Quadridirectional mode expansion modeling in integrated optics

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Kleinheubacher Tagung, Miltenberg, Germany, 29.09. – 02.10.2003

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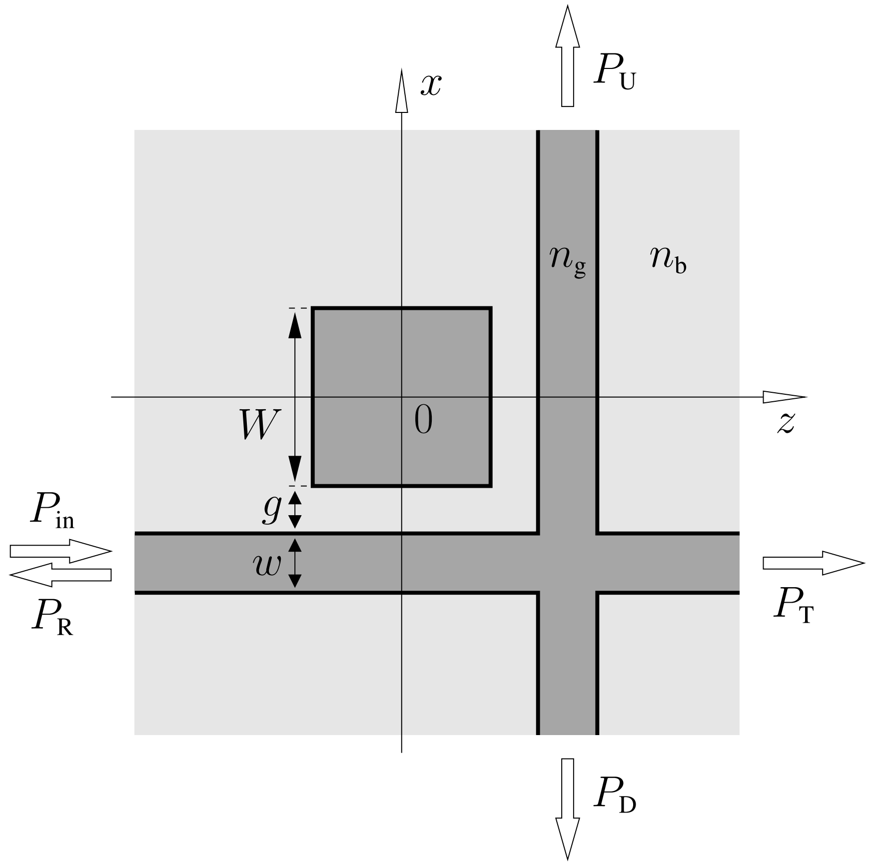
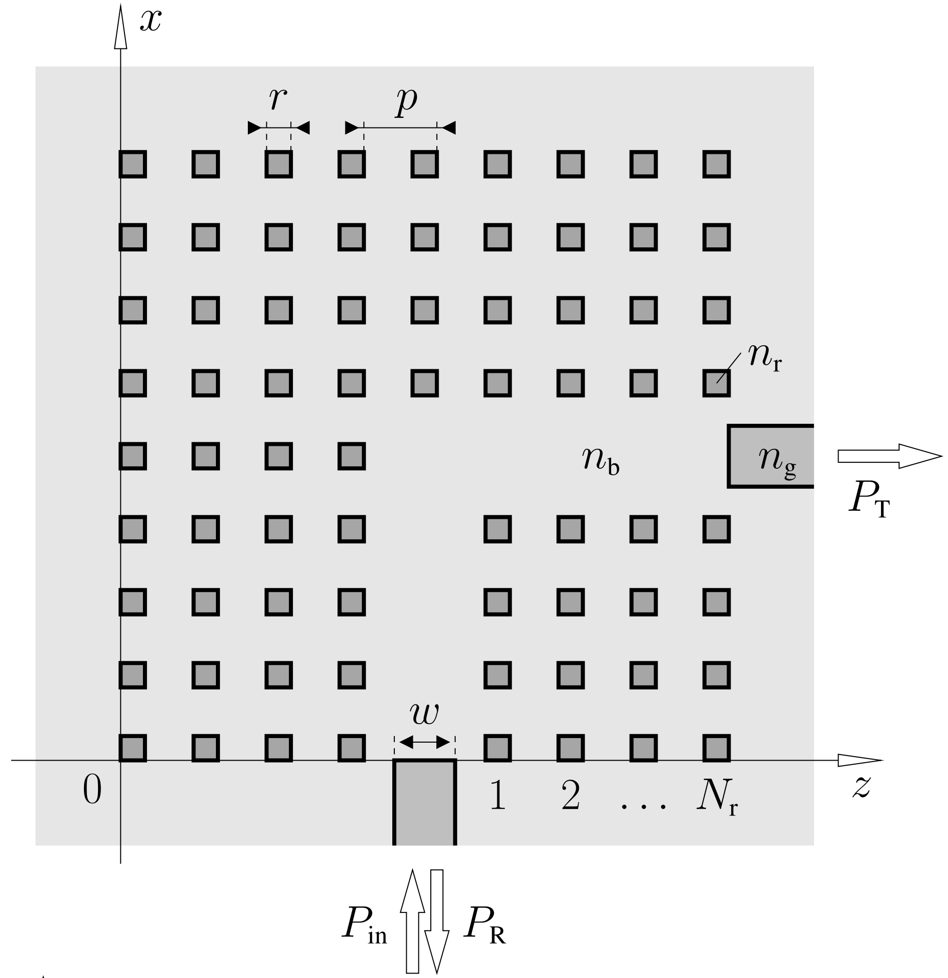
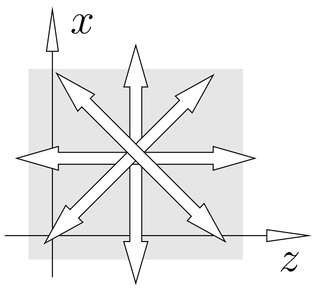
Omnidirectional light propagation

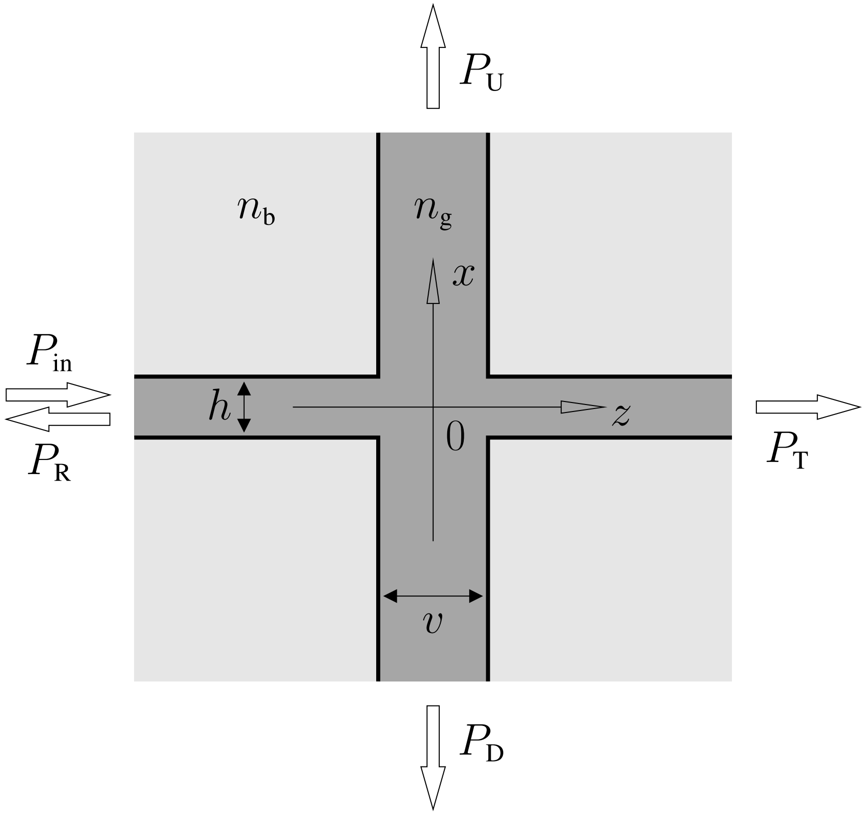




$$\Delta n \not\ll 1 \longrightarrow$$



Mode expansion modeling of omnidirectional light propagation ?



