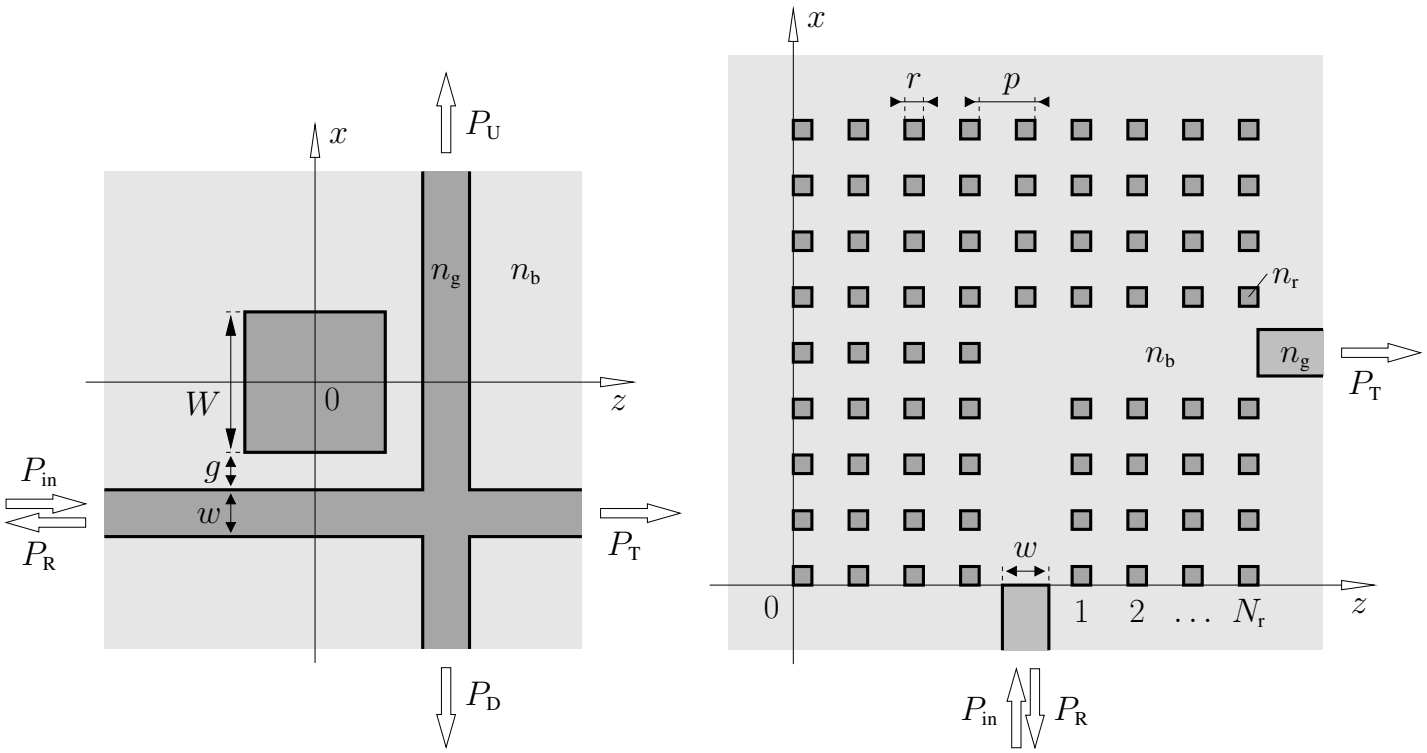

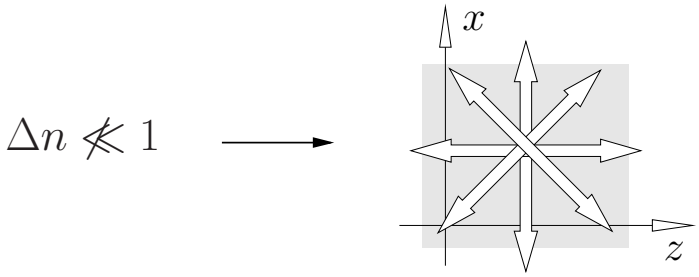

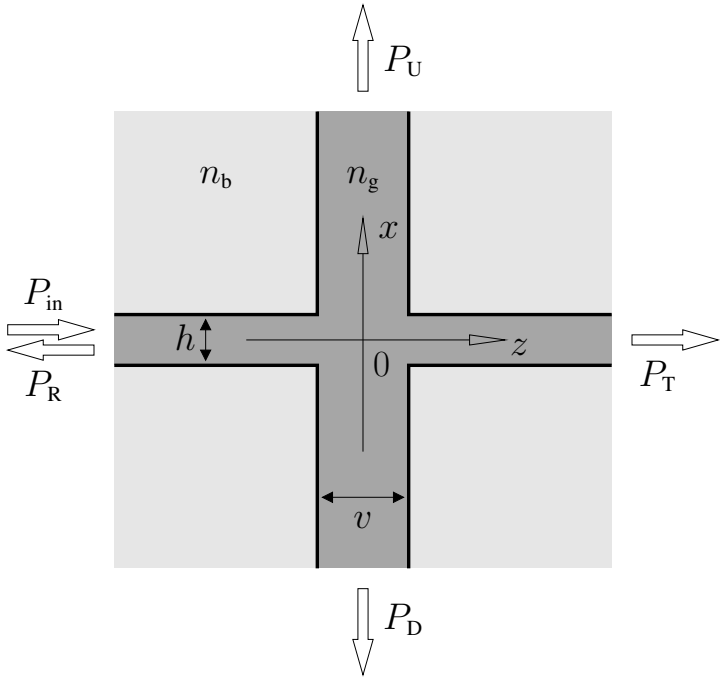

Directions x and z deserve the same treatment.

QUEP: An eigenmode expansion scheme that implements that constraint.

