.111	0.130	0.350	▷ (7)	$10\times12+12\times10$	3.696	3.696	0.108	0.420
◇ (3)	$1\times3+3\times1$	0.810	0.810	0.035	0.490	☆ (8)	8×13	2.524	4.682	0.112	0.450
▽ (4)	$2\times4+4\times2$	1.132	1.132	0.059	0.400	☆ (9)	8×18	2.524	6.423	0.112	0.450

$n_{\rm b}=1.45,\ \ n_{\rm g}=3.40,\ \ \lambda=1.55\,\mu$m.



- Integrated optical microresonators with rectangular cavities:
  Convenient trial structures for Helmholtz solvers in a 2-D setting.
- Rigorous and efficient numerical simulations by means of
  mode expansion (bidirectional eigenmode propagation) techniques.
- More insight from an approximative coupled mode theory model:
  Classification of resonances related to separated cavities;
  single mode and bimodal resonances are found to be relevant.
- Quality of some of the pronounced bimodal resonances:
  Related to a total reflection effect at facets of high contrast
  multimode slab waveguides.
- A separable (approximative) representation in terms of 1-D slab modes
  is adequate for the resonant field patterns:
  Identification of resonant cavity geometries.