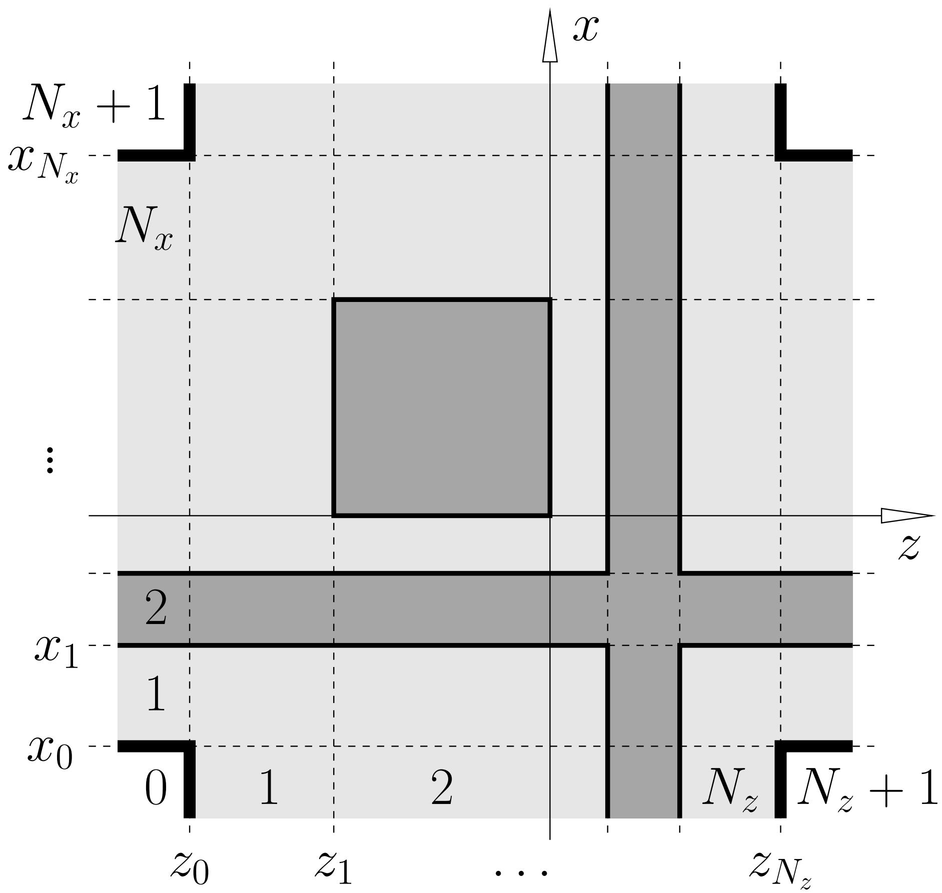
Directions x and z deserve the same treatment.

QUEP: An eigenmode expansion scheme that implements that constraint.

Outline

- Quadridirectional mode propagation:
    - Problem setting
    - Eigenmode expansion
    - Algebraic procedure
- Numerical results:
    - Gaussian beams in free space
    - Bragg grating
    - Waveguide crossing
    - Square resonator with perpendicular ports
    - Photonic crystal bend

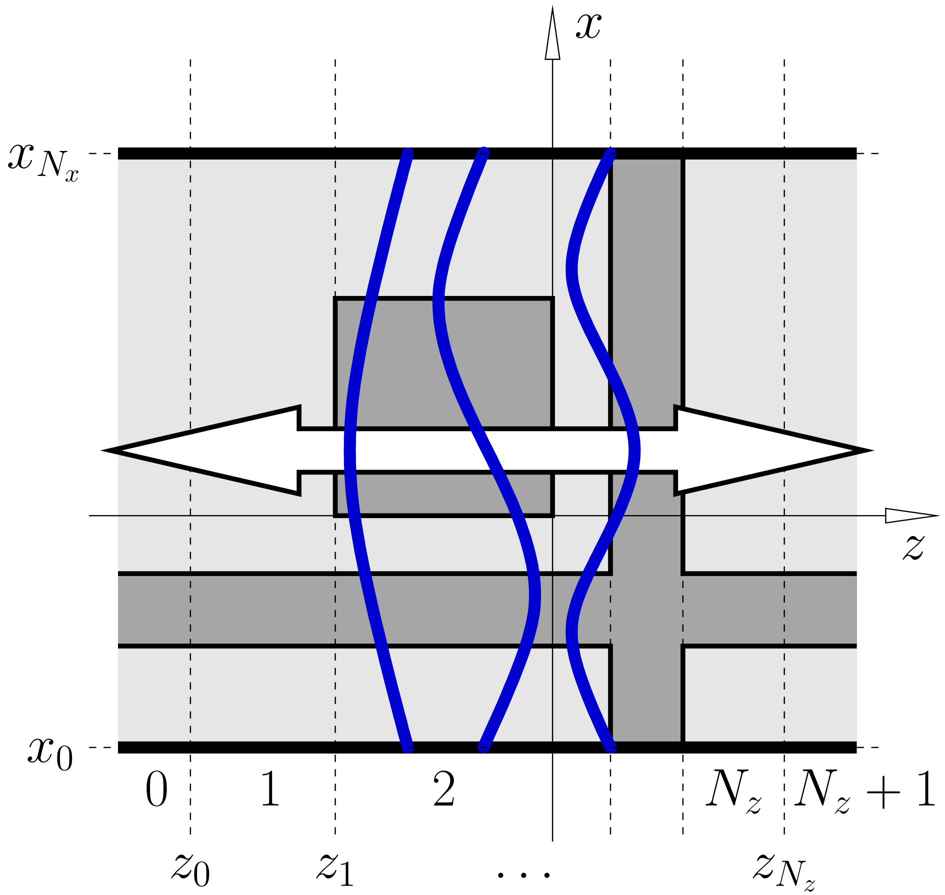
Problem setting



- 2D problem, Cartesian coordinates x, z, TE / TM polarization.
- Piecewise constant, rectangular refractive index distribution; linear, lossless materials.
- Fixed frequency simulations, vacuum wavelength $\lambda = 2\pi/k$.
- Rectangular interior computational domain, influx & outflux across all four boundaries, external regions are homogeneous along x or z.
- Assumption $E_y = 0$, $H_y = 0$ on the corner points and on the external border lines is reasonable for the problems under investigation.

Modal basis fields

Basis fields,
defined by Dirichlet boundary conditions $E_y = 0$ (TE) or $H_y = 0$ (TM):



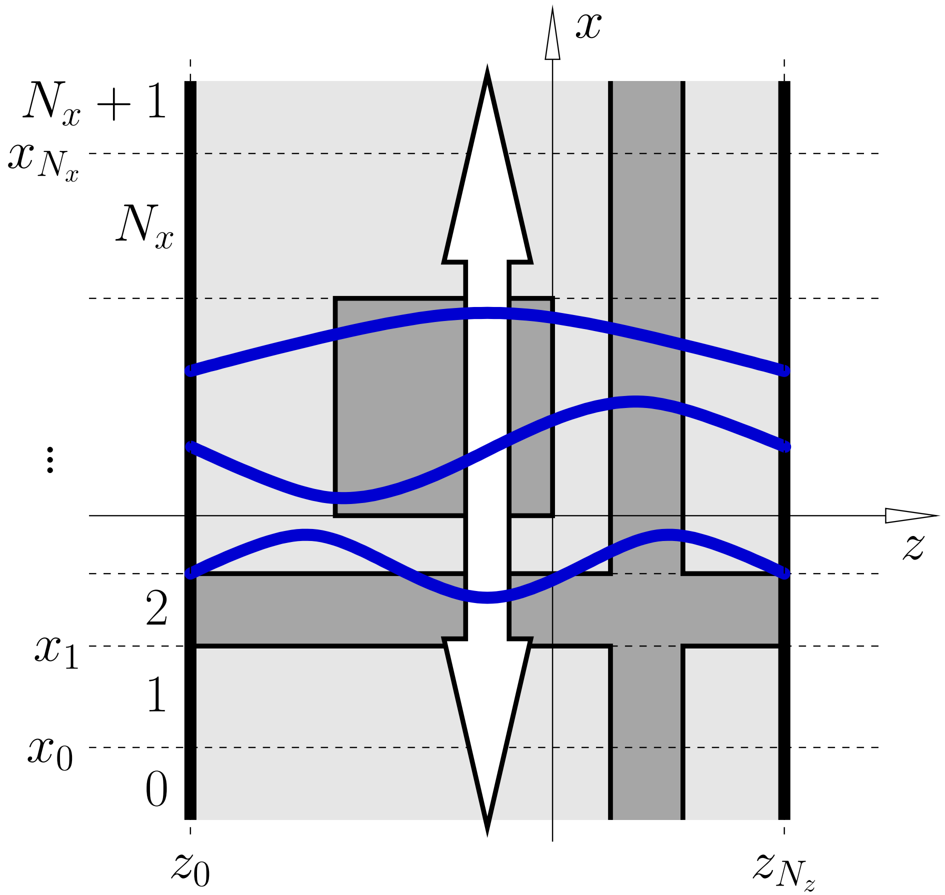
Horizontally traveling eigenmodes:

M_z profiles	$\psi^d_{s,m}(x)$
and propagation constants	$\pm\beta_{s,m}$

of order m, on slice s,
for propagation directions $d = \text{f, b}$,

Modal basis fields

Basis fields,
defined by Dirichlet boundary conditions $E_y = 0$ (TE) or $H_y = 0$ (TM):

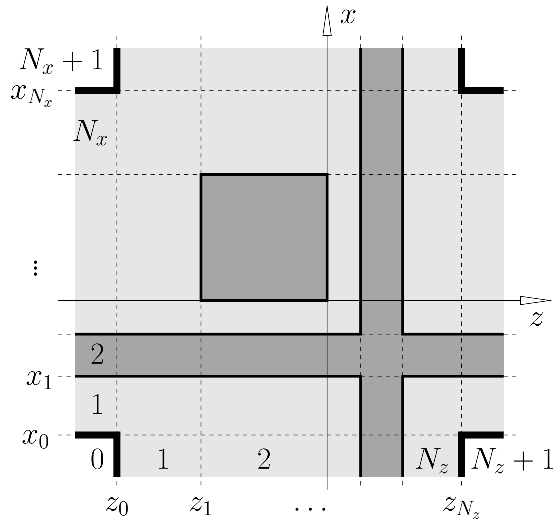


... *and* vertically traveling fields:

M_x profiles	$\phi^d_{l,m}(z)$
and propagation constants	$\pm\gamma_{l,m}$

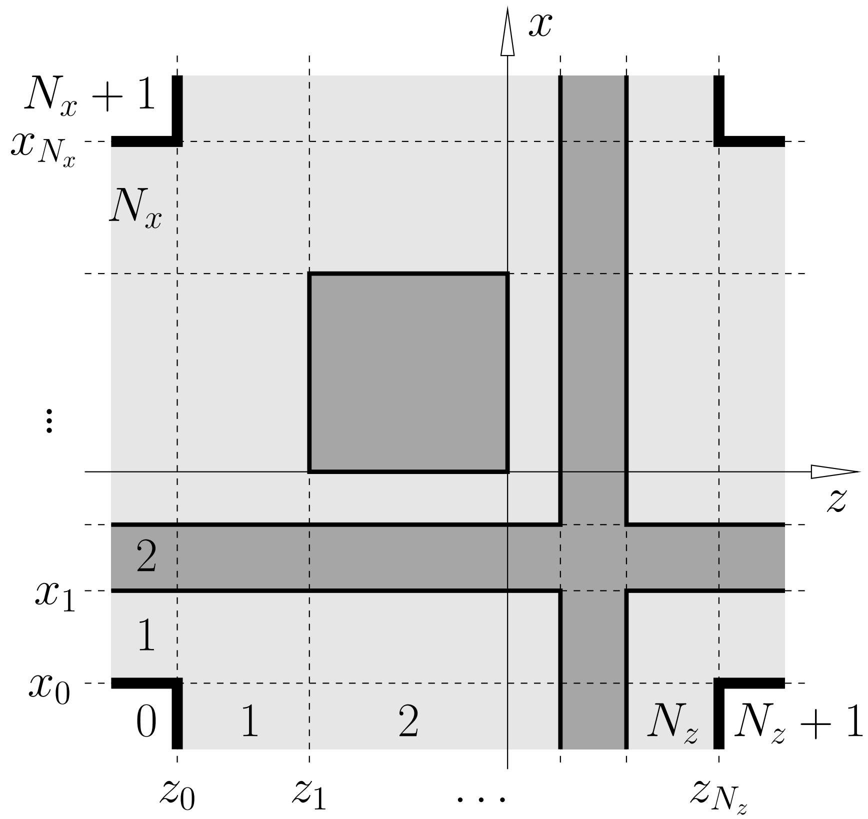
of order m, on layer l,
for propagation directions $d =$ u, d.

Eigenmode expansion



Ansatz for the optical field,
for $z_{s-1} \leq z \leq z_s,\ s = 1, \dots, N_z$,
and $x_{l-1} \leq x \leq x_l$, $l = 1, \dots, N_x$:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, z, t) = \mathrm{Re}\left\{ \sum_{m=0}^{M_z-1} F_{s,m} \boldsymbol{\psi}^{\mathrm{f}}_{s,m}(x)\, \mathrm{e}^{-\mathrm{i}\beta_{s,m}(z-z_{s-1})} + \sum_{m=0}^{M_z-1} B_{s,m} \boldsymbol{\psi}^{\mathrm{b}}_{s,m}(x)\, \mathrm{e}^{+\mathrm{i}\beta_{s,m}(z-z_{s})} + \sum_{m=0}^{M_x-1} U_{l,m} \boldsymbol{\phi}^{\mathrm{u}}_{l,m}(z)\, \mathrm{e}^{-\mathrm{i}\gamma_{l,m}(x-x_{l-1})} + \sum_{m=0}^{M_x-1} D_{l,m} \boldsymbol{\phi}^{\mathrm{d}}_{l,m}(z)\, \mathrm{e}^{+\mathrm{i}\gamma_{l,m}(x-x_{l})} \right\} \mathrm{e}^{\mathrm{i}\omega t}.$$

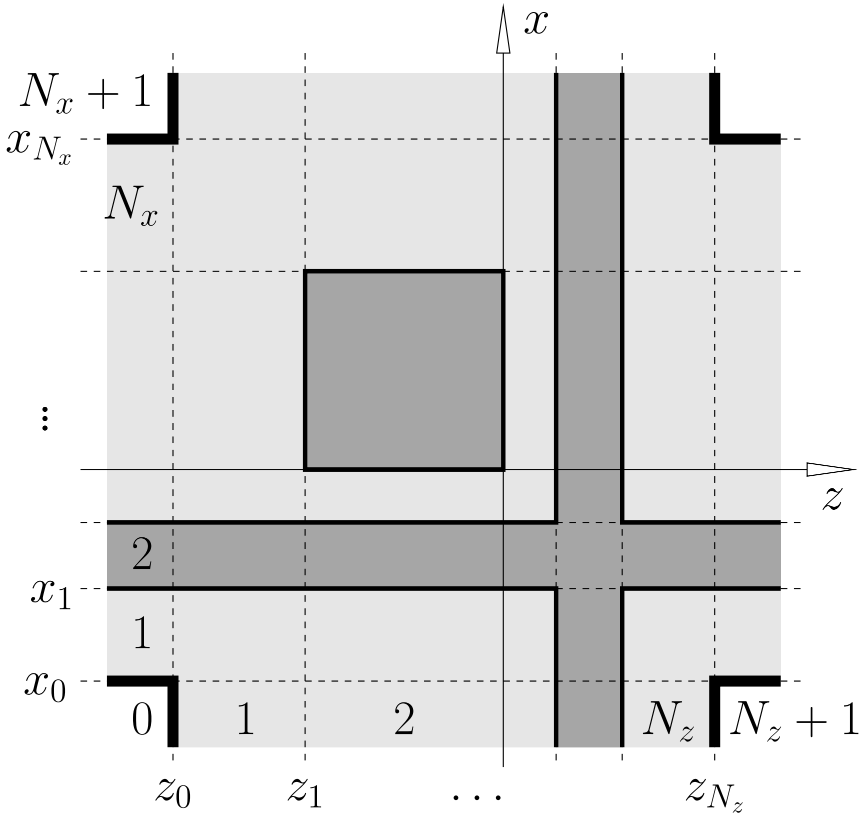


Mode products ↔ normalization, projection:

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_x = \frac{1}{4} \int (E^*_{1,x} H_{2,y} - E^*_{1,y} H_{2,x} + H^*_{1,y} E_{2,x} - H^*_{1,x} E_{2,y}) \mathrm{d}x \,,$$

$$(\boldsymbol{E}_1, \boldsymbol{H}_1; \boldsymbol{E}_2, \boldsymbol{H}_2)_z = \frac{1}{4} \int (E^*_{1,y} H_{2,z} - E^*_{1,z} H_{2,y} + H^*_{1,z} E_{2,y} - H^*_{1,y} E_{2,z}) \mathrm{d}z \,.$$

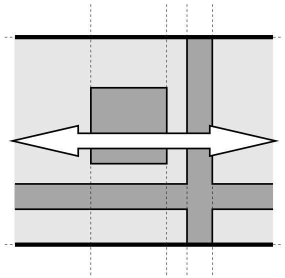
Algebraic procedure



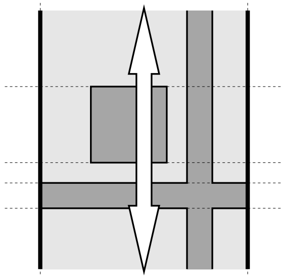
- Consistent bidirectional projection at all interfaces
  $\longrightarrow$ linear system of equations in $\{F_{s,m}, B_{s,m}, U_{l,m}, D_{l,m}\}$.
- Influx: $\boldsymbol{F}_0,\ \boldsymbol{B}_{N_x+1},\ \boldsymbol{U}_0,\ \boldsymbol{D}_{N_z+1} \longrightarrow$ RHS.
- Outflux: $\boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}$.