Quadridirectional eigenmode expansion scheme for 2D modeling of wave propagation in integrated optics

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- Quadridirectional mode propagation:
  - Problem setting
  - Eigenmode expansion
  - Algebraic procedure
- Numerical results:
  - Gaussian beams in free space
  - Bragg grating
  - Waveguide crossing
  - Square resonator with perpendicular ports
  - Photonic crystal bend

11th International Workshop on Optical Waveguide Theory and Numerical Modelling
Prague, Czech Republic, April 4 – 5, 2003

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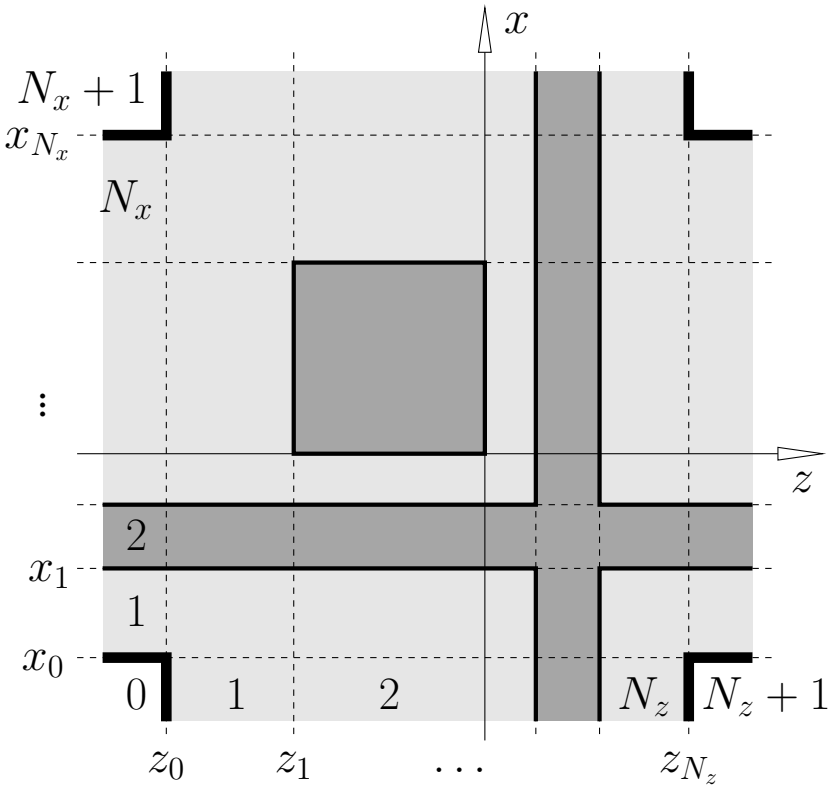
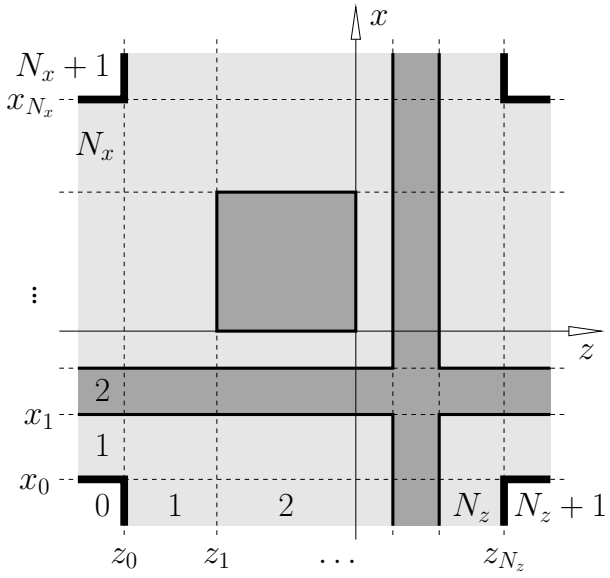
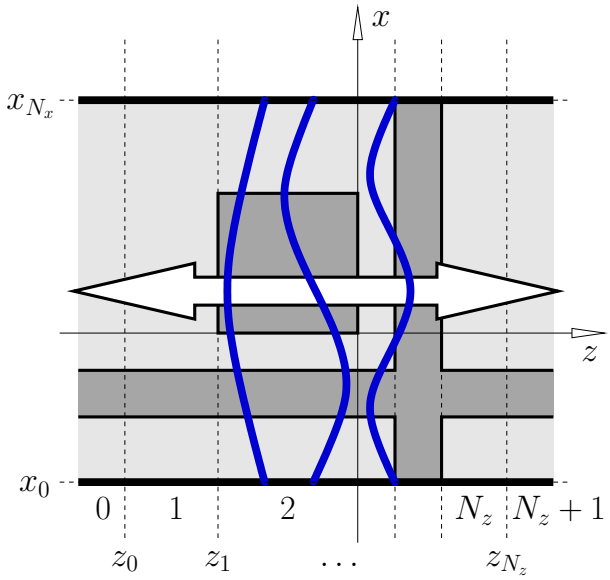


- 2D problem, Cartesian coordinates x, z, TE / TM polarization.
- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.
- Fixed frequency simulations, vacuum wavelength $\lambda = 2\pi/k$.
- Rectangular interior computational domain, influx & outflux across all four boundaries, external regions are homogeneous along x or z.
- Assumption $E_y = 0$, $H_y = 0$ on the corner points and on the external border lines is reasonable for the problems under investigation.



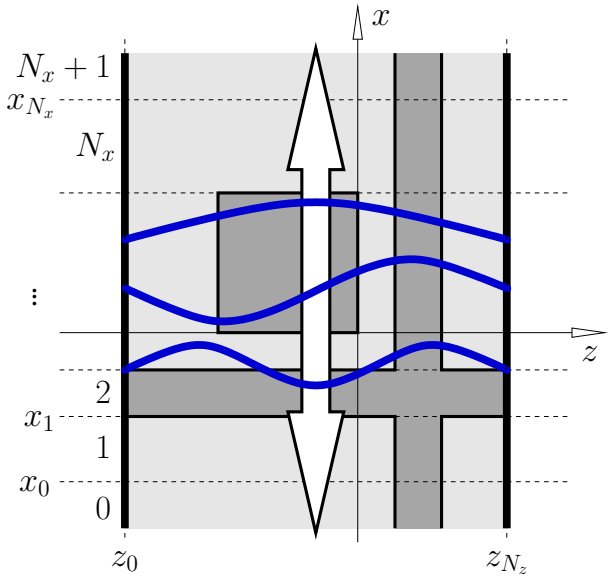
Basis fields, defined by
Dirichlet boundary conditions
$E_y = 0$ (TE) or $H_y = 0$ (TM):



Horizontally traveling eigenmodes:

M_z profiles	$\boldsymbol{\psi}^d_{s,m}(x)$
and propagation constants	$\pm\beta_{s,m}$

