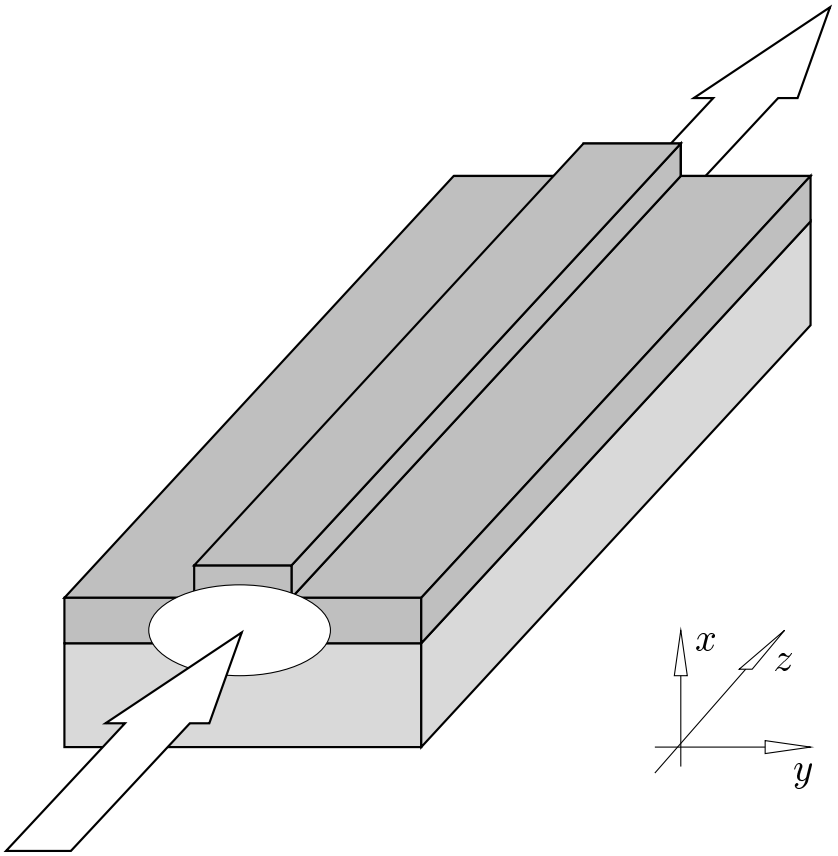
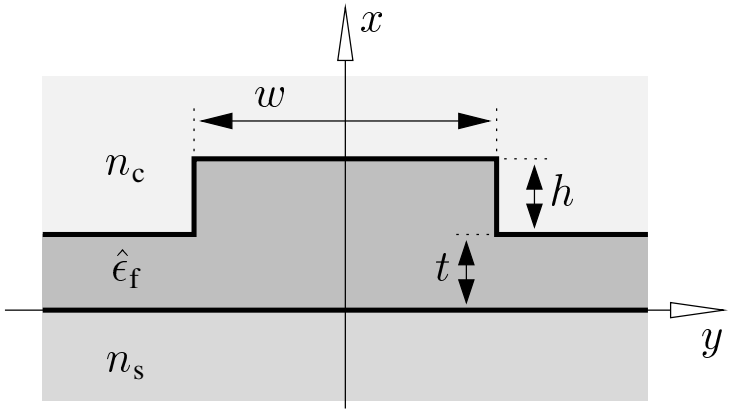
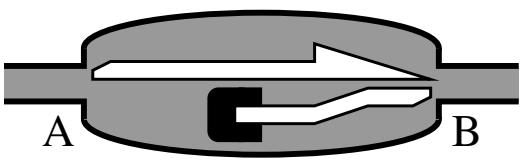
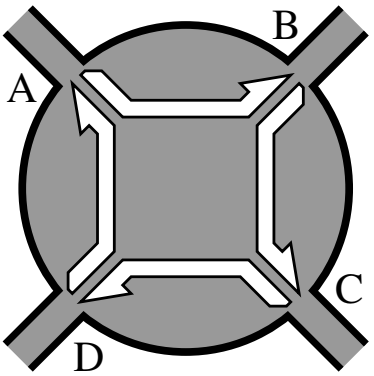




$$\hat{\epsilon}_f = n_f^2 + \delta\hat{\epsilon}_{mo}$$

$n_s = 1.9$
$n_f = 2.2$
$n_c = 1.0$
$\lambda = 1.3\,\mu\text{m}$
$t = 0.5\,\mu\text{m}$
$h = 0.5\,\mu\text{m}$
$w = 1.0\,\mu\text{m}$





Modeling of nonreciprocal integrated magnetooptic devices

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- Magnetooptic waveguides
- Guided modes
  - Wave-matching method
  - Mode profile examples
- Nonreciprocal effects
  - Magnetooptic phase shift:
    Nonreciprocal interferometry
  - Polarization conversion:
    A circulator device
- Summary

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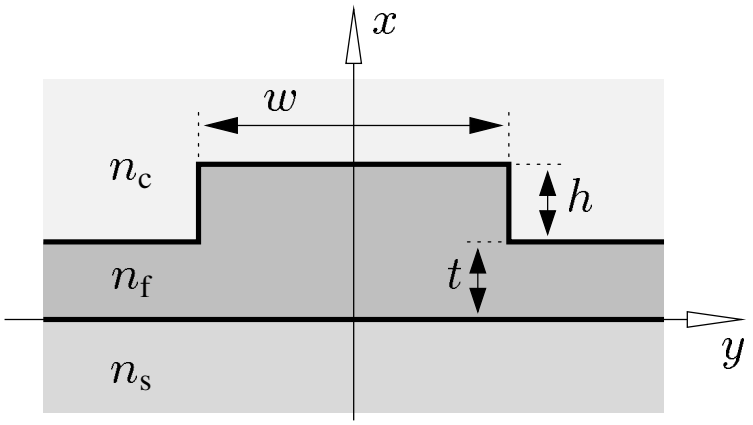
Phone: 0541/969-2641 *Fax: 0541/969-2351* *e-mail: manfred.lohmeyer@physik.uni-osnabrueck.de*



Ansatz: $\epsilon = n^2, \quad k = \omega/c = 2\pi/\lambda$

$$\boldsymbol{\mathcal{E}}(x,y,z,t) = \begin{pmatrix} E_x \\ E_y \\ \mathrm{i}E_z \end{pmatrix}(x,y)\,\mathrm{e}^{\mathrm{i}(\omega t - \beta z)}\,,$$

$$\boldsymbol{\mathcal{H}}(x,y,z,t) = \begin{pmatrix} H_x \\ H_y \\ \mathrm{i}H_z \end{pmatrix}(x,y)\,\mathrm{e}^{\mathrm{i}(\omega t - \beta z)}\,,$$

$E_x, \ldots, H_z$: real mode profile components,
β: propagation constant.

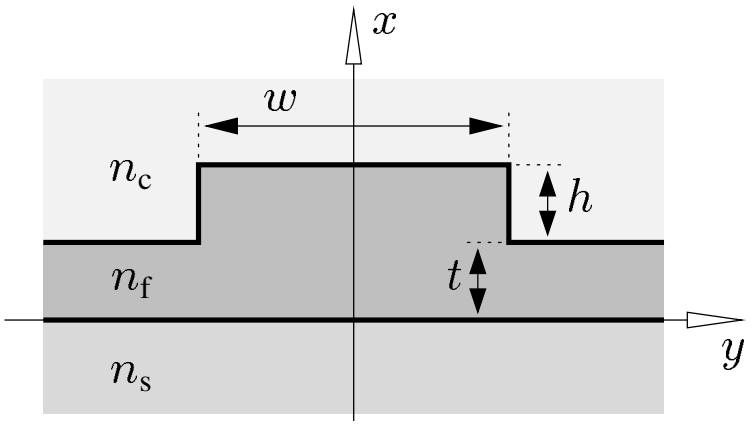
Guided modes:

$$P = \frac{1}{2}\iint (E_x H_y - E_y H_x)\,\mathrm{d}x\,\mathrm{d}y < \infty\,.$$

Mode equations: $X = \frac{1}{\epsilon}(\partial_x \epsilon), \quad Y = \frac{1}{\epsilon}(\partial_y \epsilon),$

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_x + \partial_x X E_x + \partial_x Y E_y = 0\,,$$

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_y + \partial_y X E_x + \partial_y Y E_y = 0\,.$$



Mode equations for locally constant permittivity:

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_x = 0$$

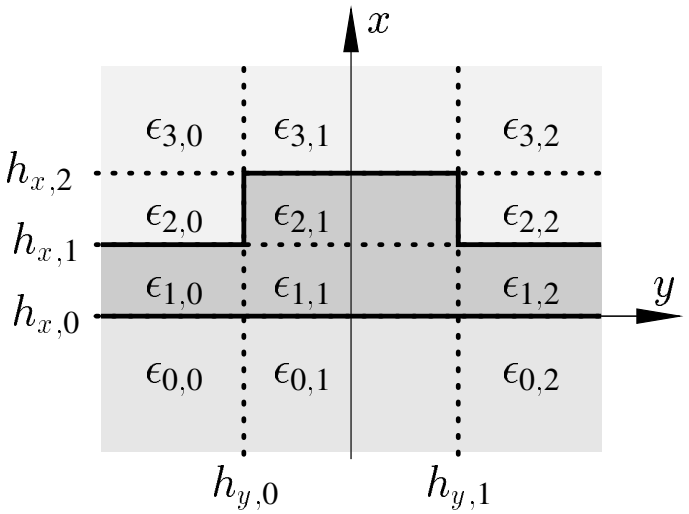
$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_y = 0$$

Continuity conditions:

orientation of the boundary	continuous quantities
horizontal	ϵE_x, E_y, E_z, H_x, H_y, H_z
vertical	E_x, ϵE_y, E_z, H_x, H_y, H_z

$$\begin{pmatrix} E_x \\ E_y \\ \mathrm{i}E_z \end{pmatrix} = \begin{pmatrix} E_x \\ E_y \\ -\mathrm{i}\beta^{-1}(\partial_x E_x + \partial_y E_y) \end{pmatrix}$$

$$\begin{pmatrix} H_x \\ H_y \\ \mathrm{i}H_z \end{pmatrix} = \frac{1}{\omega\mu_0} \begin{pmatrix} -\beta E_y + \beta^{-1}(\partial_{xy}E_x + \partial_y^2 E_y) \\ \beta E_x - \beta^{-1}(\partial_x^2 E_x + \partial_{xy}E_y) \\ \mathrm{i}(\partial_x E_y - \partial_y E_x) \end{pmatrix}$$



$$(\partial_x^2 + \partial_y^2)\phi = (\beta^2 - k^2\epsilon_{lm})\phi, \quad \begin{matrix} l = 0, \ldots, N_x + 1 \\ m = 0, \ldots, N_y + 1 \end{matrix}$$

- Ansatz:
  - separately for two basic components
  - separately for each rectangle

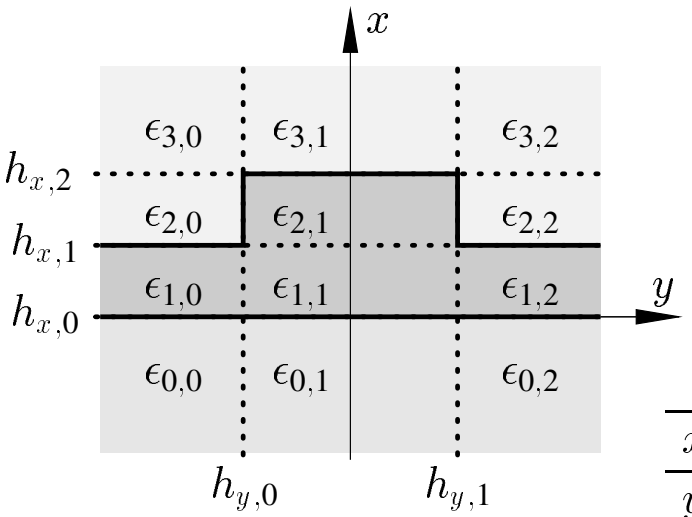
$$\phi(x, y) = \sum_j a_j F_j(p_j x) G_j(q_j y).$$

$F, G \in \{\sin, \cos, \exp\}$
p, q: transverse wavevector-components with

$$\pm p^2 \pm q^2 = \beta^2 - k^2\epsilon\,.$$

- Spectral discretization:
  For each combination of F and G select a number of parameters $\{(p_j, q_j)\}$, such that the trial functions $F(p_j x)\, G(q_j y)$
  - are representative for that set of functions,
  - are numerically linearly independent.

- On unbounded rectangles:
  Consider only outwards exponentially decaying trial functions.