Algebraic procedure

- "Exact" mode profiles $\longrightarrow$ interior problems decouple:

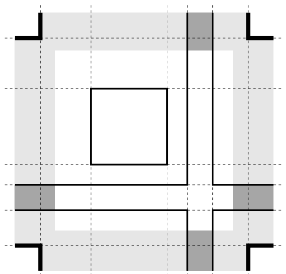


Solve for $\boldsymbol{F}_2, \ldots, \boldsymbol{F}_{N_z}$ and $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_{N_z-1}$
in terms of $\boldsymbol{F}_1$ and $\boldsymbol{B}_{N_z}$ $\longrightarrow$ BEP.



Solve for $\boldsymbol{U}_2, \ldots, \boldsymbol{U}_{N_x}$ and $\boldsymbol{D}_1, \ldots, \boldsymbol{D}_{N_x-1}$
in terms of $\boldsymbol{U}_1$ and $\boldsymbol{D}_{N_x}$ $\longrightarrow$ BEP.

- Continuity of E and H on outer interfaces:

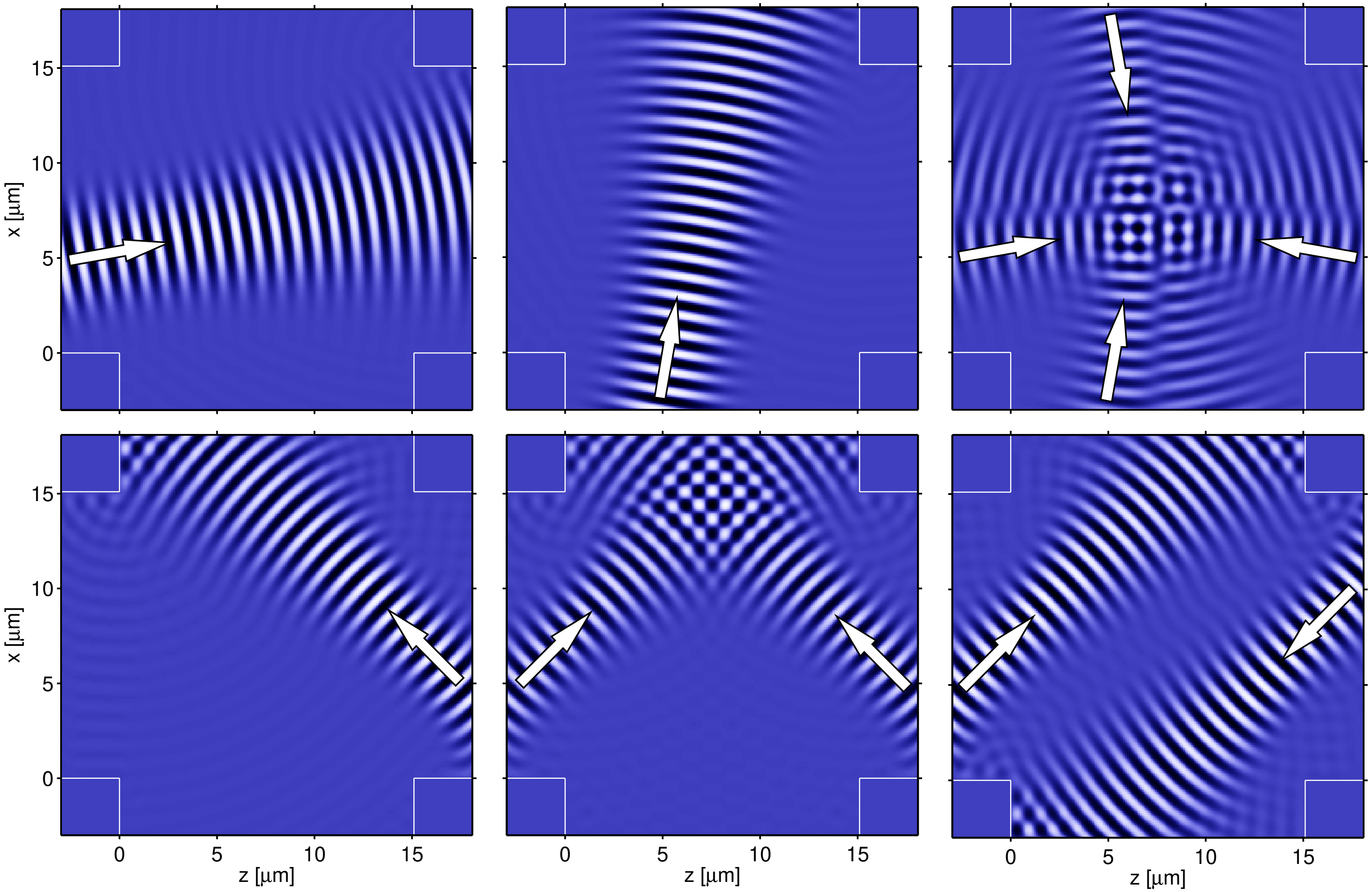


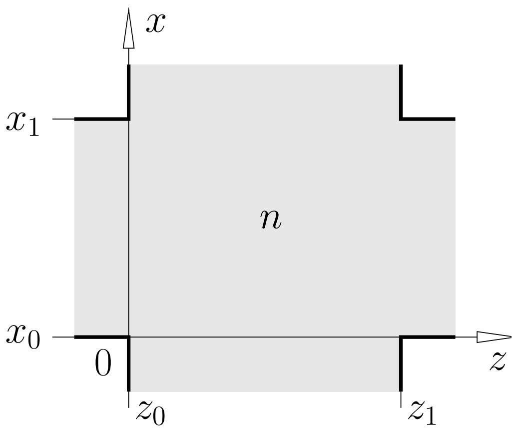
Interior BEP solutions
$+$ equations at $z = z_0, z_{N_z},\ x = x_0, x_{N_x}$
$\longrightarrow \boldsymbol{B}_0,\ \boldsymbol{F}_{N_x+1},\ \boldsymbol{D}_0,\ \boldsymbol{U}_{N_z+1}.$

"QUadridirectional Eigenmode Propagation method" (QUEP).