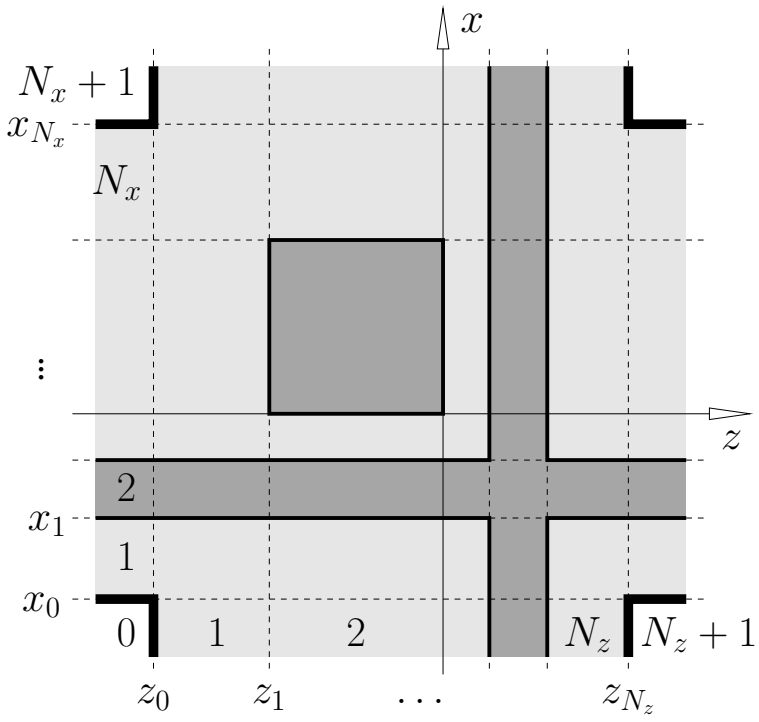
of order m, on slice s,
for propagation directions $d =$ f, b,



. . . and
vertically traveling fields:

M_x profiles	$\boldsymbol{\phi}^d_{l,m}(z)$
and propagation constants	$\pm\gamma_{l,m}$

of order m, on layer l,
for propagation directions $d =$ u, d.



Ansatz for the optical field:

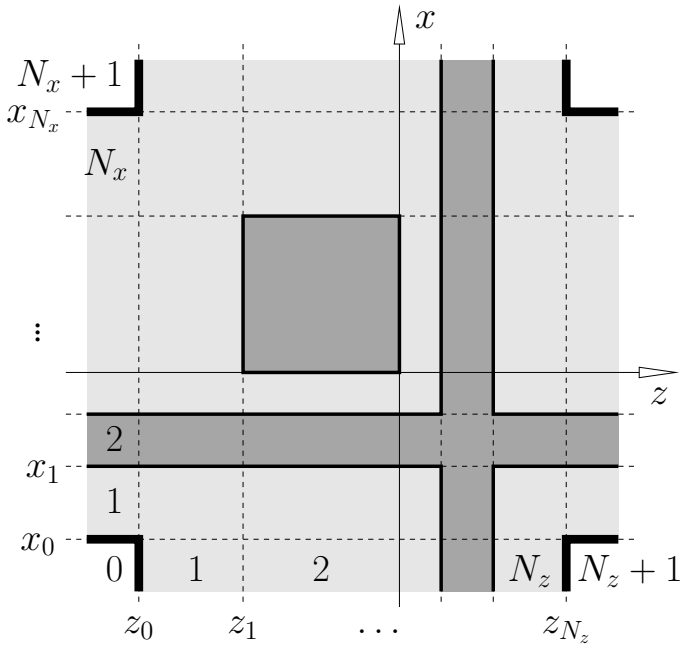
$$\begin{pmatrix}\boldsymbol{\mathcal{E}}\\ \boldsymbol{\mathcal{H}}\end{pmatrix}(x,z,t) = \mathrm{Re}\left\{\sum_{m=0}^{M_z-1} F_{s,m}\boldsymbol{\psi}^{\mathrm{f}}_{s,m}(x)\,\mathrm{e}^{-\mathrm{i}\beta_{s,m}(z-z_{s-1})} + \sum_{m=0}^{M_z-1} B_{s,m}\boldsymbol{\psi}^{\mathrm{b}}_{s,m}(x)\,\mathrm{e}^{+\mathrm{i}\beta_{s,m}(z-z_s)} + \sum_{m=0}^{M_x-1} U_{l,m}\boldsymbol{\phi}^{\mathrm{u}}_{l,m}(z)\,\mathrm{e}^{-\mathrm{i}\gamma_{l,m}(x-x_{l-1})} + \sum_{m=0}^{M_x-1} D_{l,m}\boldsymbol{\phi}^{\mathrm{d}}_{l,m}(z)\,\mathrm{e}^{+\mathrm{i}\gamma_{l,m}(x-x_l)}\right\}\mathrm{e}^{\mathrm{i}\omega t},$$

for $z_{s-1} \le z \le z_s,\ s = 1,\ldots,N_z$,
and $x_{l-1} \le x \le x_l,\ l = 1,\ldots,N_x$.

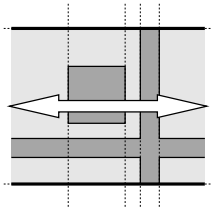
Mode products $\leftrightarrow$ normalization, projection:

$$(\boldsymbol{E}_1,\boldsymbol{H}_1;\boldsymbol{E}_2,\boldsymbol{H}_2)_x = \frac{1}{4}\int (E^*_{1,x}H_{2,y} - E^*_{1,y}H_{2,x} + H^*_{1,y}E_{2,x} - H^*_{1,x}E_{2,y})\mathrm{d}x\,,$$

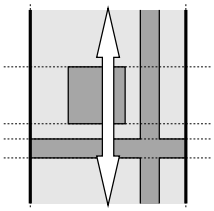
$$(\boldsymbol{E}_1,\boldsymbol{H}_1;\boldsymbol{E}_2,\boldsymbol{H}_2)_z = \frac{1}{4}\int (E^*_{1,y}H_{2,z} - E^*_{1,z}H_{2,y} + H^*_{1,z}E_{2,y} - H^*_{1,y}E_{2,z})\mathrm{d}z\,.$$



- Consistent bidirectional projection at all interfaces
  $\longrightarrow$ linear system of equations in $\{F_{s,m}, B_{s,m}, U_{l,m}, D_{l,m}\}$.
- Influx: $\boldsymbol{F}_0,\ \boldsymbol{B}_{N_x+1},\ \boldsymbol{U}_0,\ \boldsymbol{D}_{N_z+1}$ $\longrightarrow$ RHS.
- Outflux: $\boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.
- "Exact" mode profiles $\longrightarrow$ interior problems decouple:

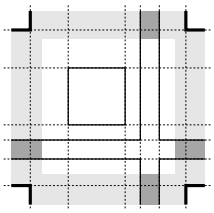


Solve for $\boldsymbol{F}_2, \ldots, \boldsymbol{F}_{N_z}$ and $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_{N_z-1}$
in terms of $\boldsymbol{F}_1$ and $\boldsymbol{B}_{N_z}$ $\longrightarrow$ BEP.