	Continuous quantities
$x = h_{x,l}$	ϵE_x, E_y, E_z, H_x, H_y, H_z
$y = h_{y,m}$	E_x, ϵE_y, E_z, H_x, H_y, H_z

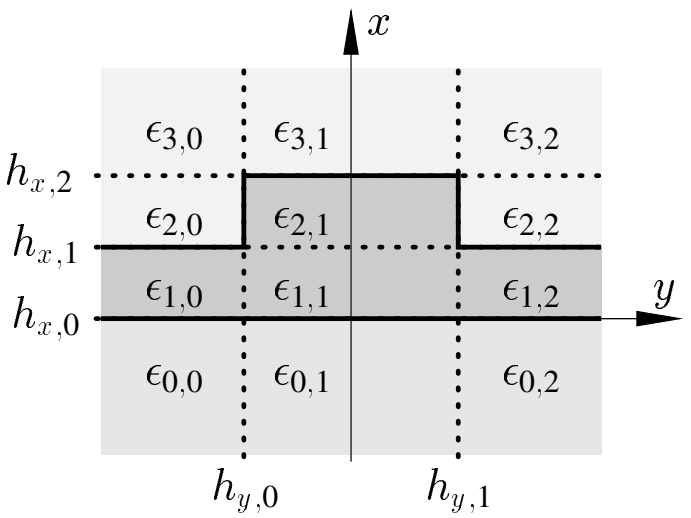
Least squares mismatch on the rectangle boundaries:

$$D_\beta(\boldsymbol{a}) = \sum_{\text{continuous quantities}} \sum_{\substack{\text{horizontal / vertical} \\ \text{boundary segments}}} \int_{\text{boundary line}} \underbrace{\text{suitable weighting factors}} \cdot \left| \underbrace{\text{difference in fields / derivatives}} \right|^2 \mathrm{d}s$$

Constraint: normalized fields

$$N_\beta(\boldsymbol{a}) = \sum_{\text{rectangles}} \iint_{\substack{\text{rectangle} \\ \text{domain}}} \underbrace{\text{suitable weighting factors}} \cdot \left| \text{field} \right|^2 \mathrm{d}x\,\mathrm{d}y$$

Guided modes: $(\beta, \boldsymbol{a})$ with $D_\beta(\boldsymbol{a}) = 0$ and $N_\beta(\boldsymbol{a}) = 1$.



- D_β and N_β are quadratic forms:

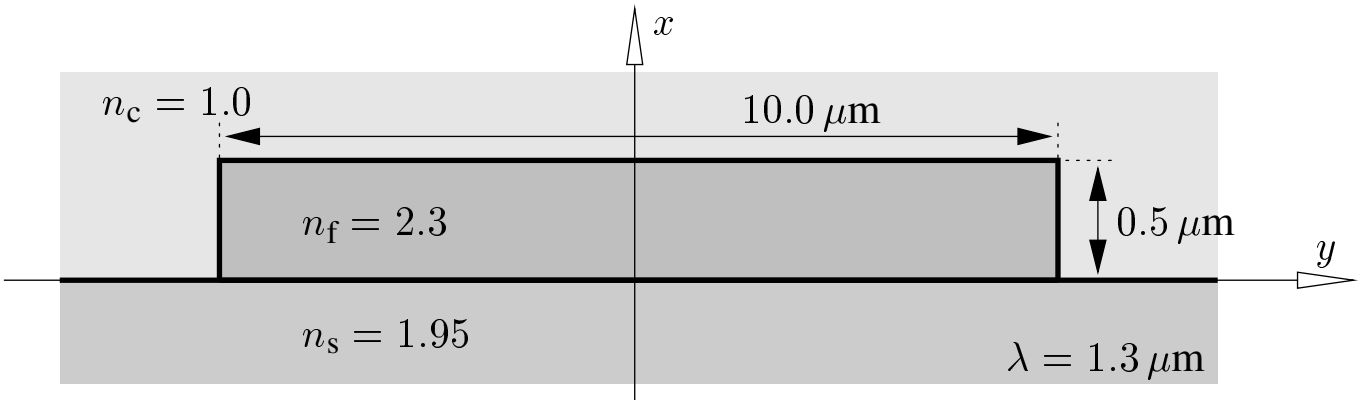
$$D_\beta(\boldsymbol{a}) = \boldsymbol{a}^{\mathsf{T}}\mathsf{D}_\beta\boldsymbol{a}\,, \qquad N_\beta(\boldsymbol{a}) = \boldsymbol{a}^{\mathsf{T}}\mathsf{N}_\beta\boldsymbol{a}\,.$$

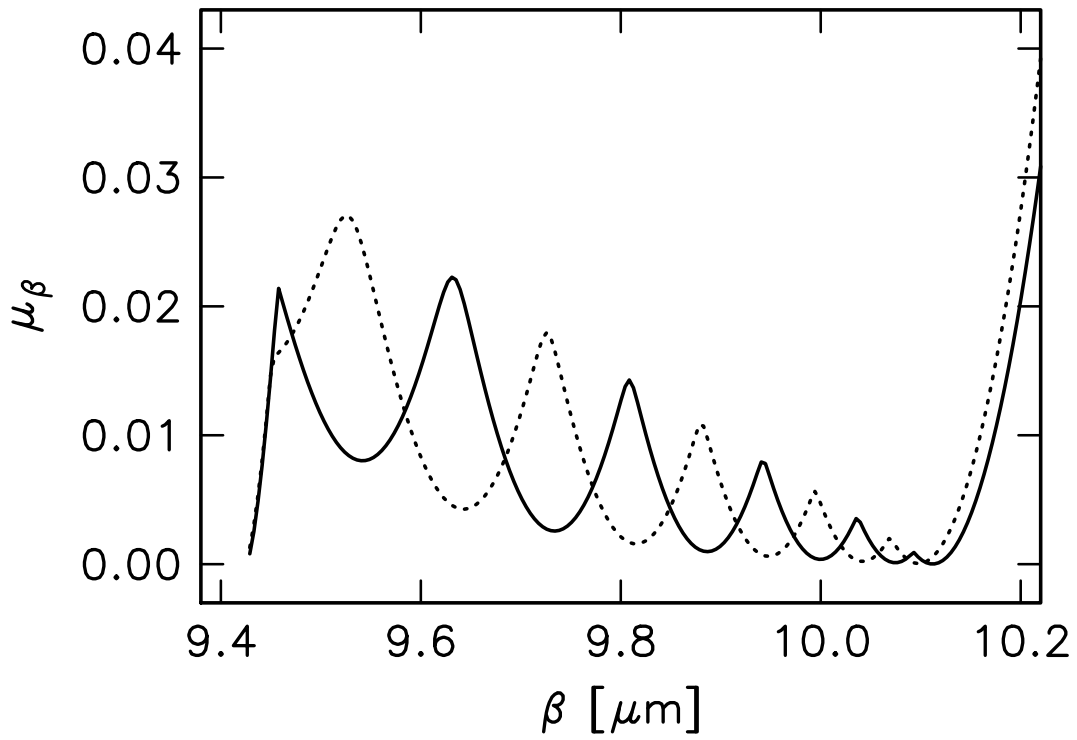
- For fixed β find the set of coefficients $\boldsymbol{a}$ with minimum error μ_β
  $\Leftrightarrow$ solve the eigenvalue equation

$$\mathsf{D}_\beta\boldsymbol{a} = \mu_\beta\mathsf{N}_\beta\boldsymbol{a}\,.$$

- Minima in μ_β with respect to β
  $\rightsquigarrow$ approximations of valid propagation constants.

$\longrightarrow$ Implementation

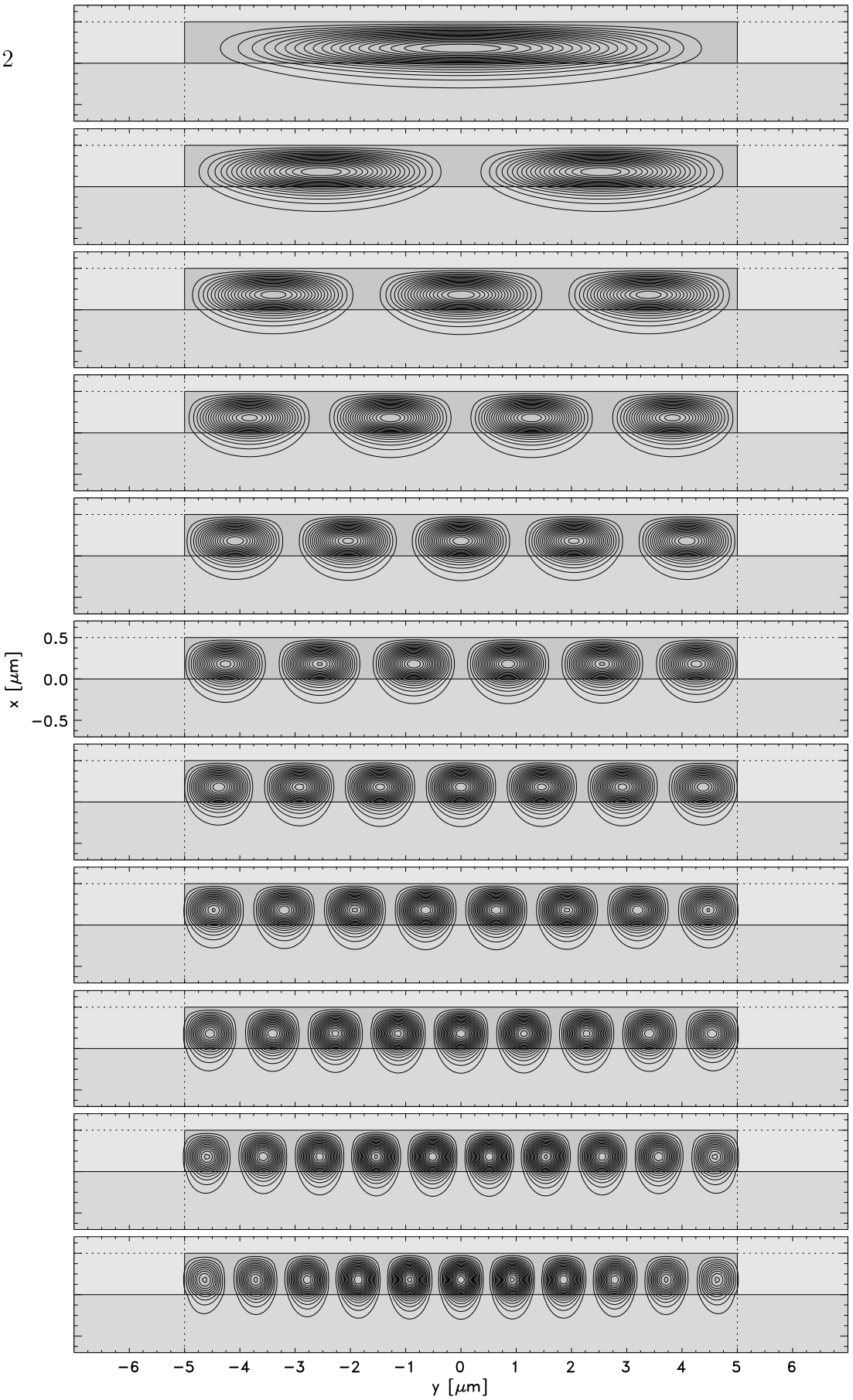


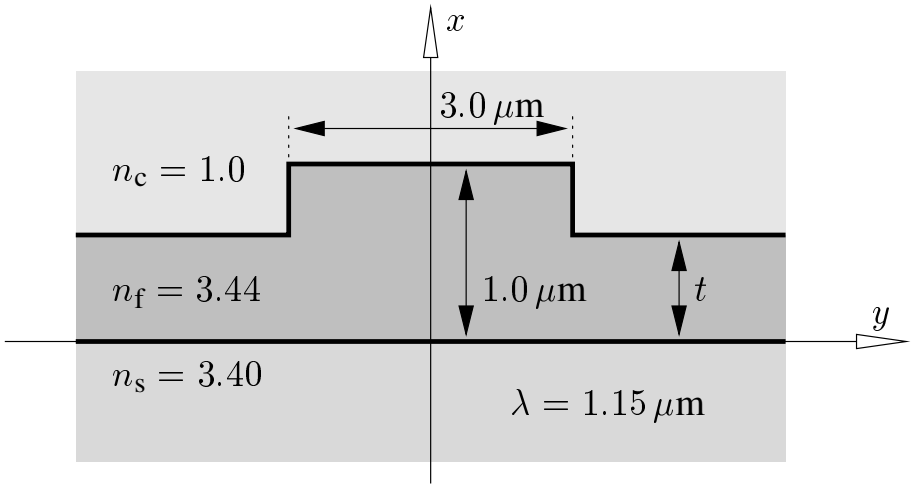


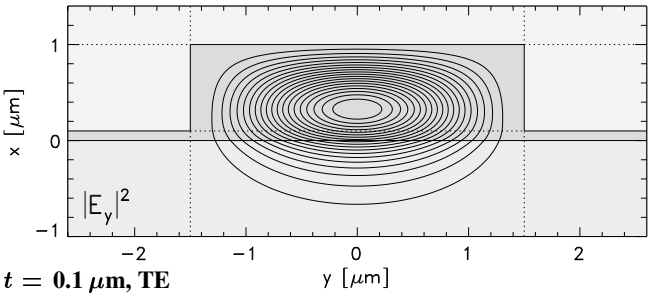
——— TM, symmetrical field $H_y(x, y) = H_y(x, -y)$