Gaussian beams in free space

$E_y(x, z):$

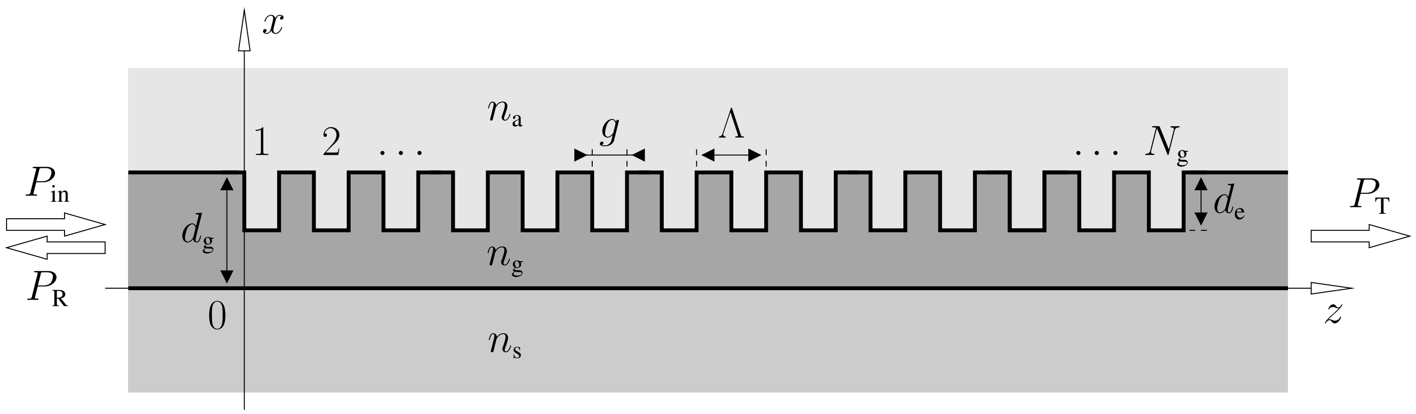




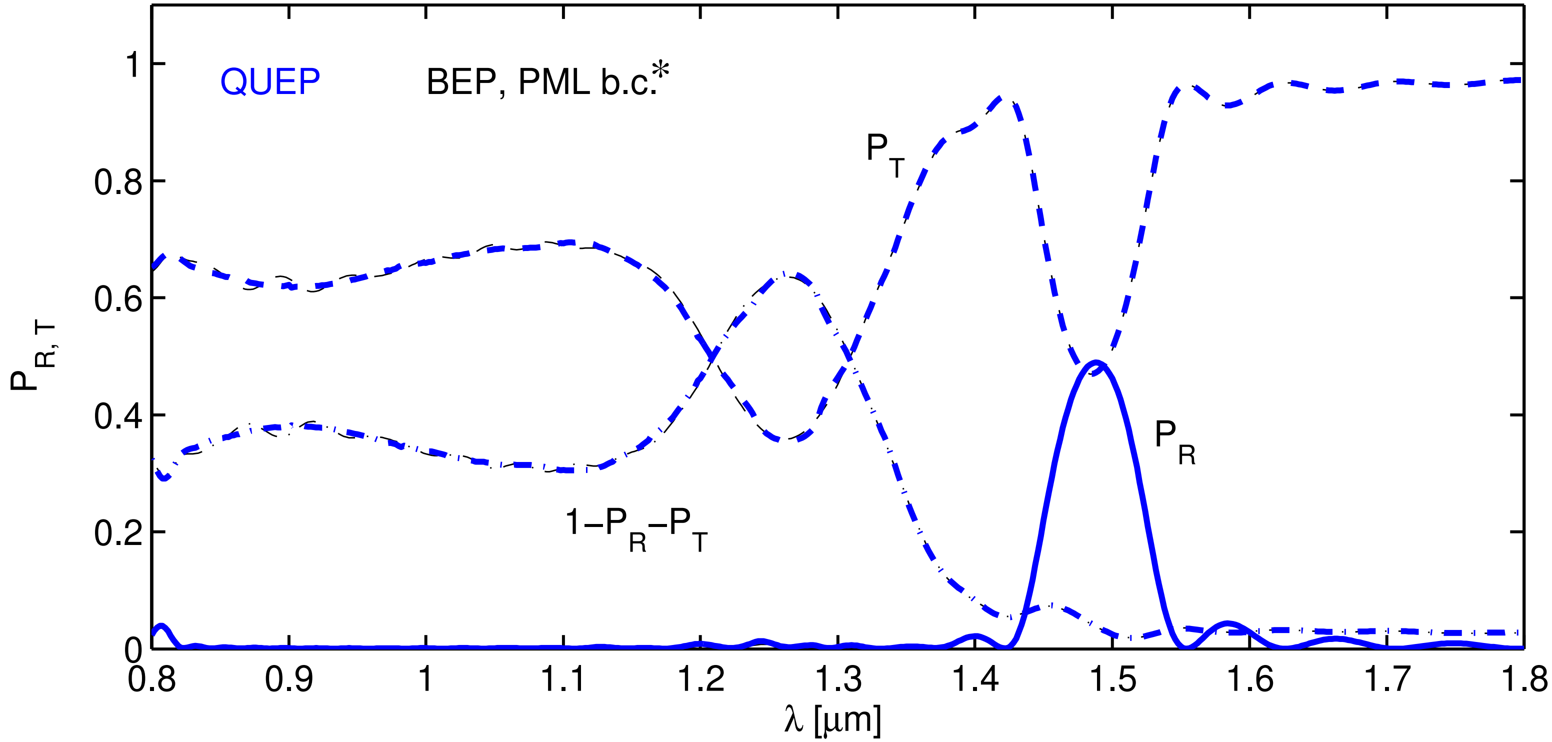
$n = 1.0$, $\lambda = 1.0\,\mu$m,
TE polarization,
$x, z \in [0, 15.1]\,\mu$m,
$M_x = M_z = 150$.

Top: $4\,\mu$m beam width, 10° tilt angle, $2\,\mu$m offset;
bottom: 45° tilt angle, initially centered beams.

Bragg grating: COST268 benchmark problem



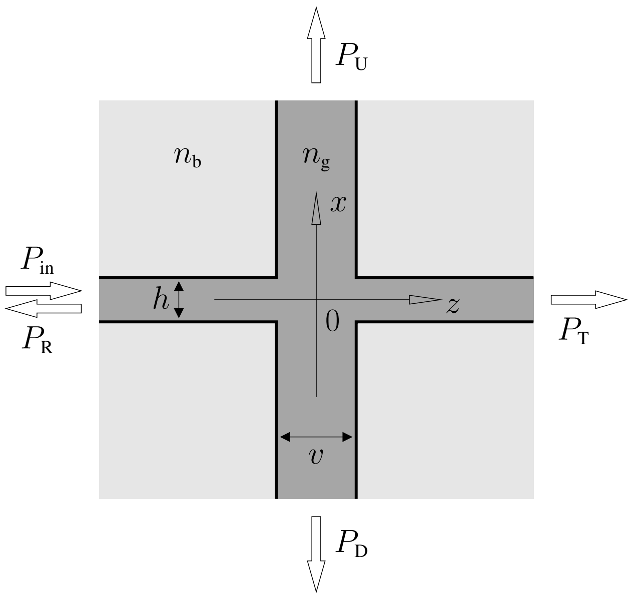
(*)
$d_g = 0.500\,\mu$m,
$d_e = 0.125\,\mu$m,
$n_s \approx 1.45$ (SiO_2),
$n_g \approx 1.99$ (Si_3N_4),
$n_a = 1.0$,
$\Lambda = 0.430\,\mu$m,
$g = \Lambda/2$, $N_g = 20$.

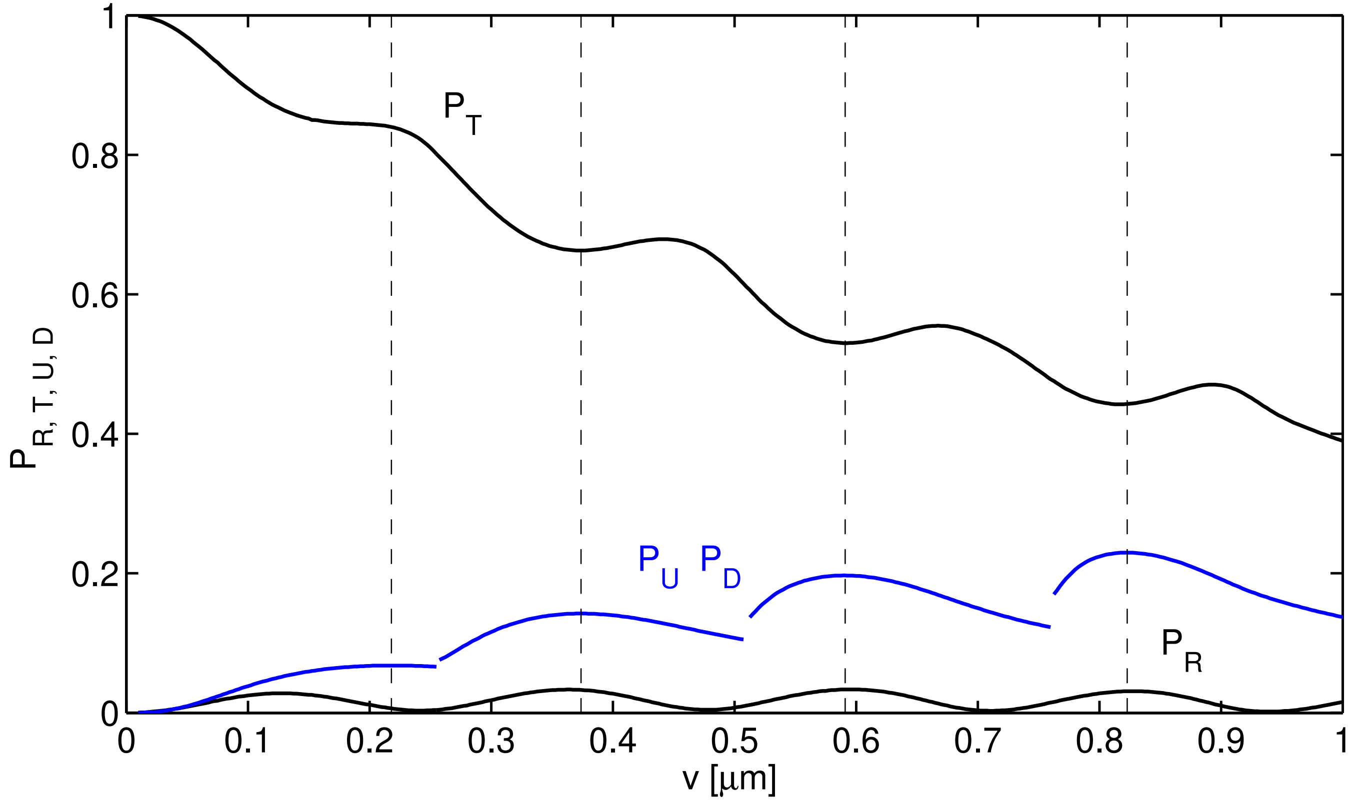


$x \in [-4, 2]\,\mu$m, 60 modes (QUEP, BEP $E_y = 0$ b.c.), $z \in [-1.8, 10.185]\,\mu$m, 120 modes (QUEP).