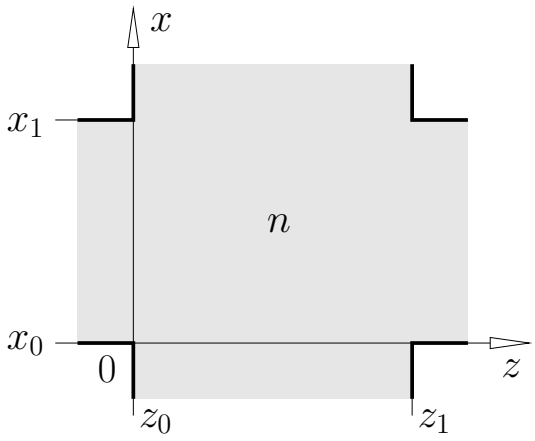


Solve for $\boldsymbol{U}_2, \ldots, \boldsymbol{U}_{N_x}$ and $\boldsymbol{D}_1, \ldots, \boldsymbol{D}_{N_x-1}$
in terms of $\boldsymbol{U}_1$ and $\boldsymbol{D}_{N_x}$ $\longrightarrow$ BEP.

- Continuity of E and H on outer interfaces:

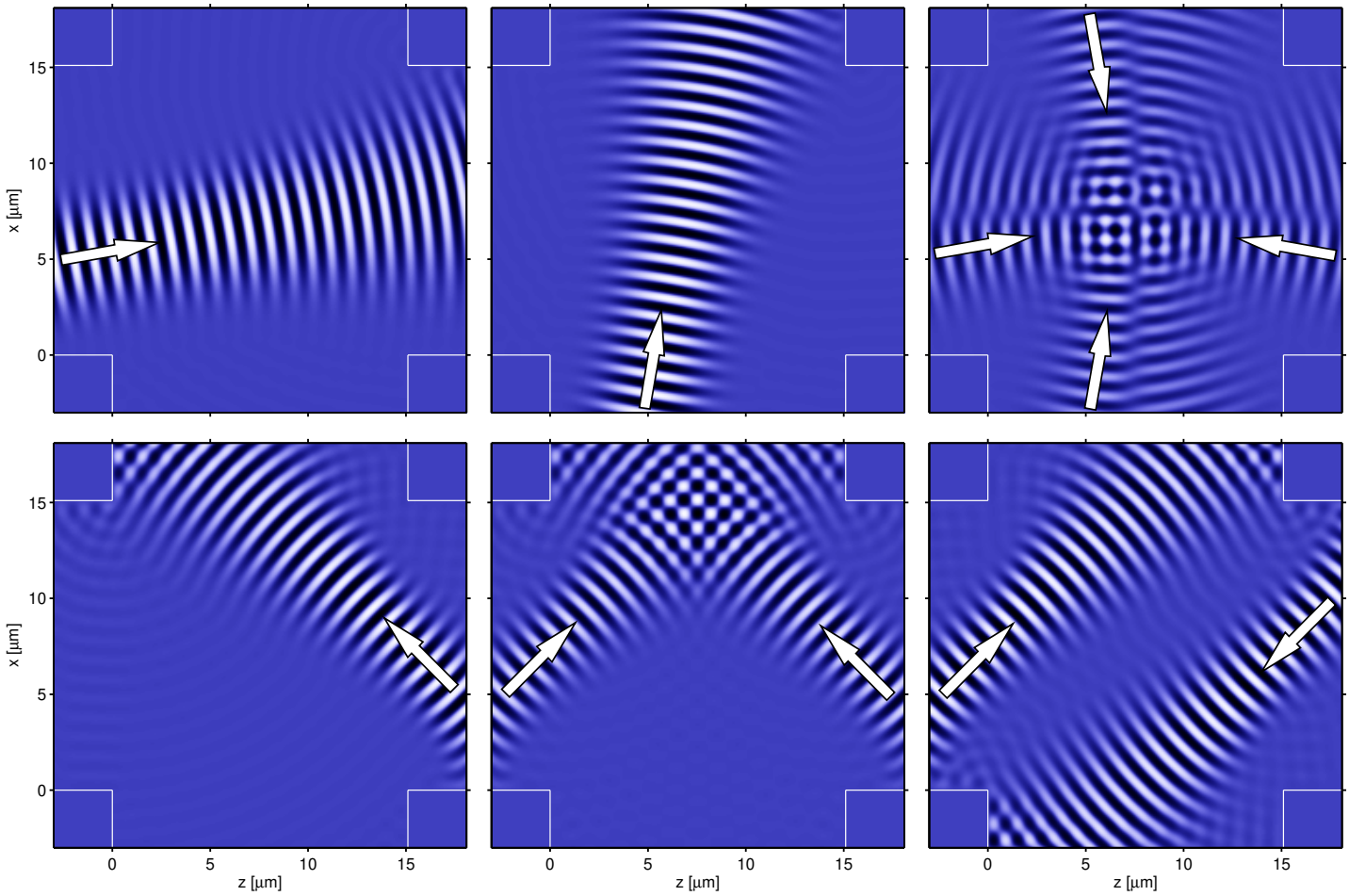


Interior BEP solutions
+ equations at $z = z_0, z_{N_z},\ x = x_0, x_{N_x}$
$\longrightarrow \boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

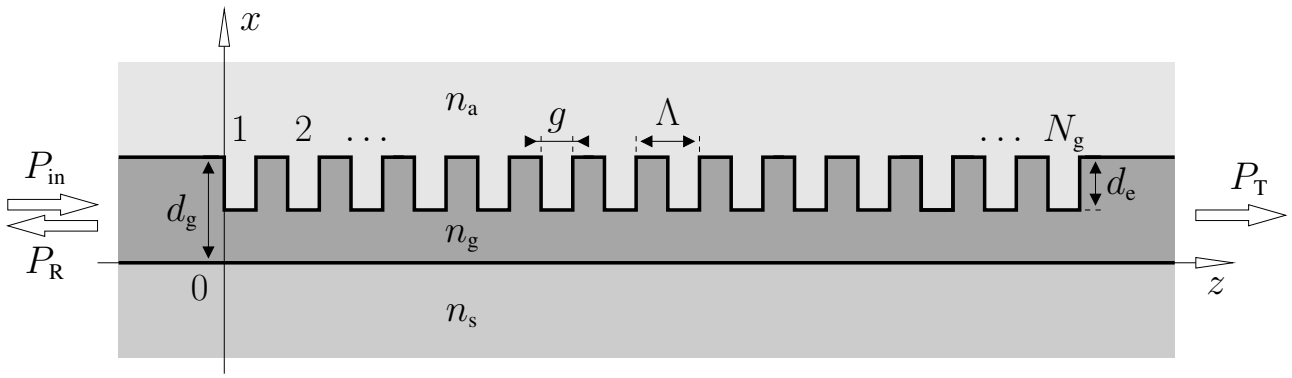


$n = 1.0,\ \lambda = 1.0\,\mu$m, TE polarization,
$x, z \in [0, 15.1]\,\mu$m, $M_x = M_z = 150$.