. . . . . TM, antisymmetrical field $H_y(x, y) = -H_y(x, -y)$

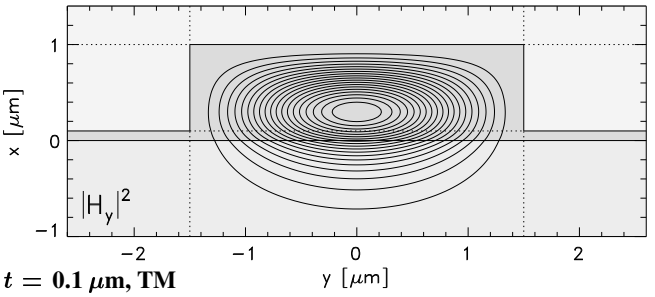
TM, $|H_y|^2$



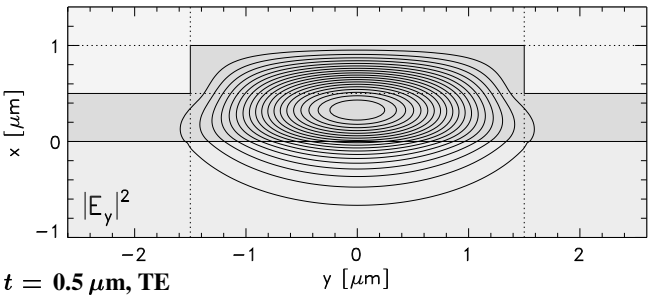




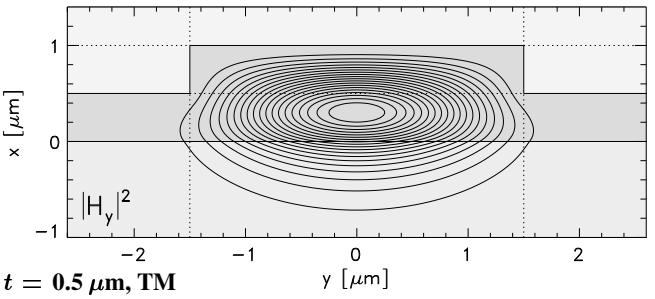
$t = 0.1\ \mu$m, TE



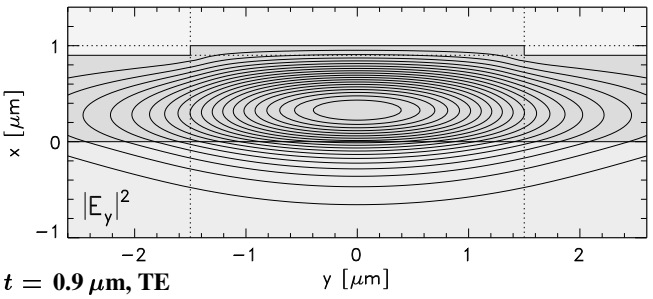
$t = 0.1\ \mu$m, TM



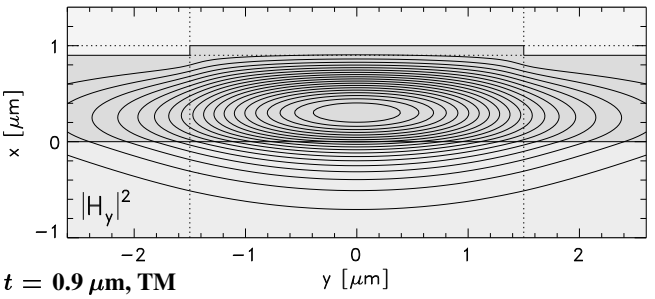
$t = 0.5\ \mu$m, TE



$t = 0.5\ \mu$m, TM



$t = 0.9\ \mu$m, TE

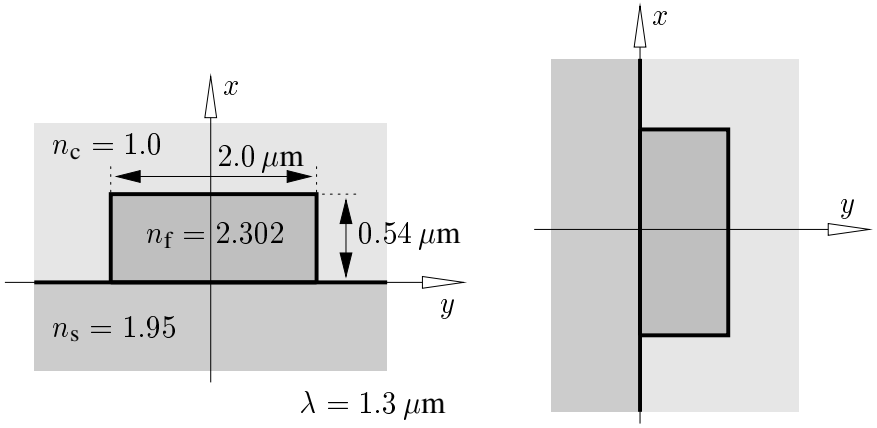


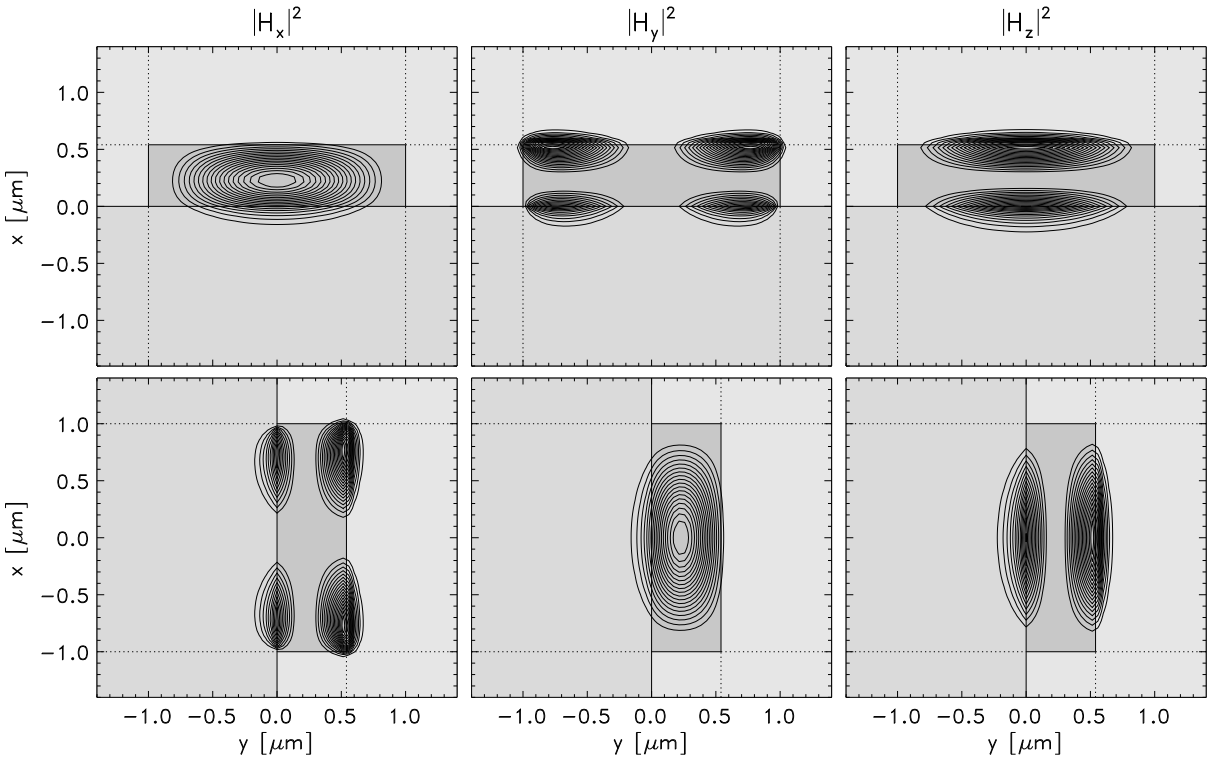
$t = 0.9\ \mu$m, TM

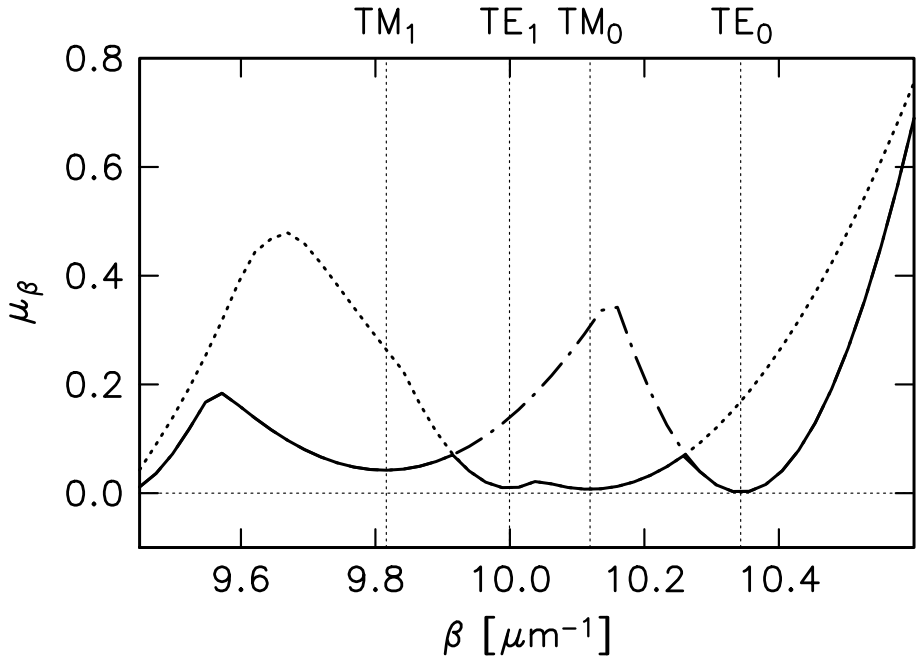
$$\frac{(\beta/k)^2 - n_s^2}{n_f^2 - n_s^2}$$

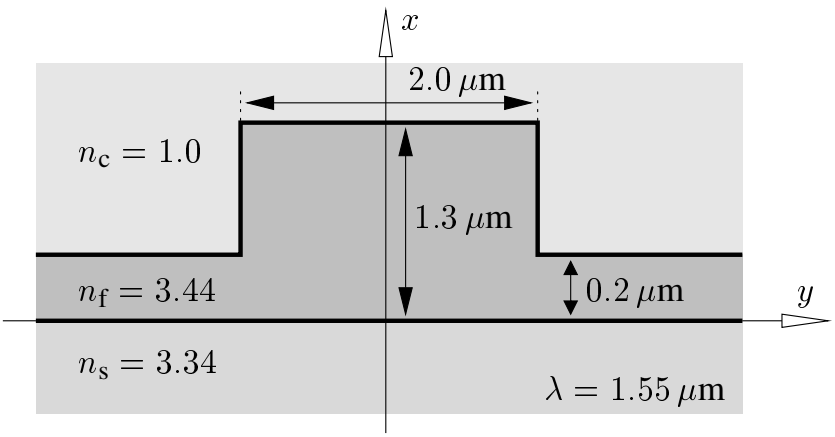
t/μm		0.1	0.3	0.5	0.7	0.9
TE	MTRM	0.3019	0.3110	0.3270	0.3512	0.3883
	WMM	0.3024	0.3112	0.3268	0.3508	0.3881
TM	MTRM	0.2674	0.2751	0.2890	0.3107	0.3455
	WMM	0.2676	0.2759	0.2892	0.3102	0.3451

MTRM: Modal Transverse Resonance Method by A. S. Sudbø;
Data from C. Vassallo, *Optical and Quantum Electronics* **29**, 1997


x
$n_c = 1.0$
2.0 μm
$n_f = 2.302$
0.54 μm
y
$n_s = 1.95$
$\lambda = 1.3$ μm
x
y

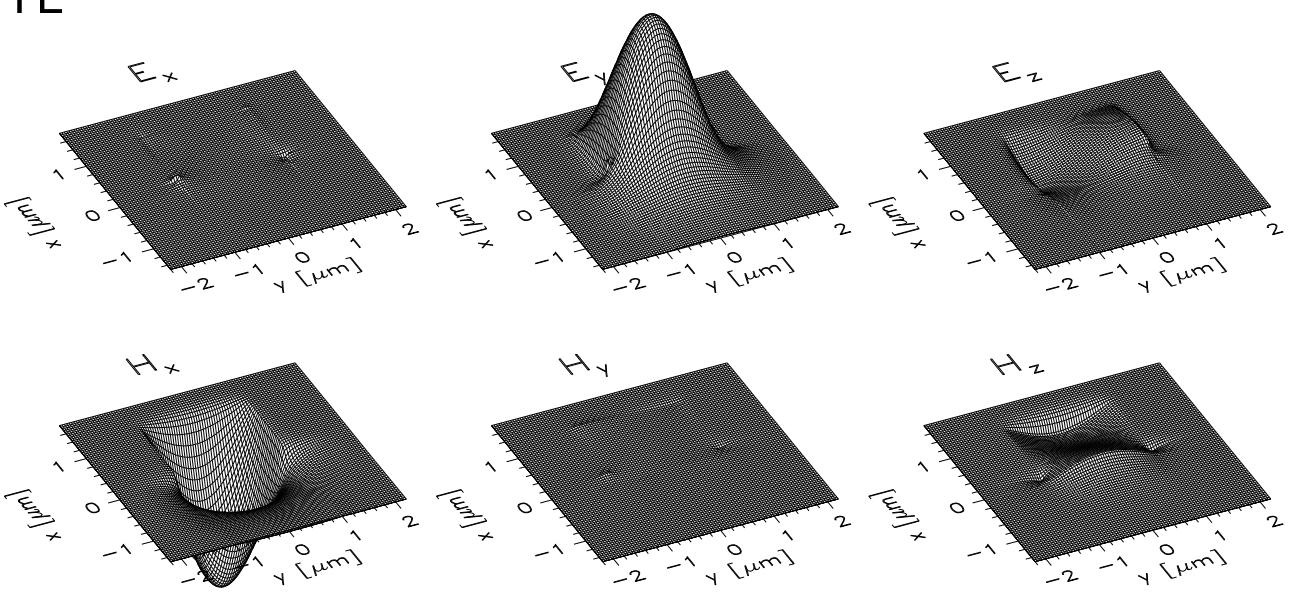

$|H_x|^2$
$|H_y|^2$
$|H_z|^2$
x [μm]
y [μm]
1.0
0.5
0.0
−0.5
−1.0


TM$_1$
TE$_1$
TM$_0$
TE$_0$
μ_β
0.8
0.6
0.4
0.2
0.0
9.6
9.8
10.0
10.2
10.4
β [μm^{-1}]