*J. Čtyroký et. al., Optical and Quantum Electronics **34**, 455–470, 2002; reference: BEP2 (PML b.c.).

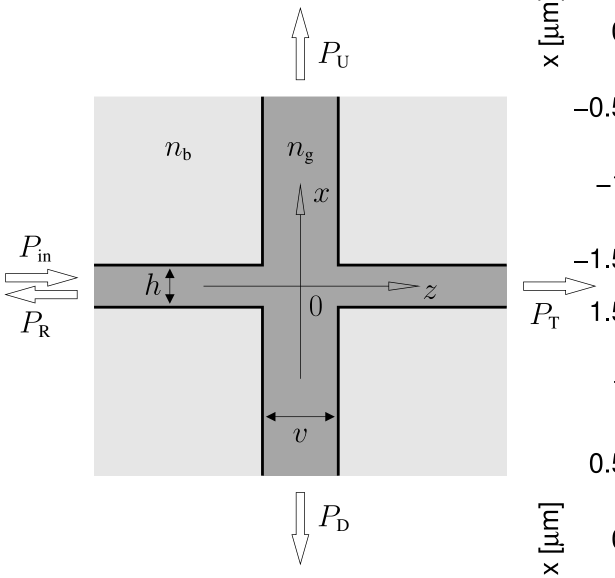
Waveguide crossings

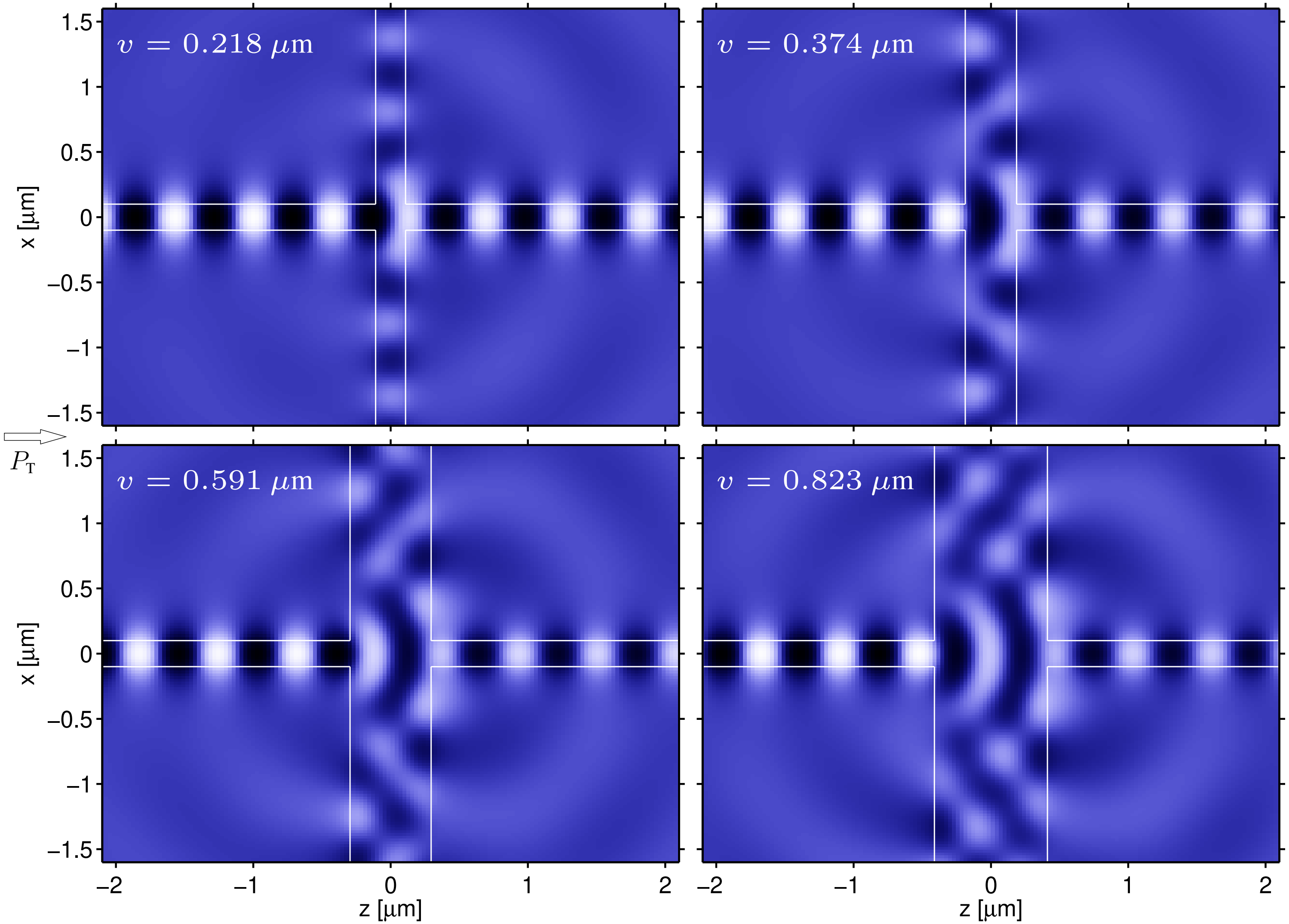




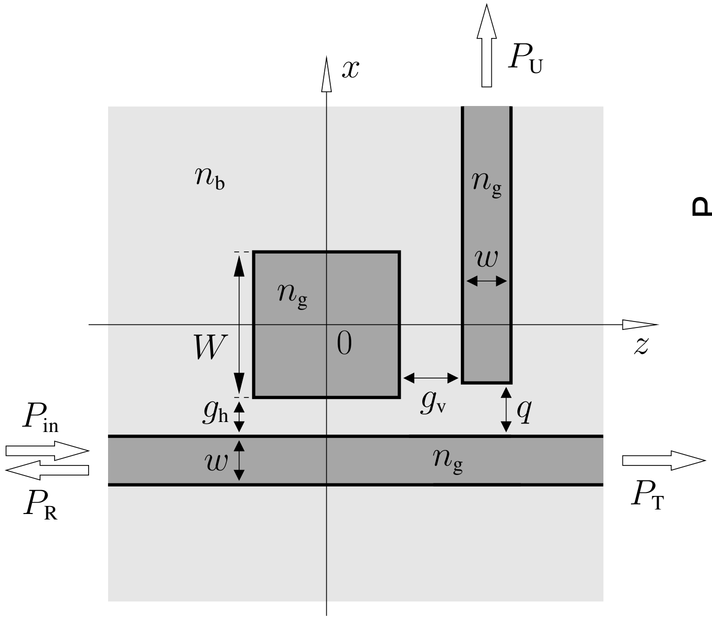
$n_g = 3.4,\ n_b = 1.45,\ \lambda = 1.55\,\mu\text{m},\ h = 0.2\,\mu\text{m},\ \text{TE},\ x, z \in [-3, 3]\,\mu\text{m},\ M_x = M_z = 120.$
Power conservation: error $< 10^{-3}$.