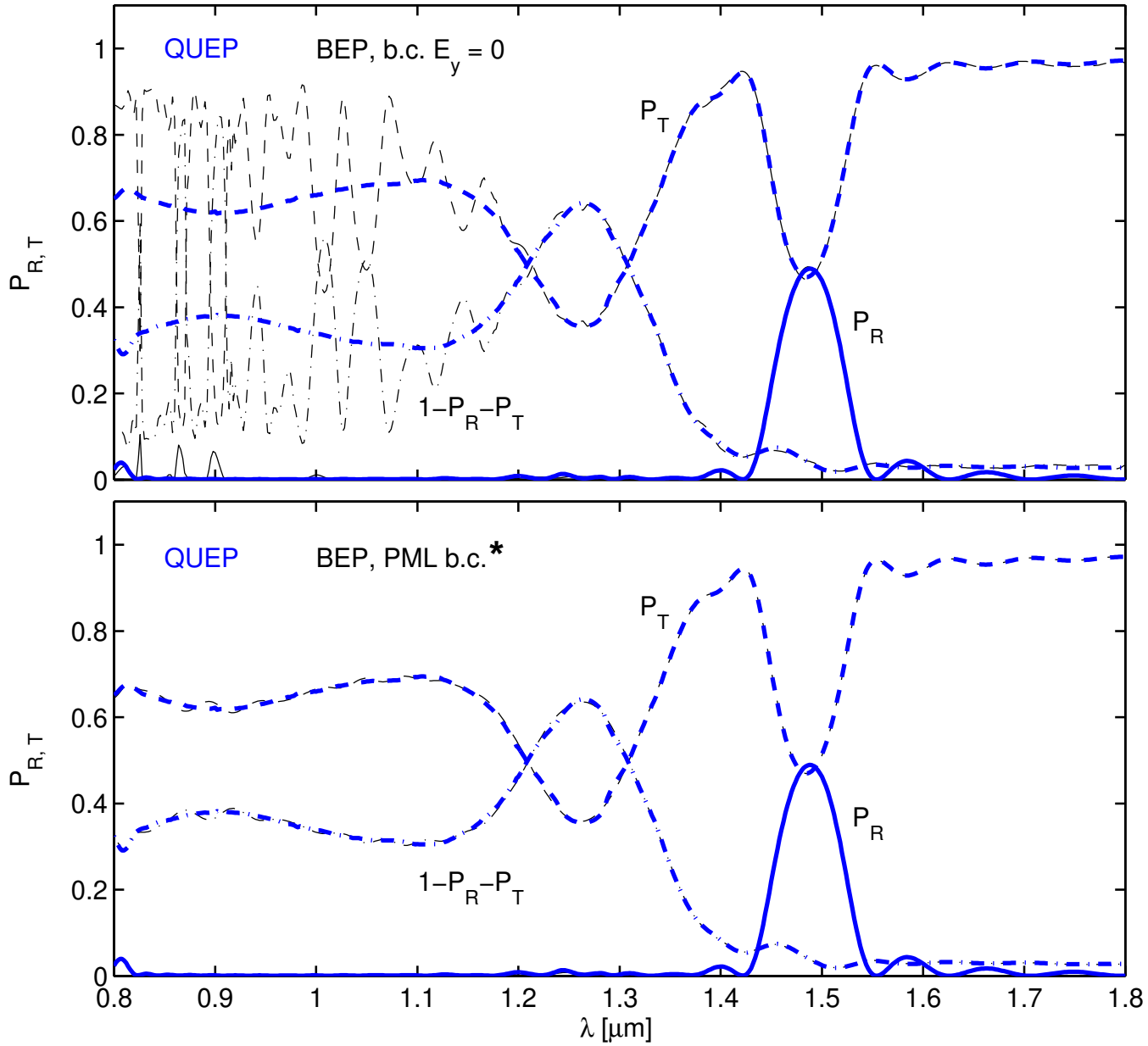
$E_y(x, z)$:



Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;
bottom: 45° tilt angle, initially centered beams.

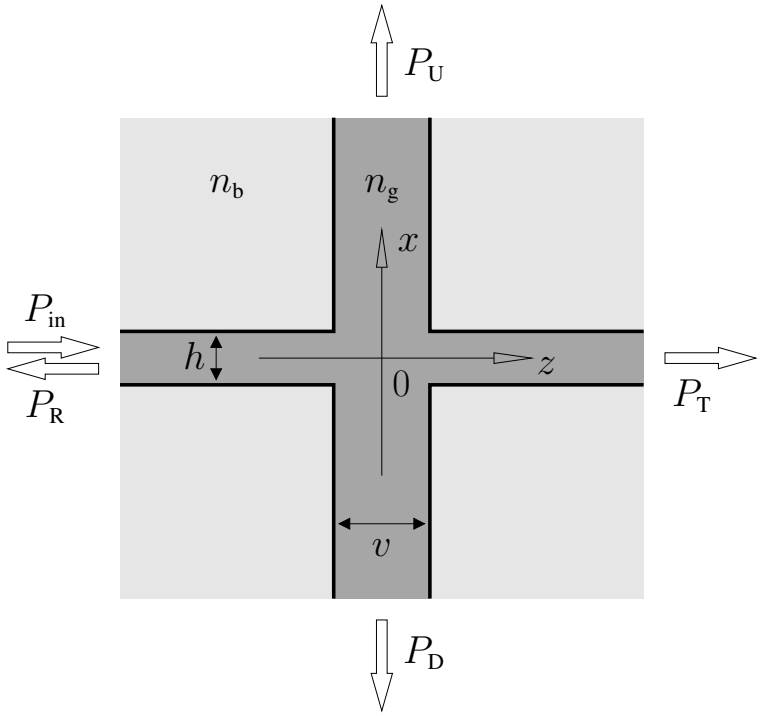


* $d_g/\,\mu$m	$d_e/\,\mu$m	n_s (SiO_2)	n_g (Si_3N_4)	n_a	$\Lambda/\,\mu$m	g	N_g
0.500	0.125	≈ 1.45	≈ 1.99	1.0	0.430 μm	$\Lambda/2$	20



$x \in [-4, 2]\,\mu$m, 60 modes (QUEP, BEP $E_y = 0$ b.c.), $z \in [-1.8, 10.185]\,\mu$m, 120 modes (QUEP).