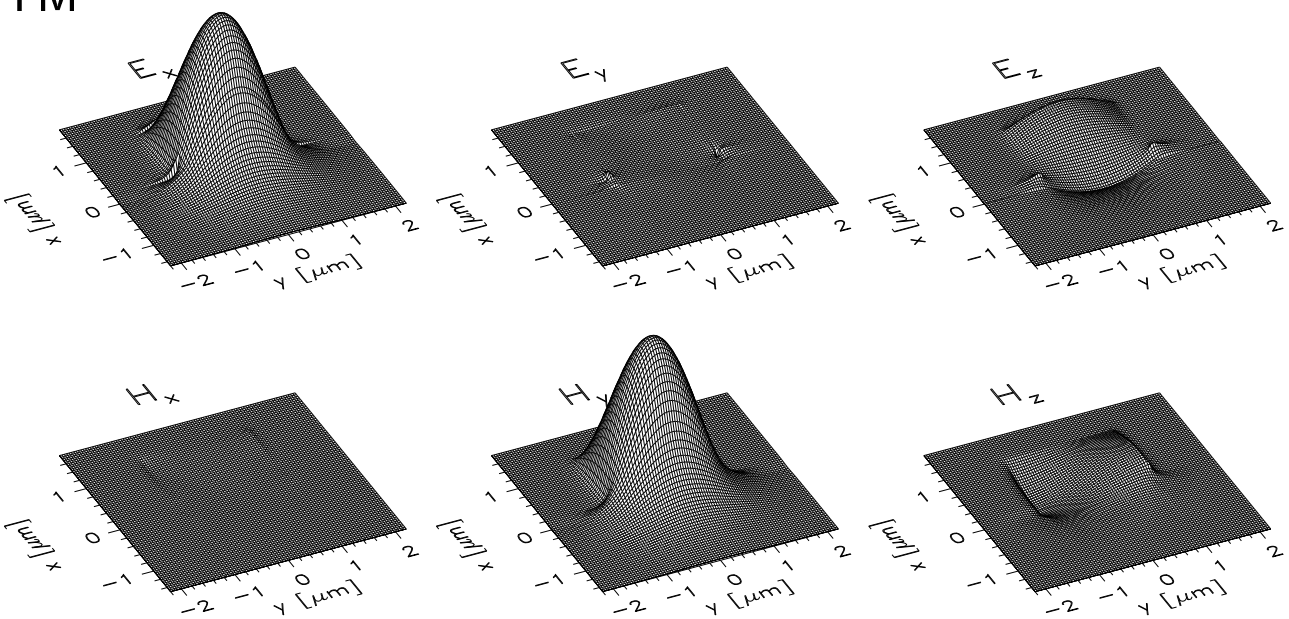


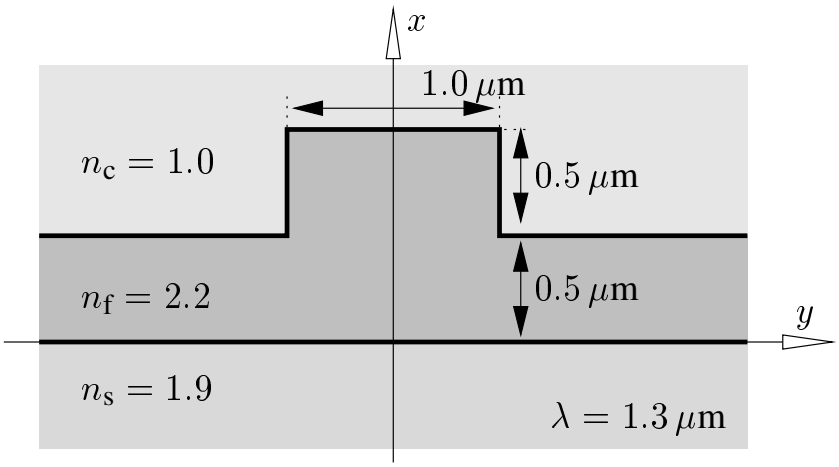
β/k	TE	TM
SV	3.38866	3.38783
VEC	3.38867	3.38782

TE



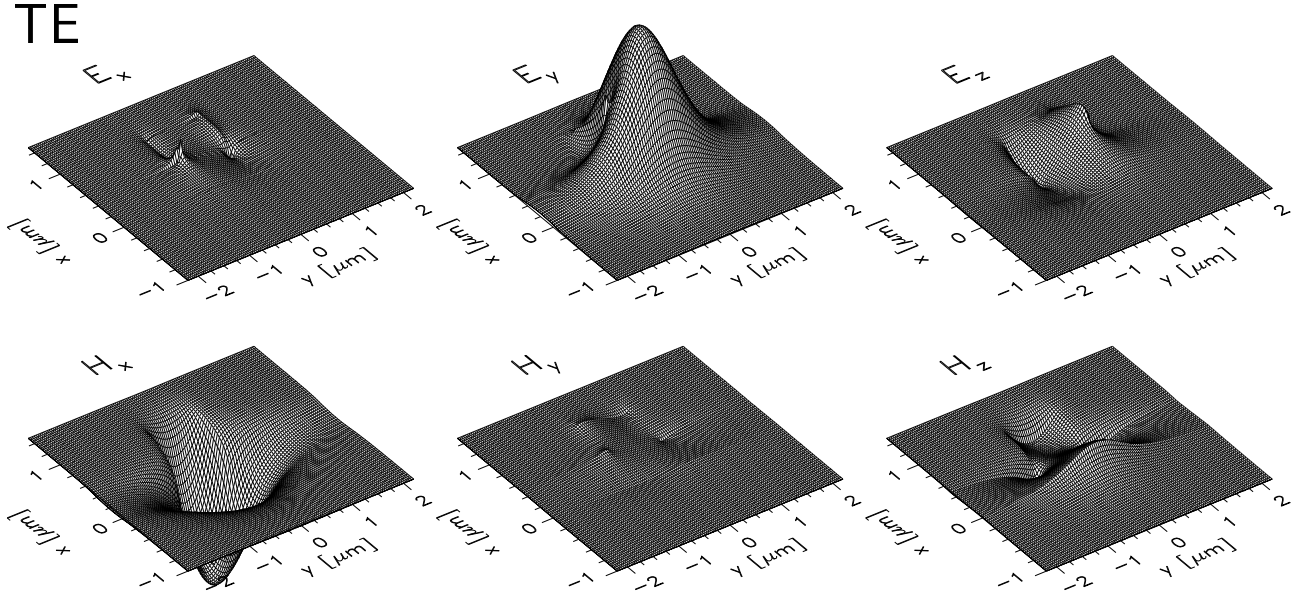
TM



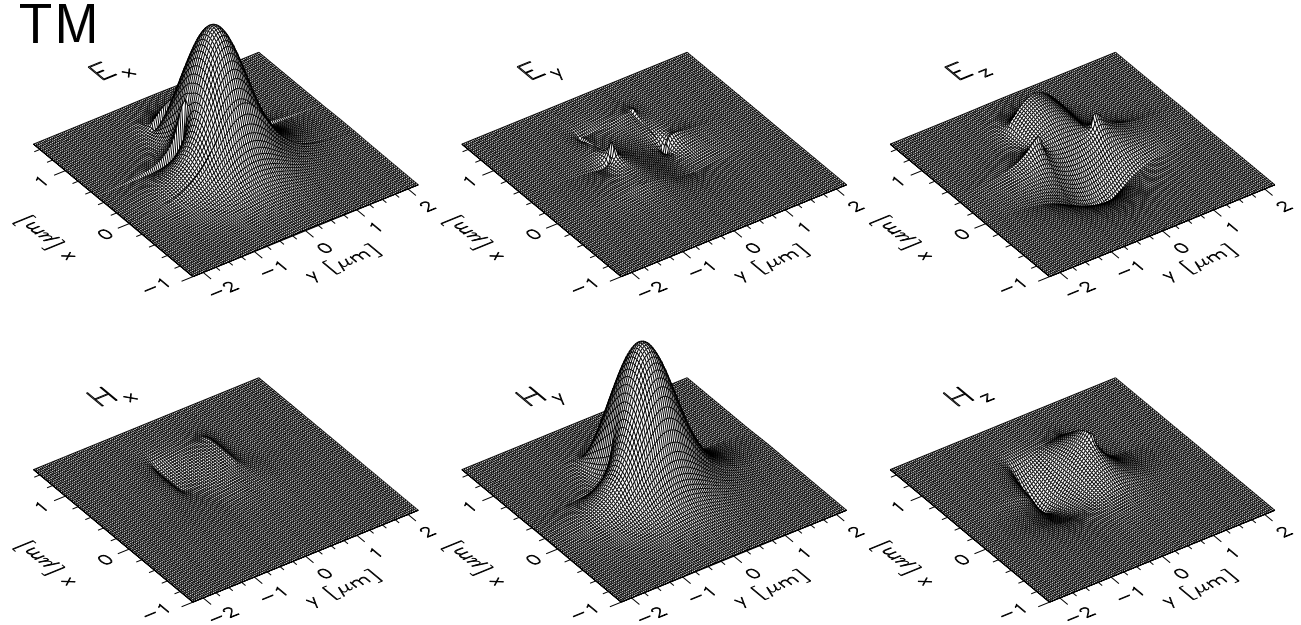


β/k	TE	TM
SV	2.08941	2.07997
VEC	2.08963	2.08009

TE



TM



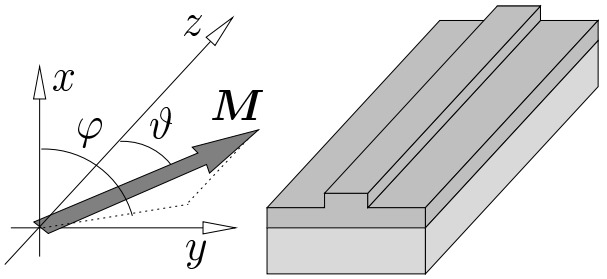
Permittivity of a waveguide made of magnetooptic garnet films:

n^2 — Guiding isotropic refractive index profile,

$+\,\delta\hat{\epsilon}_{\text{mo}}$ — Contribution due to the first order magnetooptic effect.

The magnetization

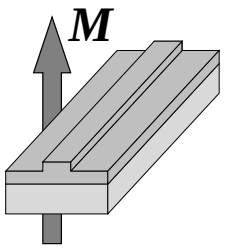
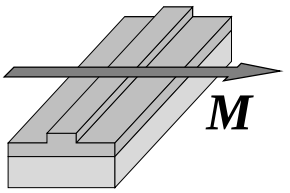
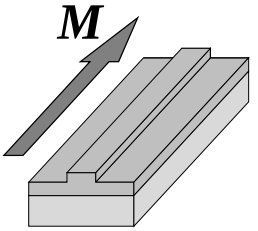
$$\boldsymbol{M} = M \begin{pmatrix} \sin\vartheta\, \cos\varphi \\ \sin\vartheta\, \sin\varphi \\ \cos\vartheta \end{pmatrix}$$



leads to the permittivity perturbation

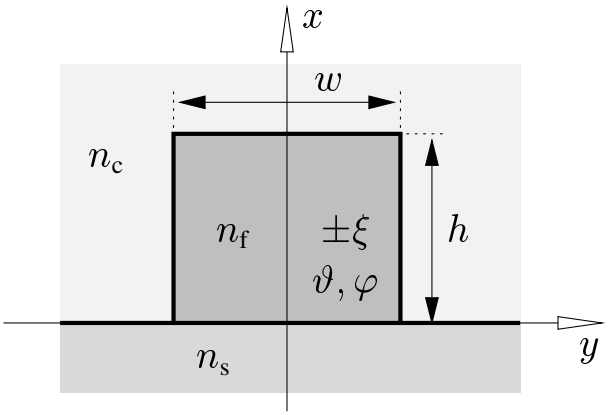
$$\delta\hat{\epsilon}_{\text{mo}} = \sin\vartheta\, \cos\varphi\, \delta\hat{\epsilon}^x_{\text{mo}} + \sin\vartheta\, \sin\varphi\, \delta\hat{\epsilon}^y_{\text{mo}} + \cos\vartheta\, \delta\hat{\epsilon}^z_{\text{mo}}\,,$$