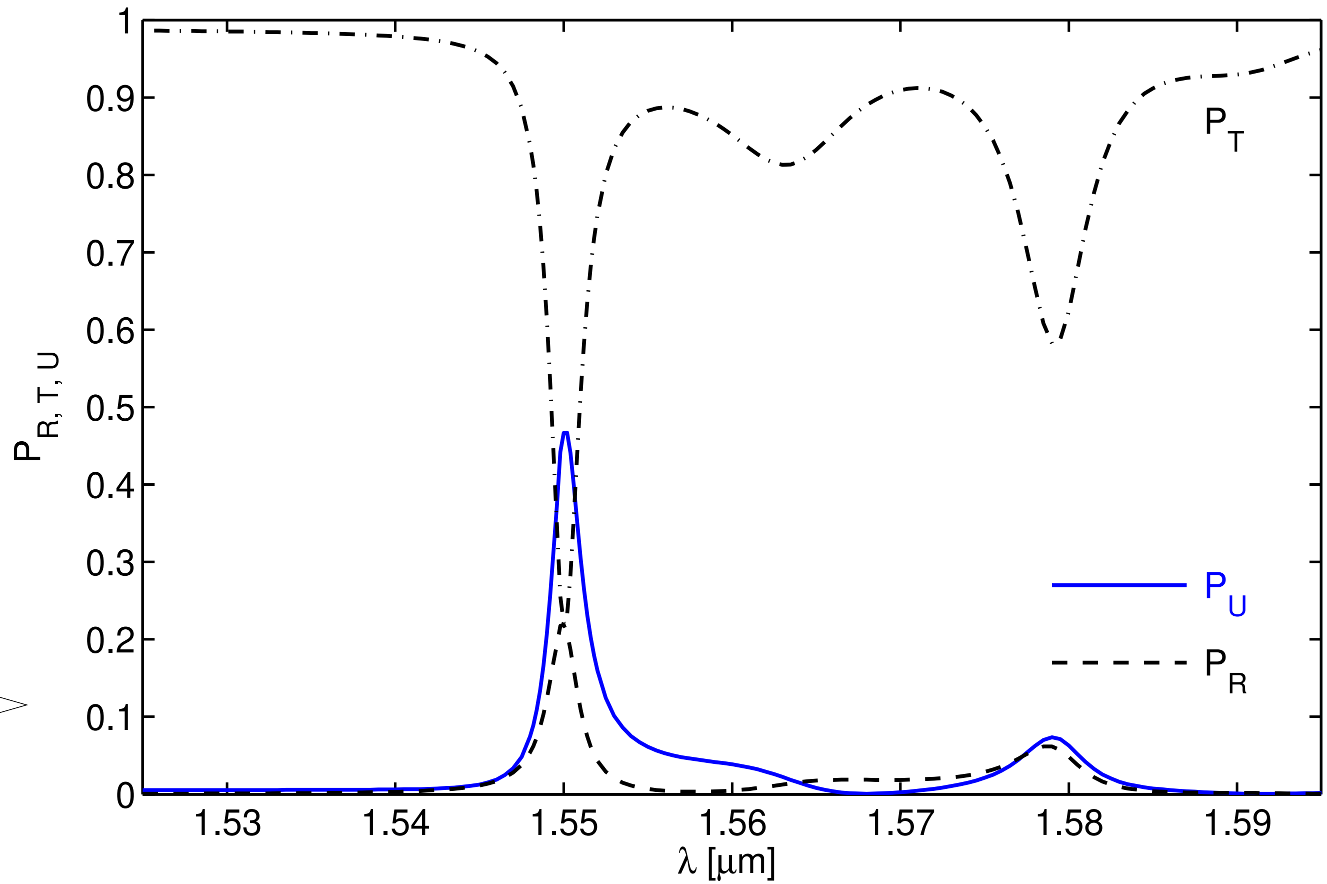
Waveguide crossings





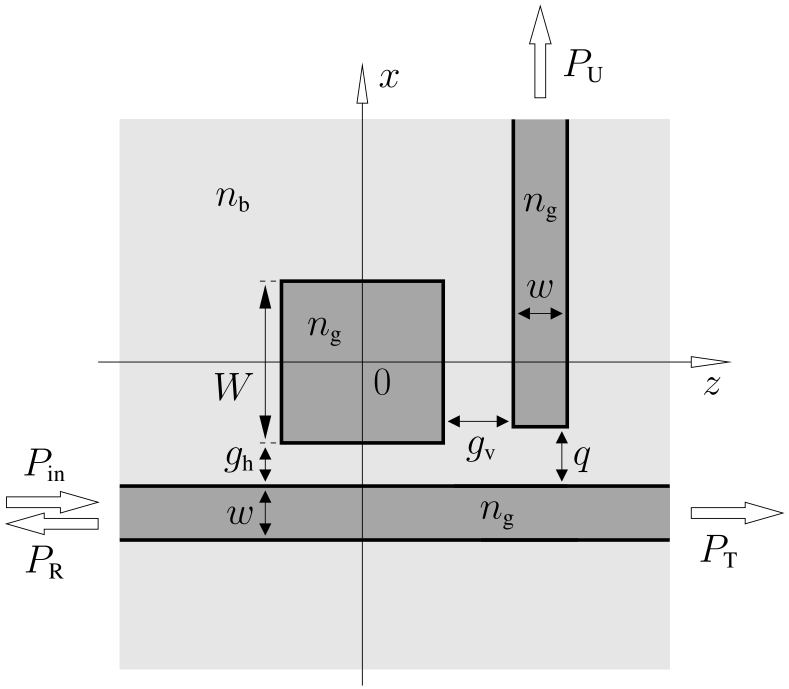
Square resonator with perpendicular ports





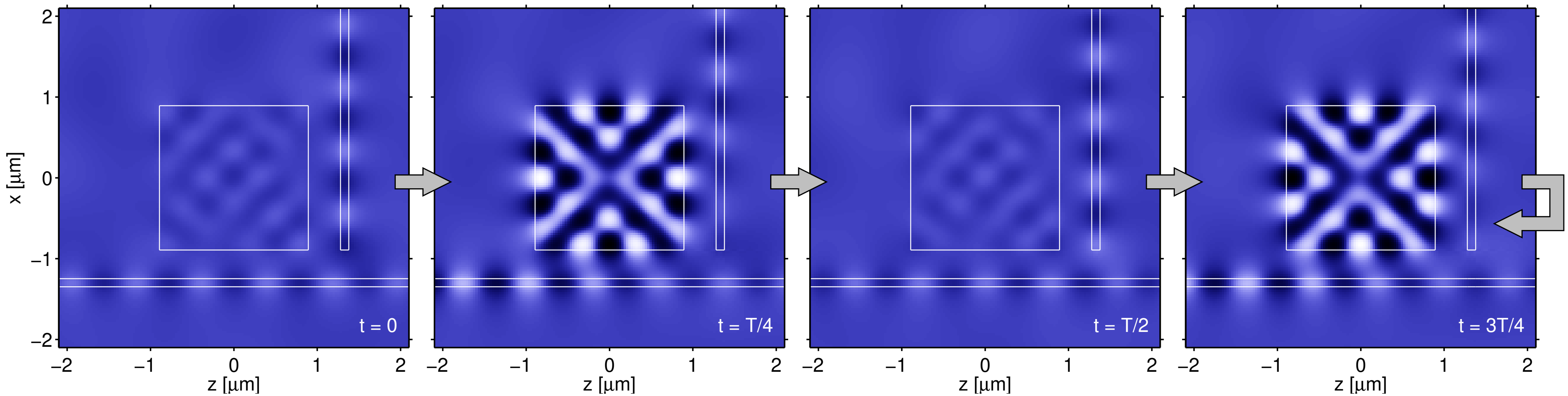
$n_g = 3.4,\ n_b = 1.0,\ W = 1.786\,\mu$m$,\ w = 0.1\,\mu$m$,\ g_h = 0.355\,\mu$m$,\ g_v = 0.385\,\mu$m$,\ q = 0.355\,\mu$m,
TE, $x, z \in [-4, 4]\,\mu$m, $M_x = M_z = 100$.