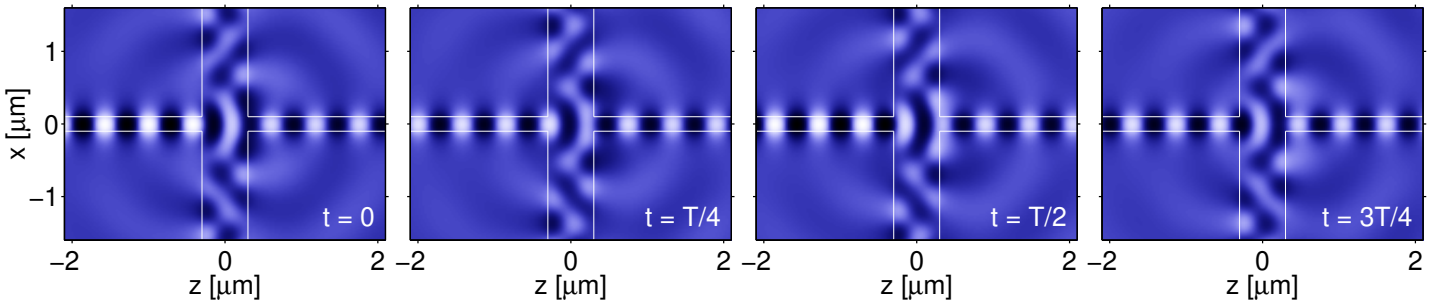
* J. Čtyroký et. al., Optical and Quantum Electronics **34**, 455–470, 2002; reference: BEP2 (PML b.c.).



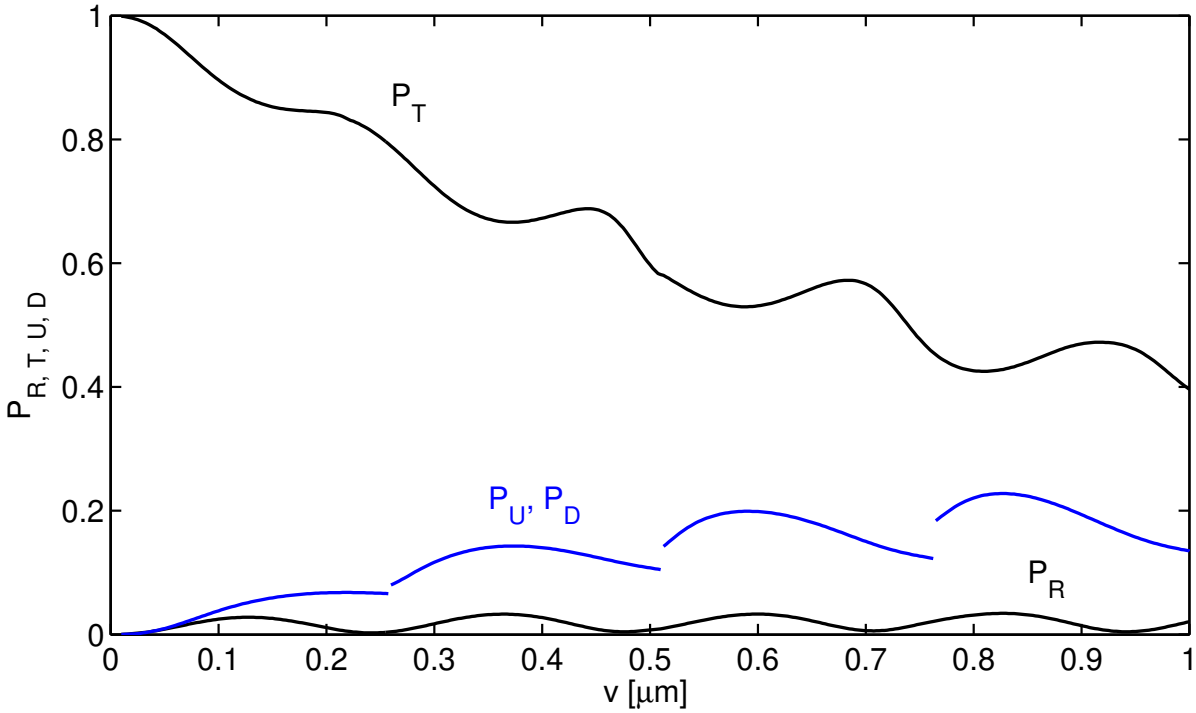
$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.45,$
$\lambda = 1.55\,\mu\mathrm{m},\ h = 0.2\,\mu\mathrm{m},$

TE, $x, z \in [-2, 2]\,\mu\mathrm{m},$
$M_x = M_z = 80.$

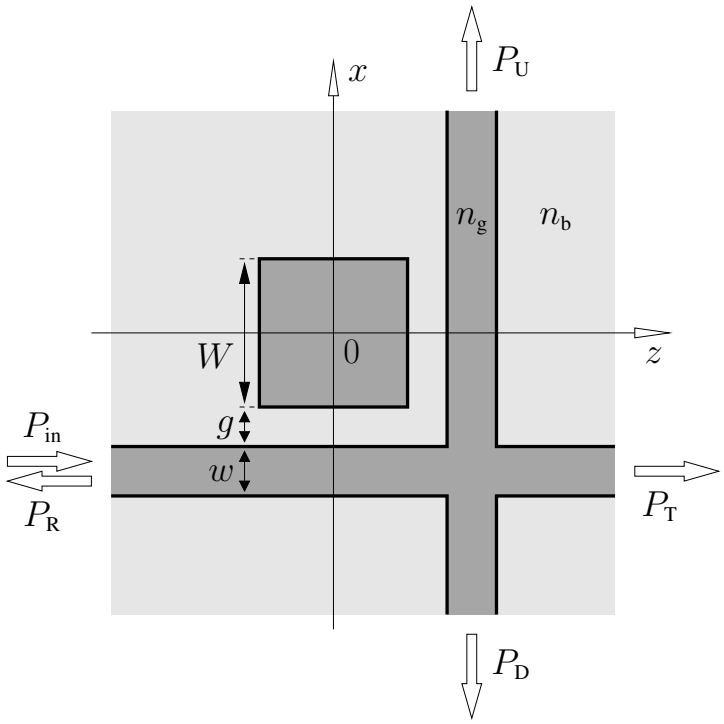
$v = 0.6\,\mu\mathrm{m}$



$v \in [0, 1]\,\mu\mathrm{m}$:



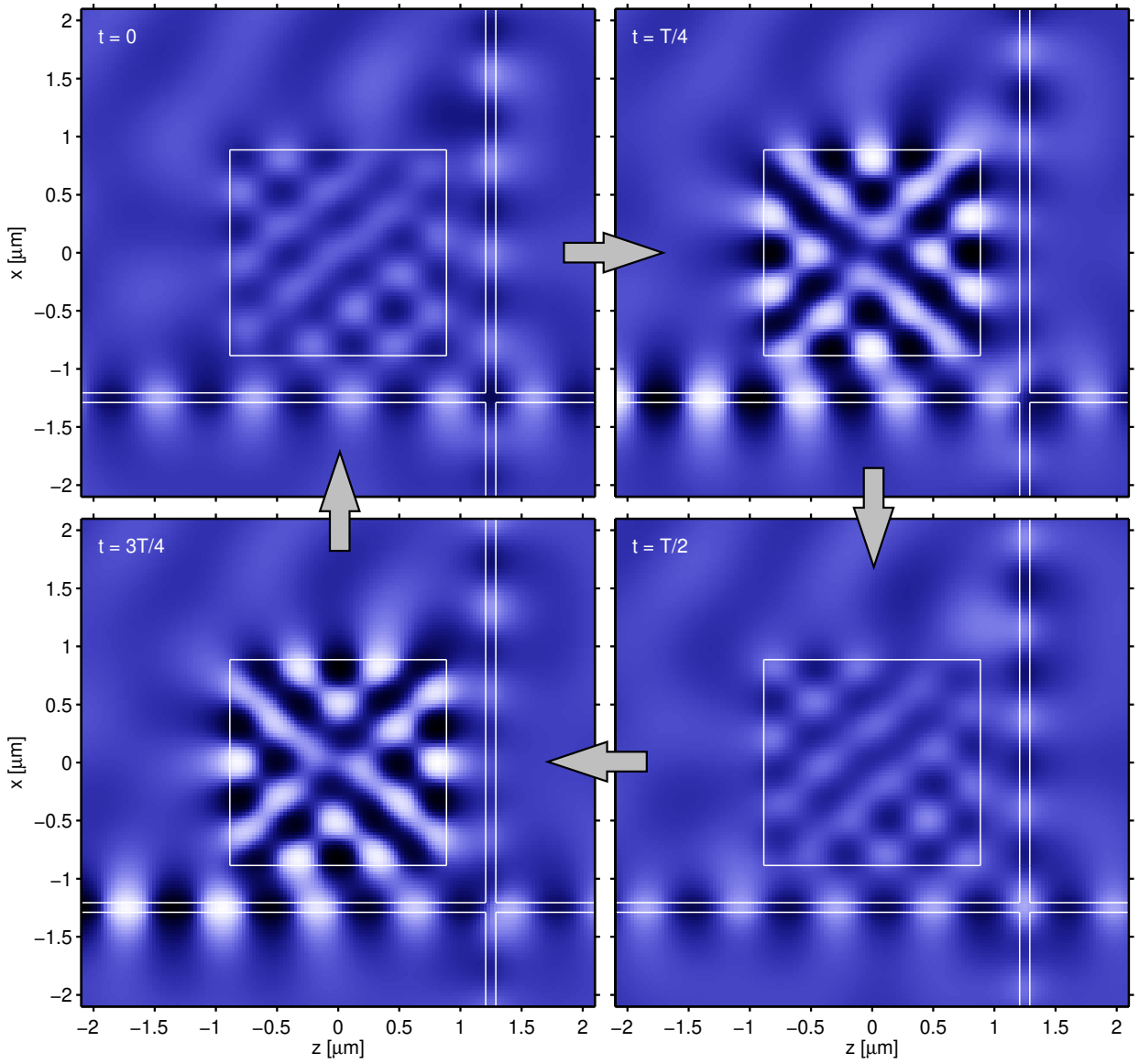
Power conservation: error $< 10^{-3}$.

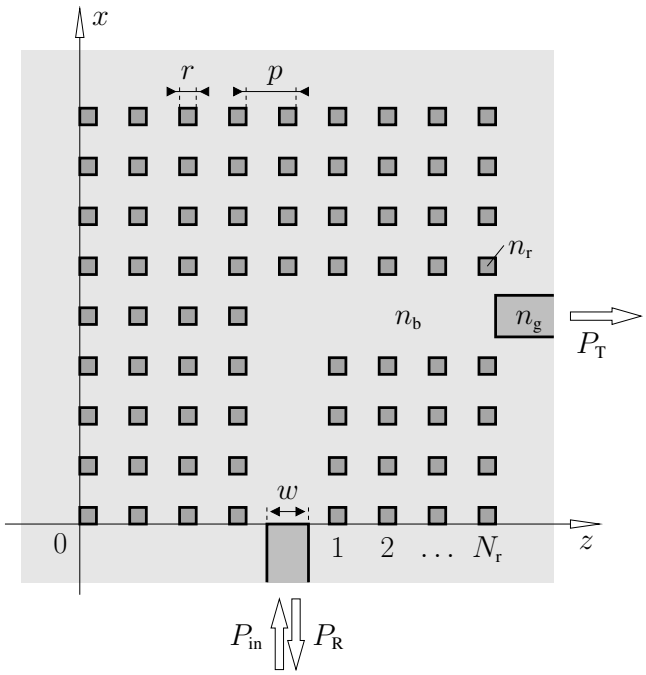


$n_{\mathrm{g}} = 3.4,\ n_{\mathrm{b}} = 1.45,$
$W = 1.772\,\mu\mathrm{m},\ w = 0.083\,\mu\mathrm{m},$
$g = 0.32\,\mu\mathrm{m},\ \lambda = 1.55\,\mu\mathrm{m},$

TE, $x, z \in [-4, 4]\,\mu\mathrm{m},$
$M_x = M_z = 120.$

$P_{\mathrm{R}} = 0.27,\ P_{\mathrm{T}} = 0.34,$
$P_{\mathrm{D}} = 0.07,\ P_{\mathrm{U}} = 0.16.$