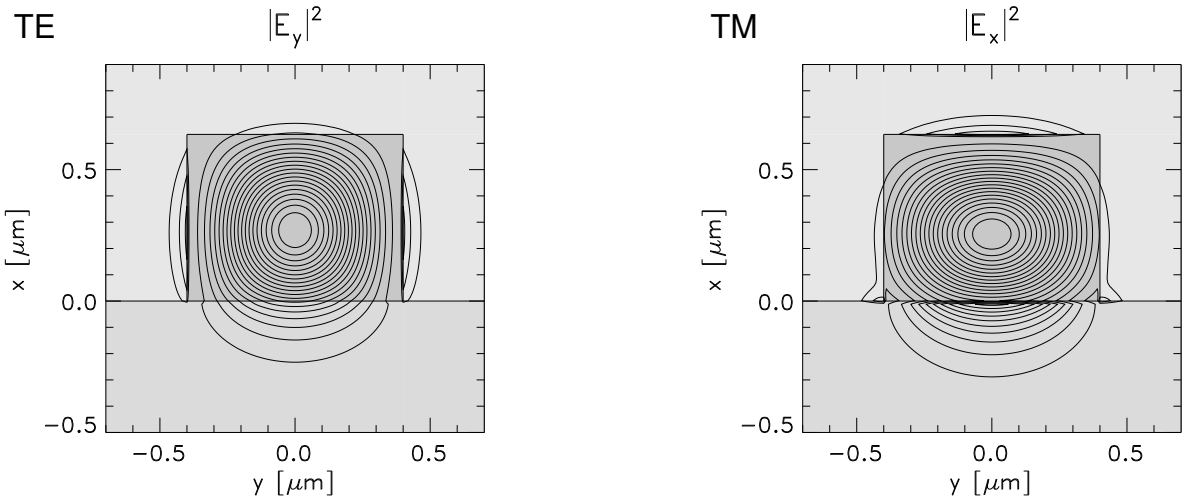
where

		
$\delta\hat{\epsilon}^x_{\text{mo}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mathrm{i}\xi \\ 0 & -\mathrm{i}\xi & 0 \end{pmatrix}$	$\delta\hat{\epsilon}^y_{\text{mo}} = \begin{pmatrix} 0 & 0 & -\mathrm{i}\xi \\ 0 & 0 & 0 \\ \mathrm{i}\xi & 0 & 0 \end{pmatrix}$	$\delta\hat{\epsilon}^z_{\text{mo}} = \begin{pmatrix} 0 & \mathrm{i}\xi & 0 \\ -\mathrm{i}\xi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$KM/\mathrm{i} = \xi = n\lambda\Theta_{\text{F}}/\pi$ with vacuum wavelength λ and specific Faraday-rotation Θ_{F}.

$$|\xi| \ll n^2 \quad \longrightarrow \quad \text{perturbation theory} \ldots$$





$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x,y,z,t) = \sum_m C_m(z) \frac{1}{\sqrt{P_m}} \begin{pmatrix} \boldsymbol{E_m} \\ \boldsymbol{H_m} \end{pmatrix}(x,y)\, \mathrm{e}^{\mathrm{i}\omega t}, \quad m = \mathrm{TE}, \mathrm{TM}\,,$$

$$\boldsymbol{C} = \begin{pmatrix} C_{\mathrm{TE}} \\ C_{\mathrm{TM}} \end{pmatrix}.$$

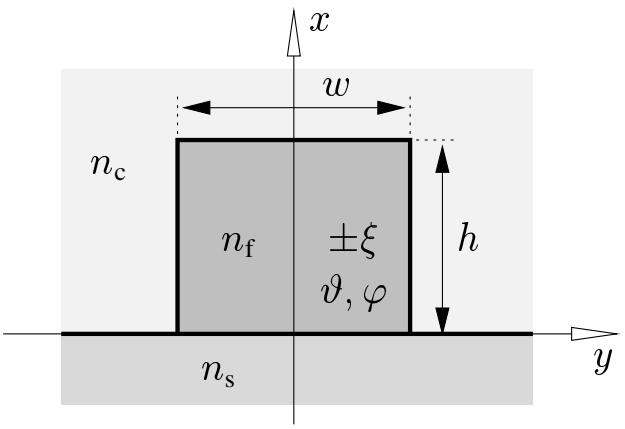
• Light propagation in the isotropic waveguide: $(\delta\hat{\epsilon}_{\mathrm{mo}} = 0)$:

$$\mathrm{i}\partial_z \boldsymbol{C} = \begin{pmatrix} \beta_{\mathrm{TE}} & 0 \\ 0 & \beta_{\mathrm{TM}} \end{pmatrix} \boldsymbol{C} \quad \rightarrow \quad \boldsymbol{C}(z) = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\beta_{\mathrm{TE}} z} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\beta_{\mathrm{TM}} z} \end{pmatrix} \boldsymbol{C}(0)\,.$$

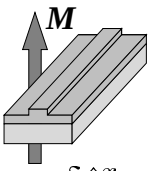
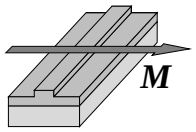
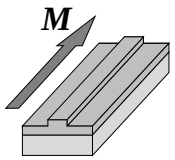
• Magnetooptic effects included:

$$\mathrm{i}\partial_z \boldsymbol{C} = \begin{pmatrix} \beta_{\mathrm{TE}} & 0 \\ 0 & \beta_{\mathrm{TM}} \end{pmatrix} \boldsymbol{C} + \begin{pmatrix} \kappa_{\mathrm{TE,TE}} & \kappa_{\mathrm{TE,TM}} \\ \kappa^*_{\mathrm{TE,TM}} & \kappa_{\mathrm{TM,TM}} \end{pmatrix} \boldsymbol{C}\,,$$

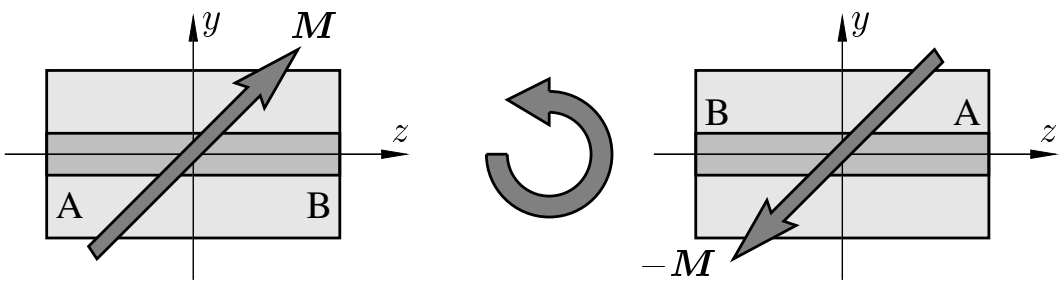
$$\kappa_{l,m} = \frac{\omega\epsilon_0}{4\sqrt{P_l P_m}} \iint \boldsymbol{E}^*_l \delta\hat{\epsilon}_{\mathrm{mo}} \boldsymbol{E}_m \,\mathrm{d}x\,\mathrm{d}y\,.$$



$$\begin{pmatrix} n^2 \\ \delta\hat{\epsilon}^j_{\text{mo}} \end{pmatrix}(x,y) = \begin{pmatrix} n^2 \\ s\,\delta\hat{\epsilon}^j_{\text{mo}} \end{pmatrix}(x,-y), \qquad \gamma = \frac{\omega\epsilon_0}{4\sqrt{P_l P_m}}.$$

	s	TE-PS $\delta\beta_{\text{TE}} = \kappa_{\text{TE,TE}}$	TM-PS $\delta\beta_{\text{TM}} = \kappa_{\text{TM,TM}}$	TE/TM-CV $\kappa = \kappa_{\text{TE,TM}}$
 $\delta\hat{\epsilon}^x_{\text{mo}}$	$+$	0	0	$\approx -\gamma \iint \xi E_y^{\text{TE}} E_z^{\text{TM}}\,\mathrm{d}x\,\mathrm{d}y$
	$-$	$-2\gamma \iint \xi E_y^{\text{TE}} E_z^{\text{TE}}\,\mathrm{d}x\,\mathrm{d}y$	≈ 0	0
 $\delta\hat{\epsilon}^y_{\text{mo}}$	$+$	≈ 0	$2\gamma \iint \xi E_x^{\text{TM}} E_z^{\text{TM}}\,\mathrm{d}x\,\mathrm{d}y$	0
	$-$	0	0	$\approx \gamma \iint \xi E_z^{\text{TE}} E_x^{\text{TM}}\,\mathrm{d}x\,\mathrm{d}y$
 $\delta\hat{\epsilon}^z_{\text{mo}}$	$+$	0	0	$\approx -\mathrm{i}\gamma \iint \xi E_y^{\text{TE}} E_x^{\text{TM}}\,\mathrm{d}x\,\mathrm{d}y$
	$-$	0	0	0

Propagation in forward/backward direction $\leftrightarrow$ signs of ξ:

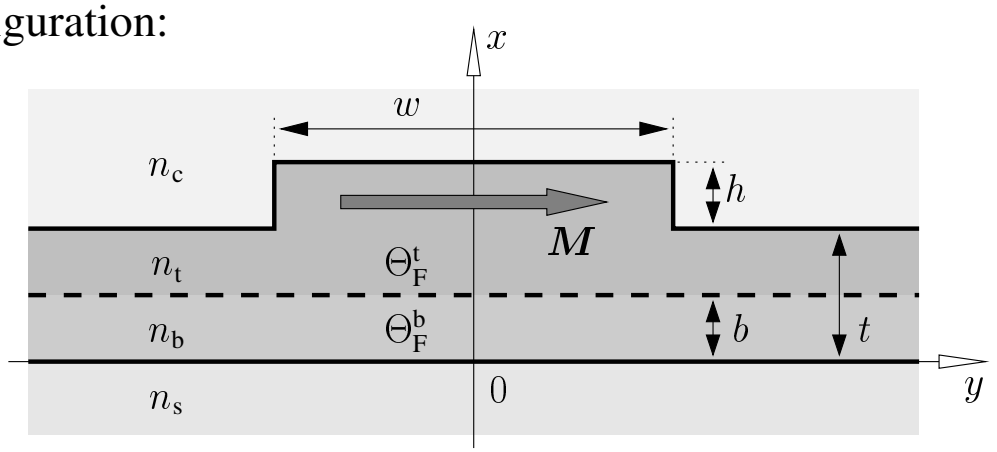


Propagation constants of modes propagating in forward (f) and backward direction (b) differ:

$$\Delta\beta = \beta^{\mathrm{f}} - \beta^{\mathrm{b}} = (\beta + \delta\beta) - (\beta - \delta\beta) = 2\,\delta\beta\,.$$

TM, equatorial configuration:

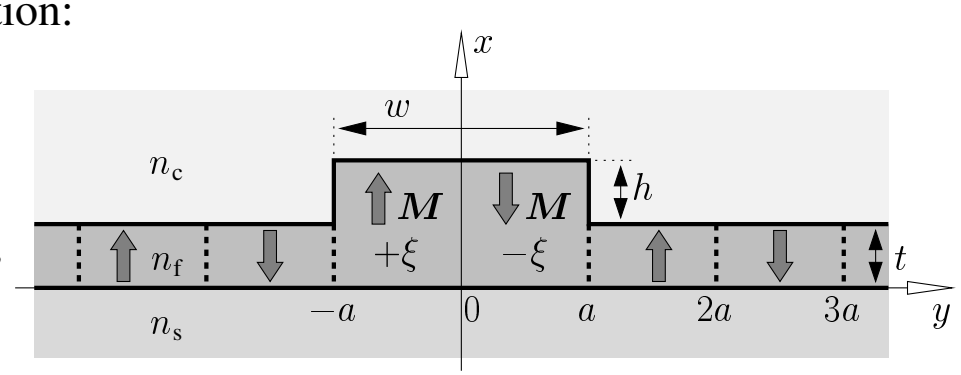
$(\xi, n)(x, y) = (\xi_j, n_j)(y)$,
if $h_{j-1} < x < h_j$,
$j = 0, \ldots N + 1$.



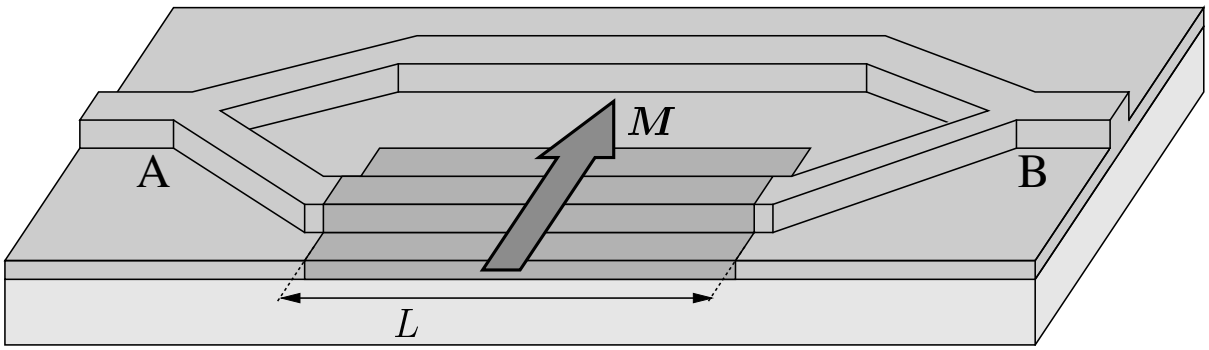
$$\Delta\beta_{\mathrm{TM}} = \frac{\displaystyle\sum_{j=0}^{N} \int \left(\frac{\xi_{j+1}}{n_{j+1}^4}(y) - \frac{\xi_j}{n_j^4}(y) \right) H_y^2(h_j, y)\,\mathrm{d}y}{\displaystyle\iint \frac{1}{n^2} H_y^2\,\mathrm{d}x\,\mathrm{d}y}$$

TE, polar configuration:

$(\xi, n)(x, y) = (\xi_j, n_j)(x)$,
if $b_{j-1} < y < b_j$,
$j = 0, \ldots N + 1$.



$$\Delta\beta_{\mathrm{TE}} = -\frac{k^2}{\beta^2} \frac{\displaystyle\sum_{j=0}^{N} \int \left(\frac{\xi_{j+1}}{n_{j+1}^4}(x) - \frac{\xi_j}{n_j^4}(x) \right) (\epsilon E_y)^2(x, b_j)\,\mathrm{d}x}{\displaystyle\iint \left(E_y - \frac{1}{\beta^2} \partial_y^2 E_y \right) E_y\,\mathrm{d}x\,\mathrm{d}y}$$