Square resonator with perpendicular ports

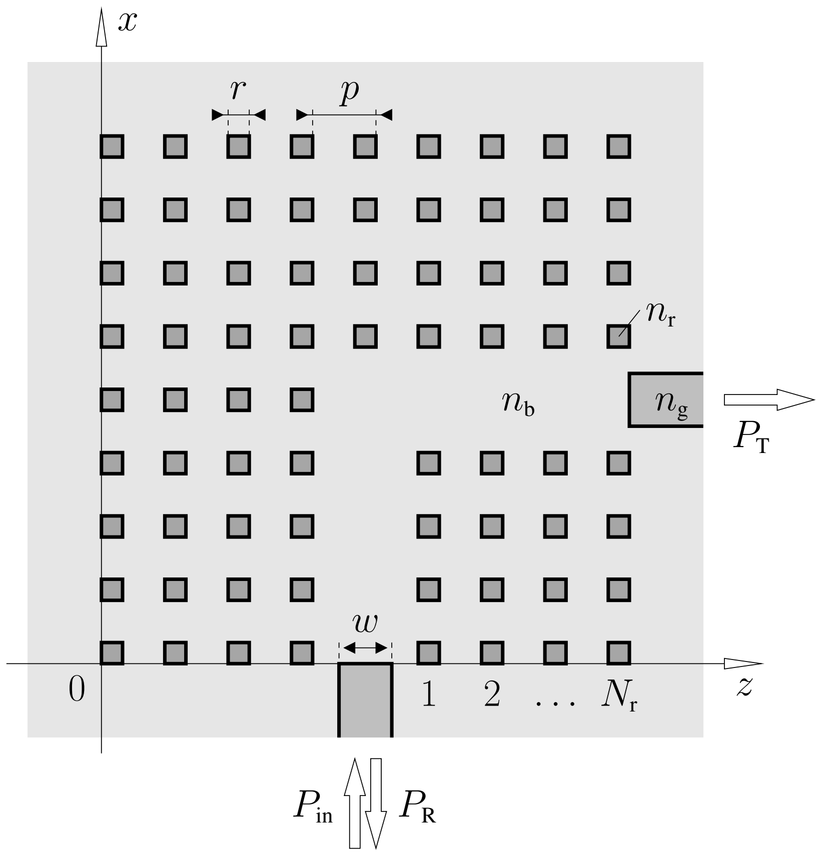


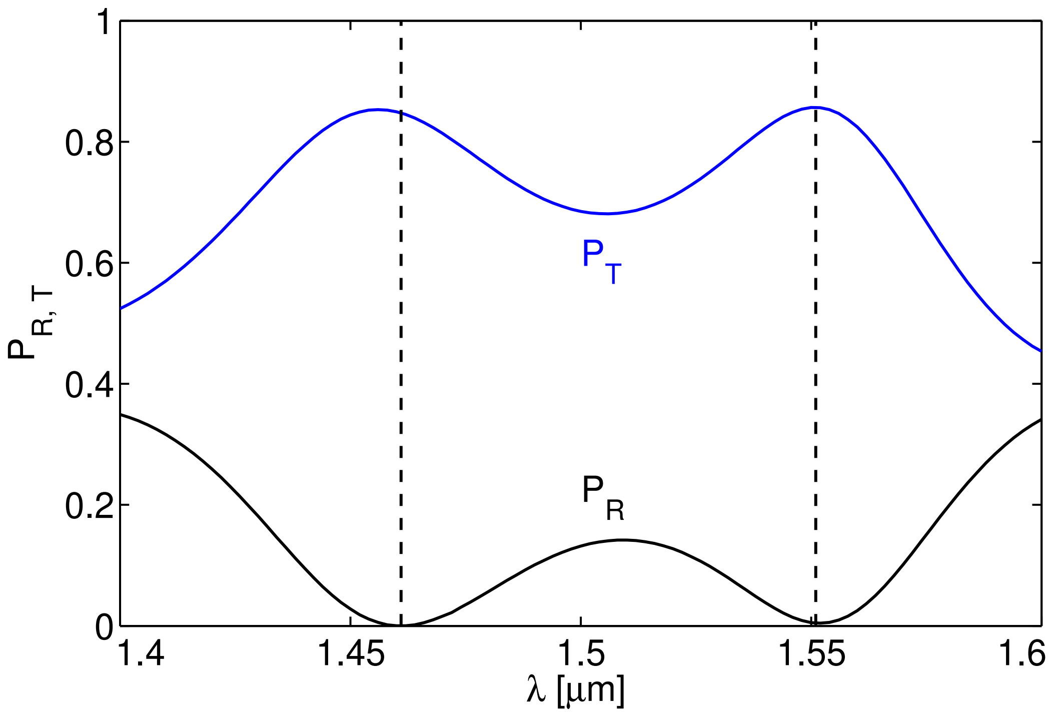
$n_\mathrm{g} = 3.4,\ n_\mathrm{b} = 1.0,\ W = 1.786\,\mu\mathrm{m},\ w = 0.1\,\mu\mathrm{m},$
$g_\mathrm{h} = 0.355\,\mu\mathrm{m},\ g_\mathrm{v} = 0.385\,\mu\mathrm{m},\ q = 0.355\,\mu\mathrm{m},$
$\mathrm{TE},\ x, z \in [-4, 4]\,\mu\mathrm{m},\ M_x = M_z = 100.$

$\lambda = 1.55\,\mu\mathrm{m},\ T = 5.17\,\mathrm{fs},\ E_y(x, z, t):$



Photonic crystal bend

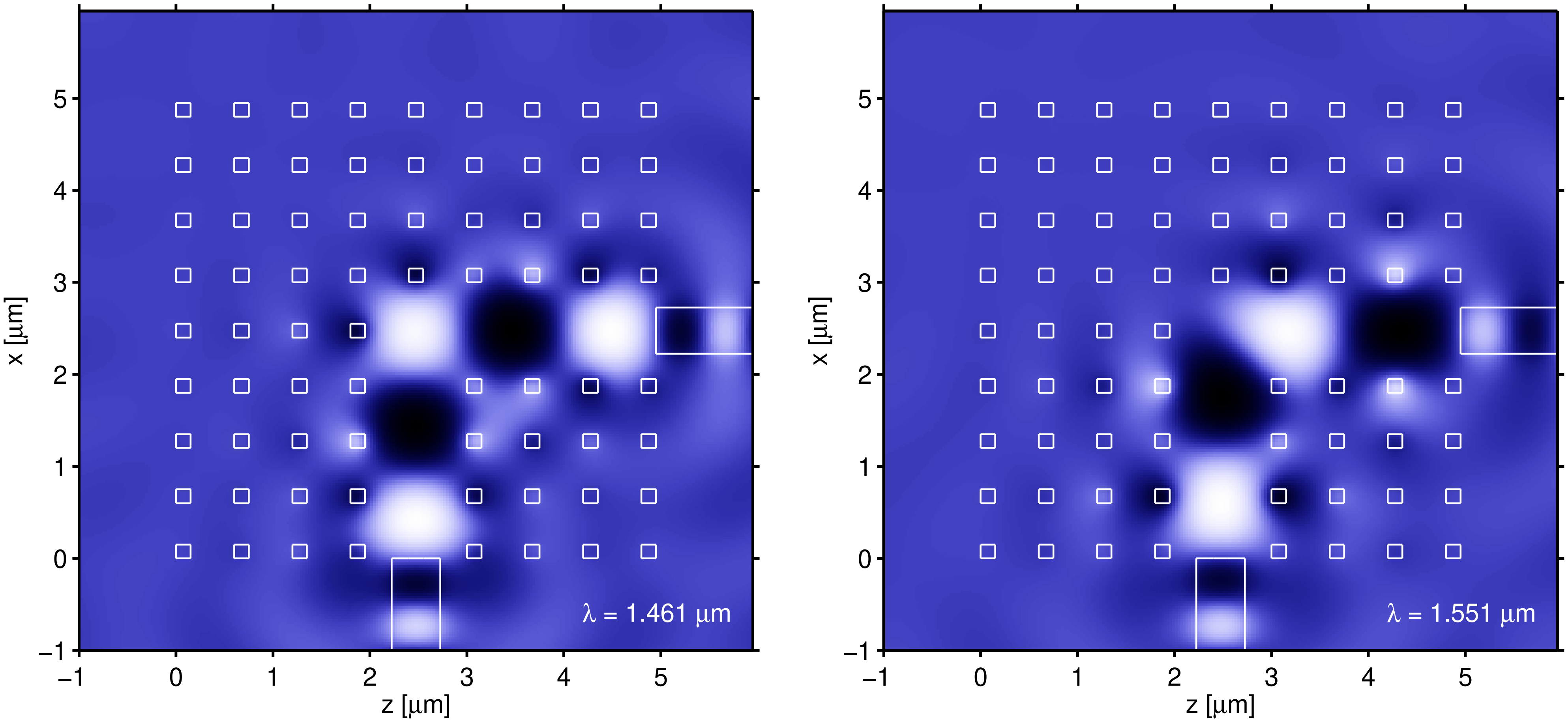




$n_r = 3.4,\ n_b = 1.0,\ r = 0.15\,\mu\text{m},\ p = 0.6\,\mu\text{m},\ n_g = 1.8,\ w = 0.5\,\mu\text{m},\ N_r = 4,^*$
TE, $x, z \in [-1, 5.95]\,\mu\text{m},\ M_x = M_z = 120.$

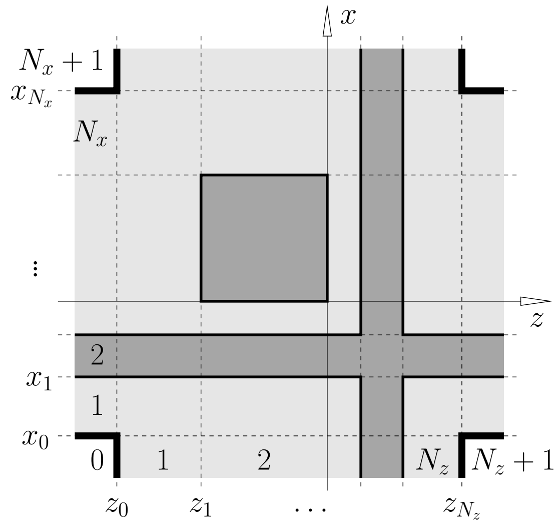
* R. Stoffer et. al., Optical and Quantum Electronics **32**, 947–961, 2000
J. D. Joannopoulos et. al., *Photonic crystals: Molding the Flow of Light*, Princeton UP, New Jersey, 1995.

Photonic crystal bend



Quadridirectional mode expansion

QUEP scheme:



- Eigenmode expansion technique, 2D Helmholtz problems with piecewise constant, rectangular permittivity.
- Equivalent treatment of the propagation along the two relevant axes.
- Way to realize transparent boundaries for the interior region on a cross-shaped computational domain.
- Basis modes can be restricted to simple Dirichlet boundary conditions.
- Applications: ...