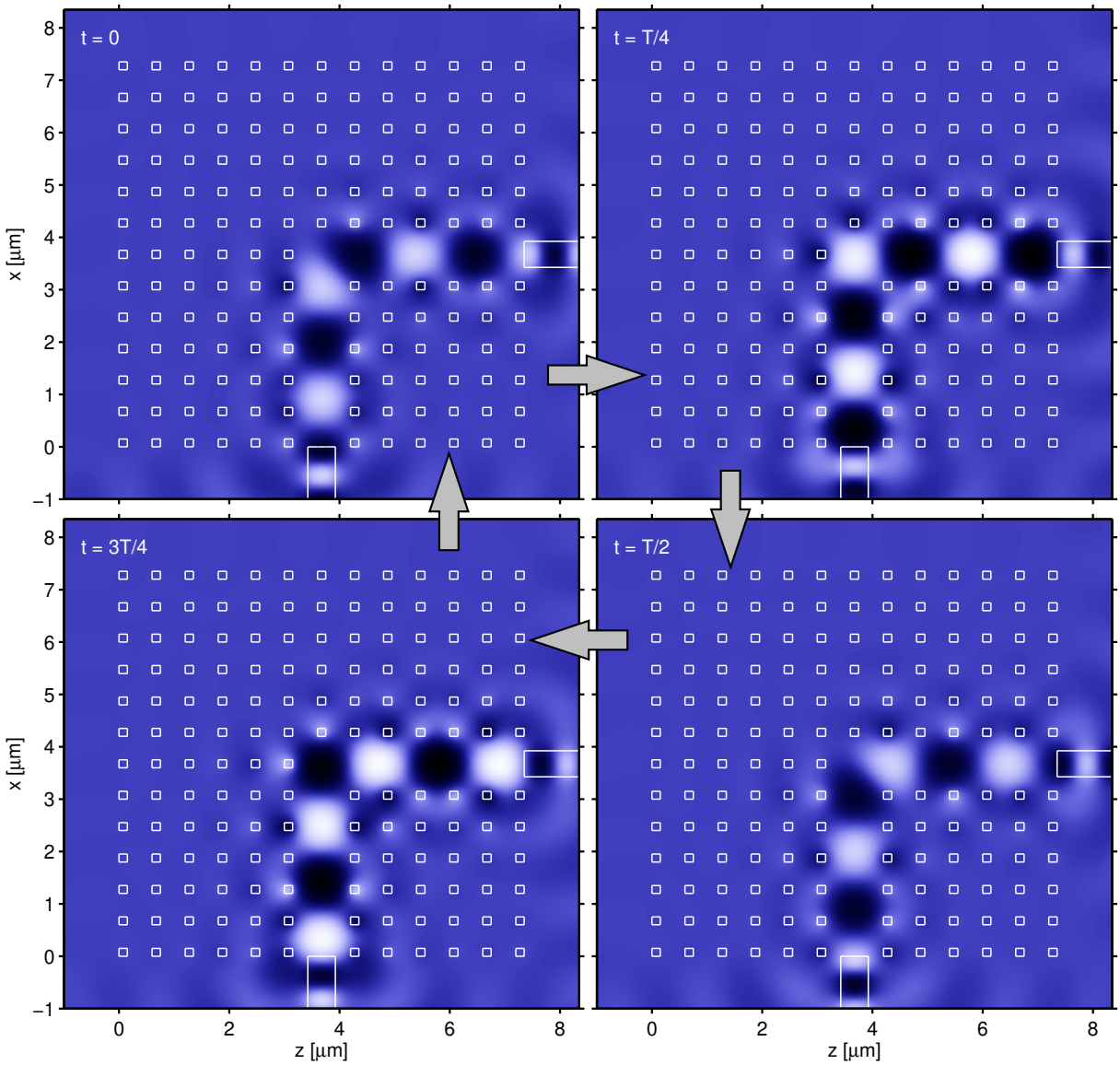




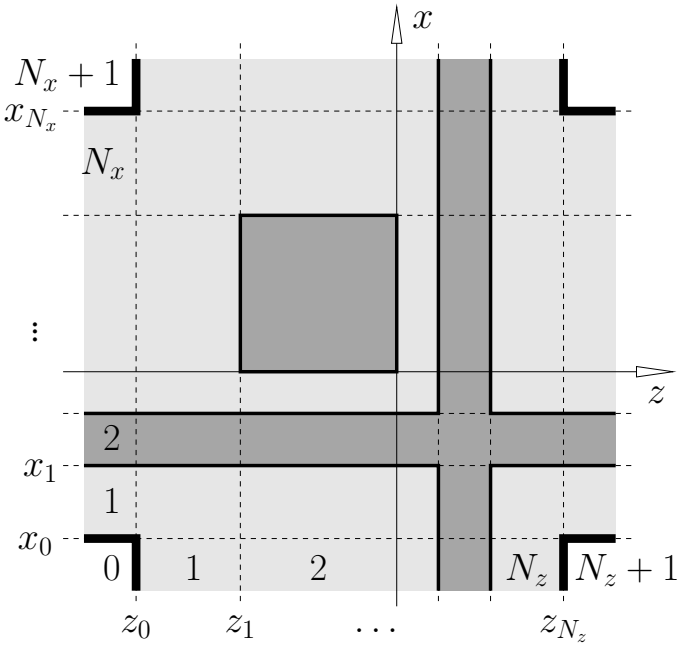
$n_r = 3.4,\ n_b = 1.0,$
$r = 0.15\,\mu\text{m},\ p = 0.6\,\mu\text{m},$
$n_g = 1.8,\ w = 0.5\,\mu\text{m},$
$\lambda = 1.5\,\mu\text{m},\ N_r = 6,$*

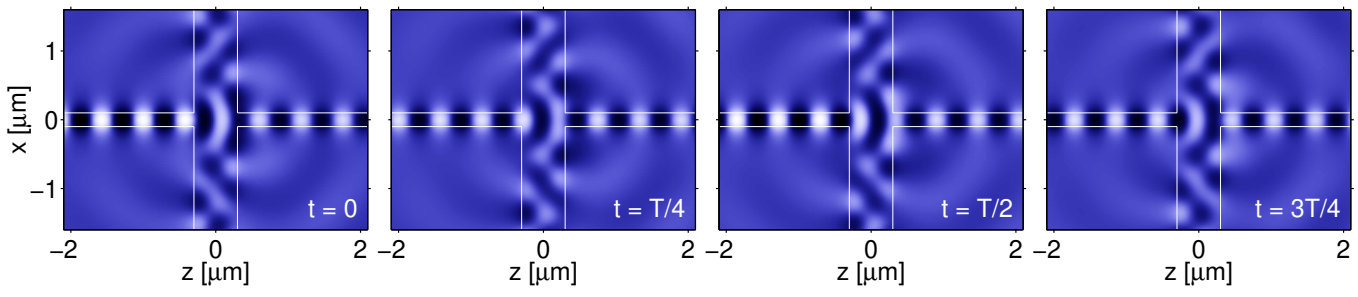
TE, $x, z \in [-1, 8.35]\,\mu\text{m},$
$M_x = M_z = 180.$

$P_R = 0.05,\ P_T = 0.79.$



* R. Stoffer et. al., Optical and Quantum Electronics **32**, 947–961, 2000
J. D. Joannopoulos et. al., *Photonic crystals: Molding the Flow of Light*, Princeton UP, New Jersey, 1995.



QUEP scheme:

- Eigenmode expansion technique, 2D Helmholtz problems with piecewise constant, rectangular permittivity.
- Equivalent treatment of the propagation along the two relevant axes.
- Way to realize transparent boundaries for the interior region on a cross-shaped computational domain.
- Basis modes can be restricted to simple Dirichlet boundary conditions.
- Applications: . . .