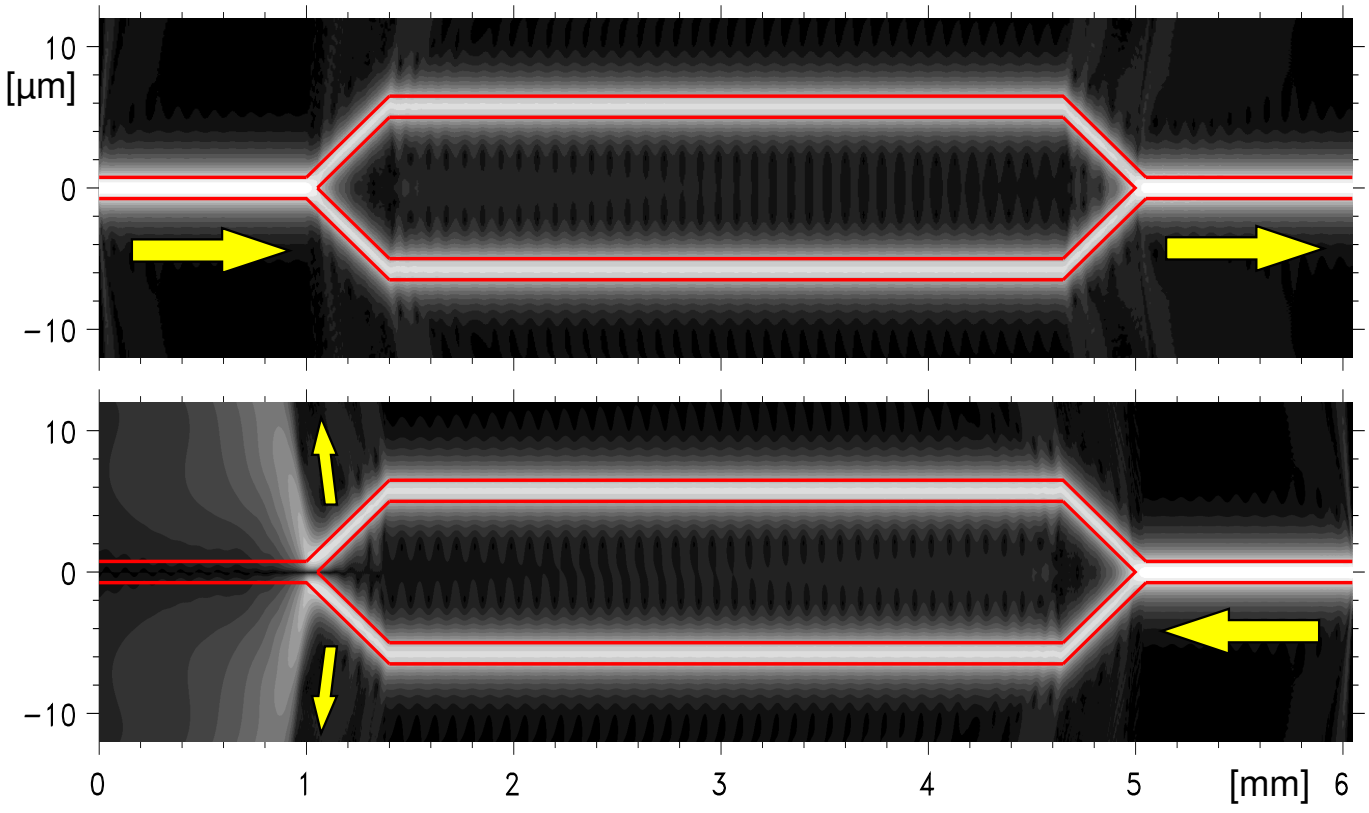


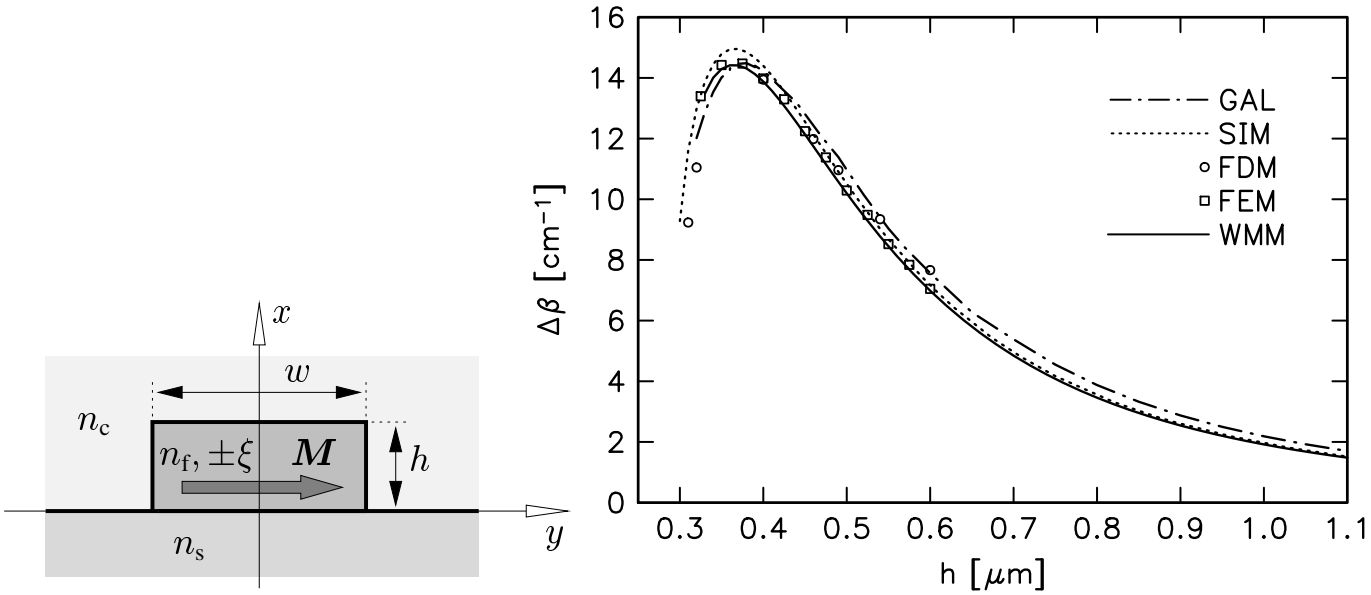
$$L = \begin{cases} \pi/\Delta\beta & \text{(one nonreciprocal arm)} \\ \pi/2\Delta\beta & \text{(two nonreciprocal arms)} \end{cases}$$

$\Longrightarrow$, input at port A: In-phase superposition at junction B, constructive interference

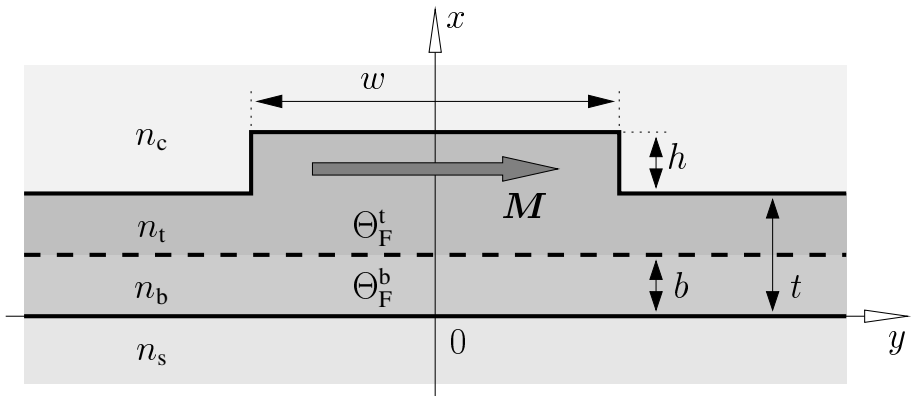
$\Longleftarrow$, input at port B: Phase difference of π at junction A, destructive interference

EIM-BPM simulation:

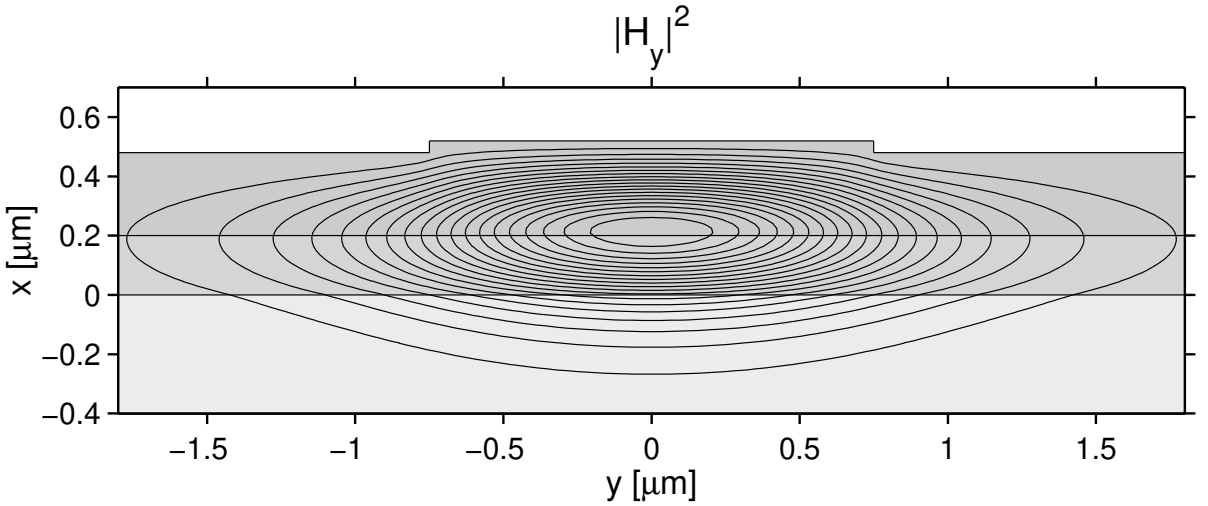




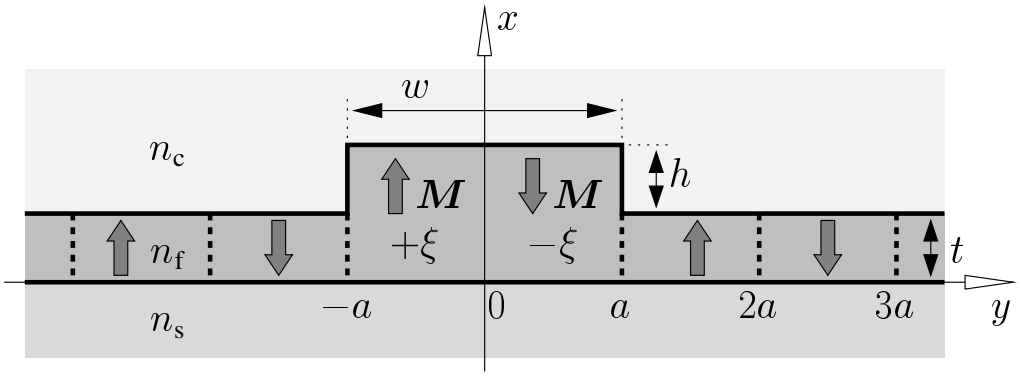
$w = 2.0\,\mu\text{m}$, $\lambda = 1.3\,\mu\text{m}$, $n_\text{s} = 1.95$, $n_\text{f} = 2.302$, $n_\text{c} = 1.0$, $\xi = 0.005$, $\Theta_\text{F} = 3000°/\,\text{cm}$.

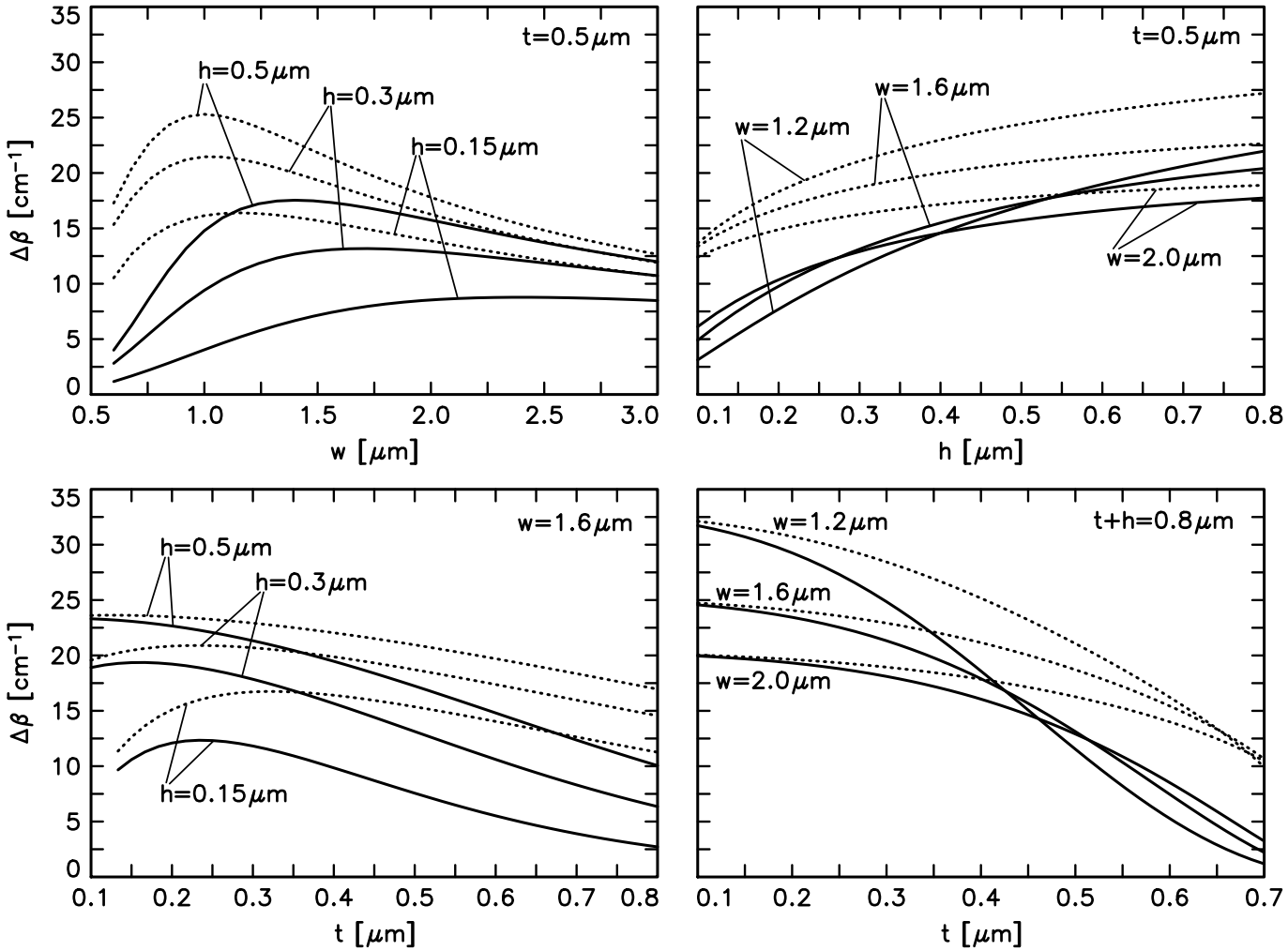


$\lambda = 1.3\,\mu\text{m}$,
$w = 1.5\,\mu\text{m}$, $h = 0.04\,\mu\text{m}$,
$t = 0.48\,\mu\text{m}$, $b = 0.2\,\mu\text{m}$,
$n_\text{s} = 1.95$, $n_\text{b} = 2.27$,
$n_\text{t} = 2.33$, $n_\text{c} = 1.0$,
$\Theta_\text{F}^\text{b} = 350°/\,\text{cm}$,
$\Theta_\text{F}^\text{t} = -1450°/\,\text{cm}$.



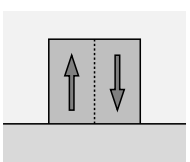
$\longrightarrow \quad \Delta\beta = 12.5\,\text{cm}^{-1}, \;\; L = 2.5\,\text{mm}.$

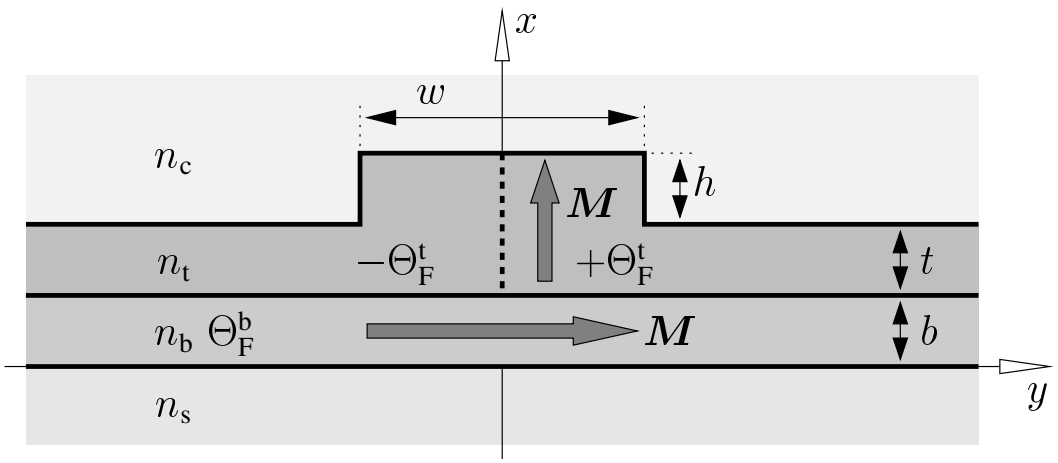




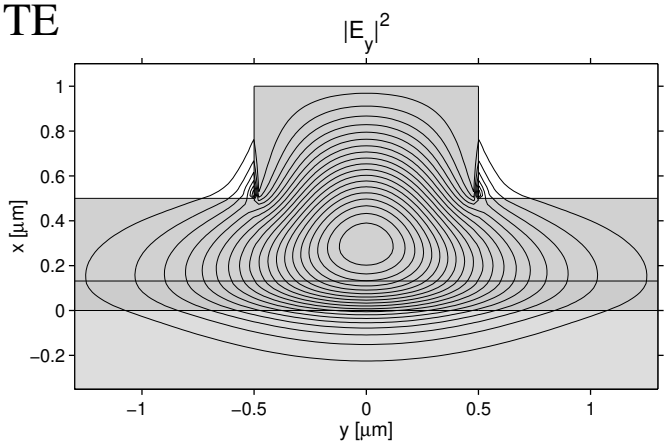
———————: domain lattice · · · · · · · · · · · · : compensation wall

$\lambda = 1.3\,\mu$m, $n_s = 1.9$, $n_f = 2.2$, $n_c = 1.0$, $\xi = \pm 0.005$, $\Theta_F = 3147°/\,$cm.

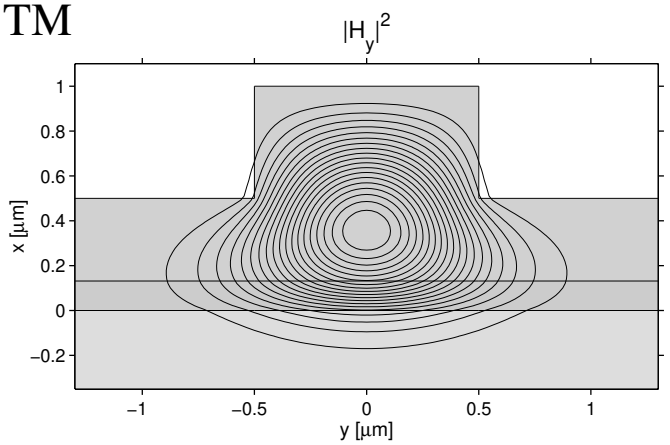
$\longrightarrow$ Optimum TE phase shifter geometry: 



TE

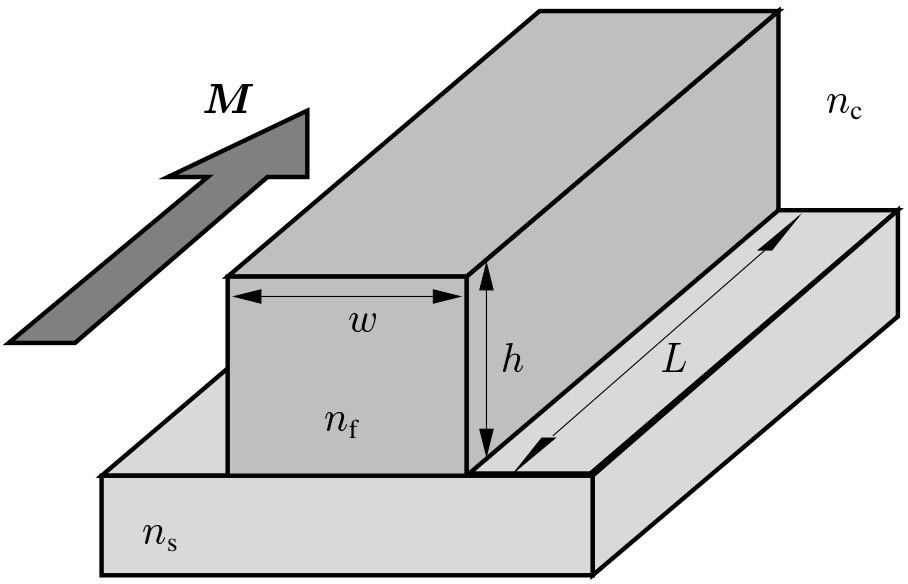


TM



$\lambda = 1.3\,\mu$m, $w = 1.0\,\mu$m, $b = 0.132\,\mu$m, $t = 0.386\,\mu$m, $h = 0.5\,\mu$m,
$n_s = 1.9$, $n_b = 2.33$, $n_t = 2.208$, $n_c = 1.0$, $\Theta_F^b = -1429°/\,\text{cm}$, $\Theta_F^t = 590°/\,\text{cm}$.

$\longrightarrow\ \Delta\beta_{TE} = \Delta\beta_{TM} = 3.4\,\text{cm}^{-1}$, $L = 9.2\,\text{mm}$.



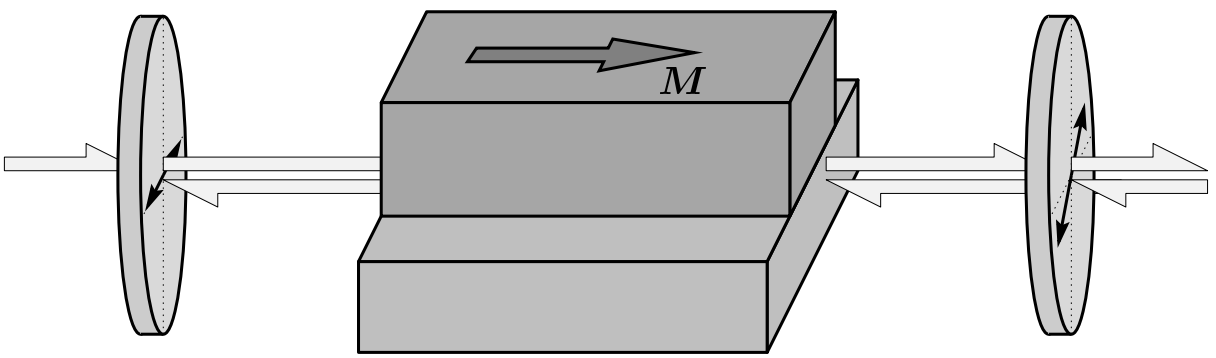
Degree of polarization conversion:

$$\eta(z) = \left| \frac{C_{\mathrm{TM}}(z)}{C_{\mathrm{TE}}(0)} \right|^2 = \frac{|\kappa|^2}{(\Delta\beta/2)^2 + |\kappa|^2} \sin^2\left(\frac{\pi}{2}\frac{z}{L_c}\right),$$

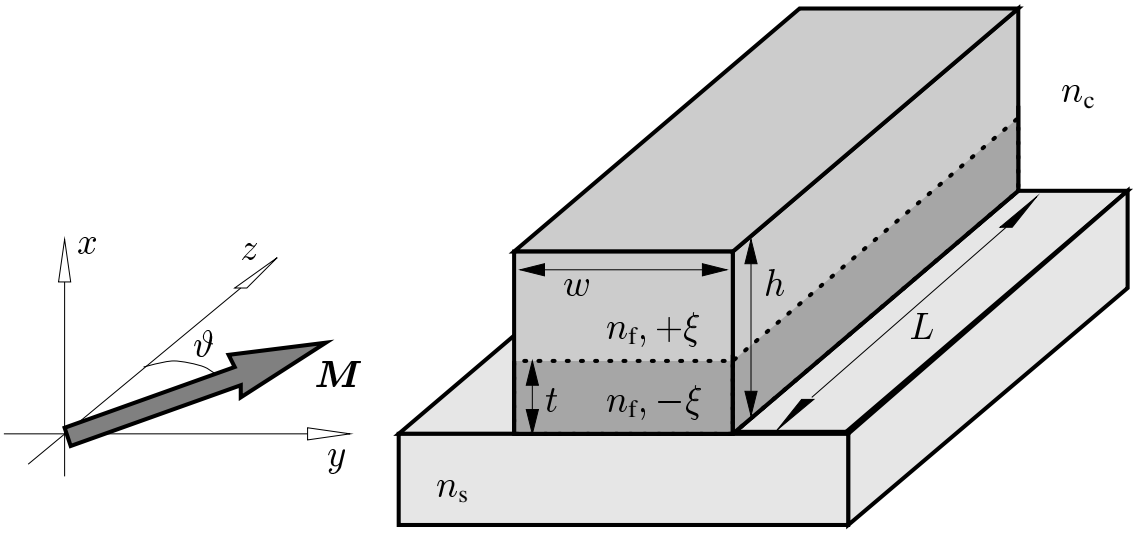
$C_{\mathrm{TM}}(0) = 0, \qquad \Delta\beta = \beta_{\mathrm{TE}} - \beta_{\mathrm{TM}}$.

Conversion length: $\qquad L_c = \dfrac{\pi}{2\sqrt{(\Delta\beta/2)^2 + |\kappa|^2}}$.

Constraint for complete conversion: Phase matching $\Delta\beta = 0$.



Polarizer, TE position | 45° Faraday rotator | Polarizer, 45° position

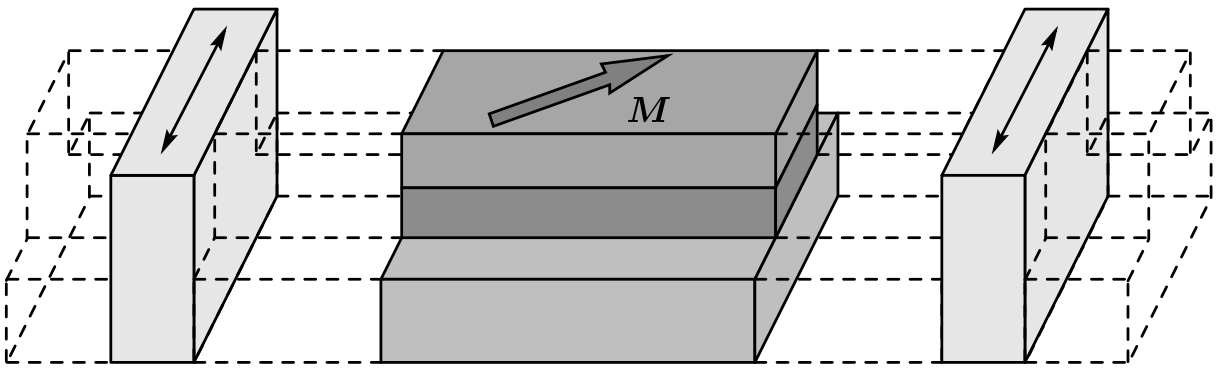


$$\Rightarrow \qquad\qquad \Leftarrow$$

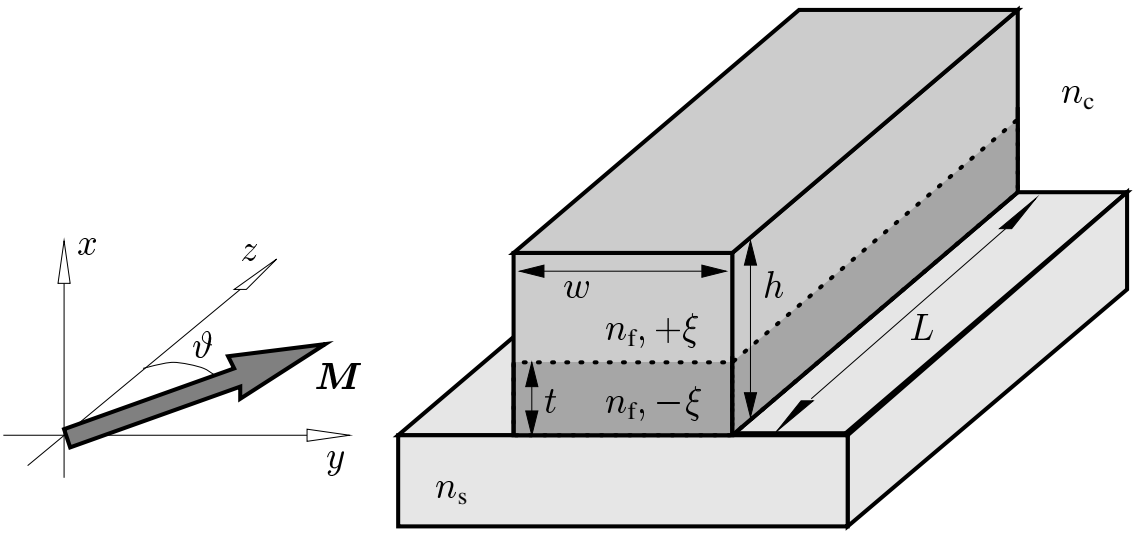
$$\left.\begin{array}{c} \Delta\beta^{\mathrm{f}} = \beta_{\mathrm{TE}} - \beta_{\mathrm{TM}} - \delta\beta_{\mathrm{TM}} \\ L_{\mathrm{c}}^{\mathrm{f}} \\ \eta_{\mathrm{max}}^{\mathrm{f}} \end{array}\right\} \neq \left\{\begin{array}{c} \Delta\beta^{\mathrm{b}} = \beta_{\mathrm{TE}} - \beta_{\mathrm{TM}} + \delta\beta_{\mathrm{TM}} \\ L_{\mathrm{c}}^{\mathrm{b}} \\ \eta_{\mathrm{max}}^{\mathrm{b}} \end{array}\right.$$

Choose parameters with $\Delta\beta^{\mathrm{b}} = 0$, i.e. $\eta_{\mathrm{max}}^{\mathrm{b}} = 1$, and $L = L_{\mathrm{c}}^{\mathrm{b}} = 2L_{\mathrm{c}}^{\mathrm{f}}$.
$\longrightarrow$ No polarization conversion in forward direction,
but complete conversion for backwards propagating light !

Ideal performance: $|\delta\beta_{\mathrm{TM}}| = \sqrt{3}|\kappa|\,,\ \ \Delta\beta^{\mathrm{b}} = 0\,,\ \ L = \dfrac{\pi}{2|\kappa|}\,.$

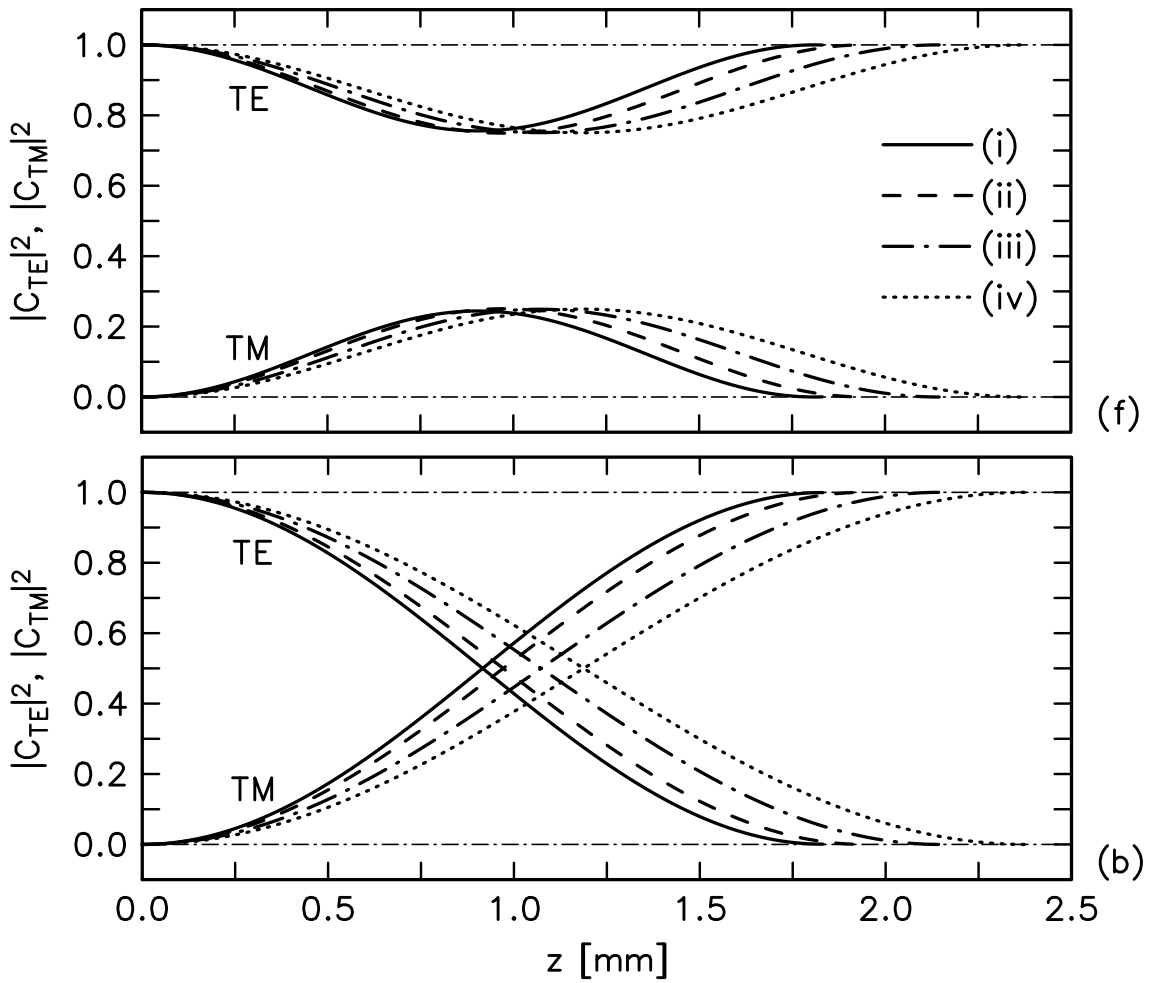


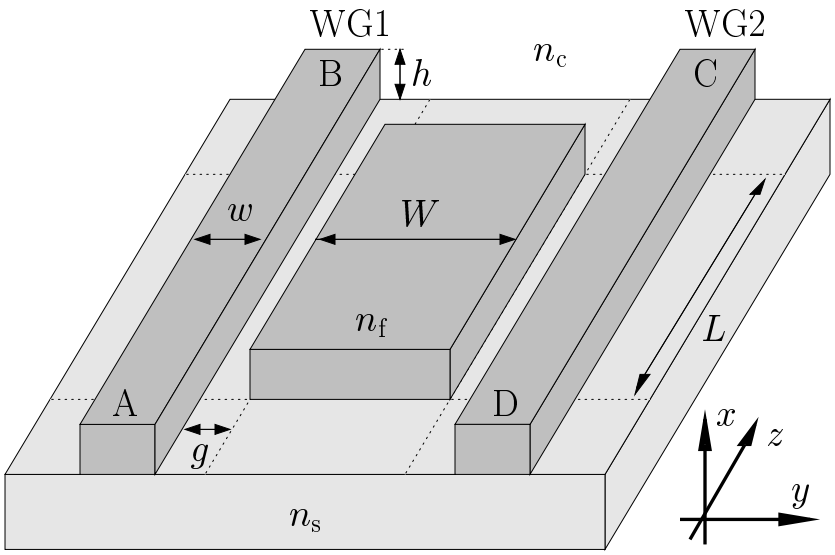
Polarizer, transparent for TE light	Unidirectional magnetooptic polarization converter TE $\longrightarrow$ TE TM $\longleftarrow$ TE	Polarizer, transparent for TE light



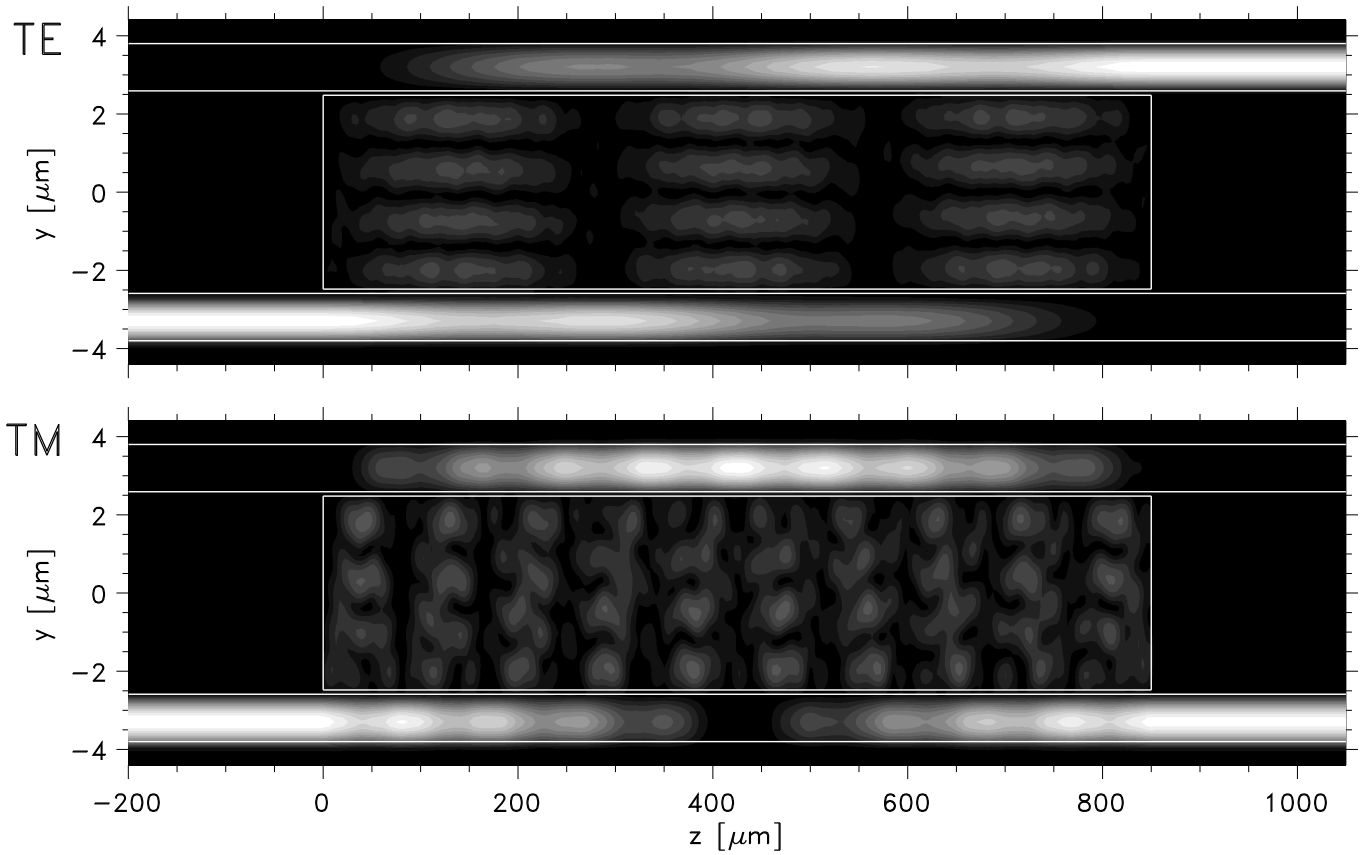
	$h/\,\mu$m	$w/\,\mu$m	$t/\,\mu$m	$\vartheta/^\circ$	$L/$ mm
(i)	0.634	0.789	0.181	63	1.83
(ii)	0.826	0.995	0.264	65	1.94
(iii)	1.012	1.191	0.349	66	2.14
(iv)	1.198	1.387	0.431	68	2.38

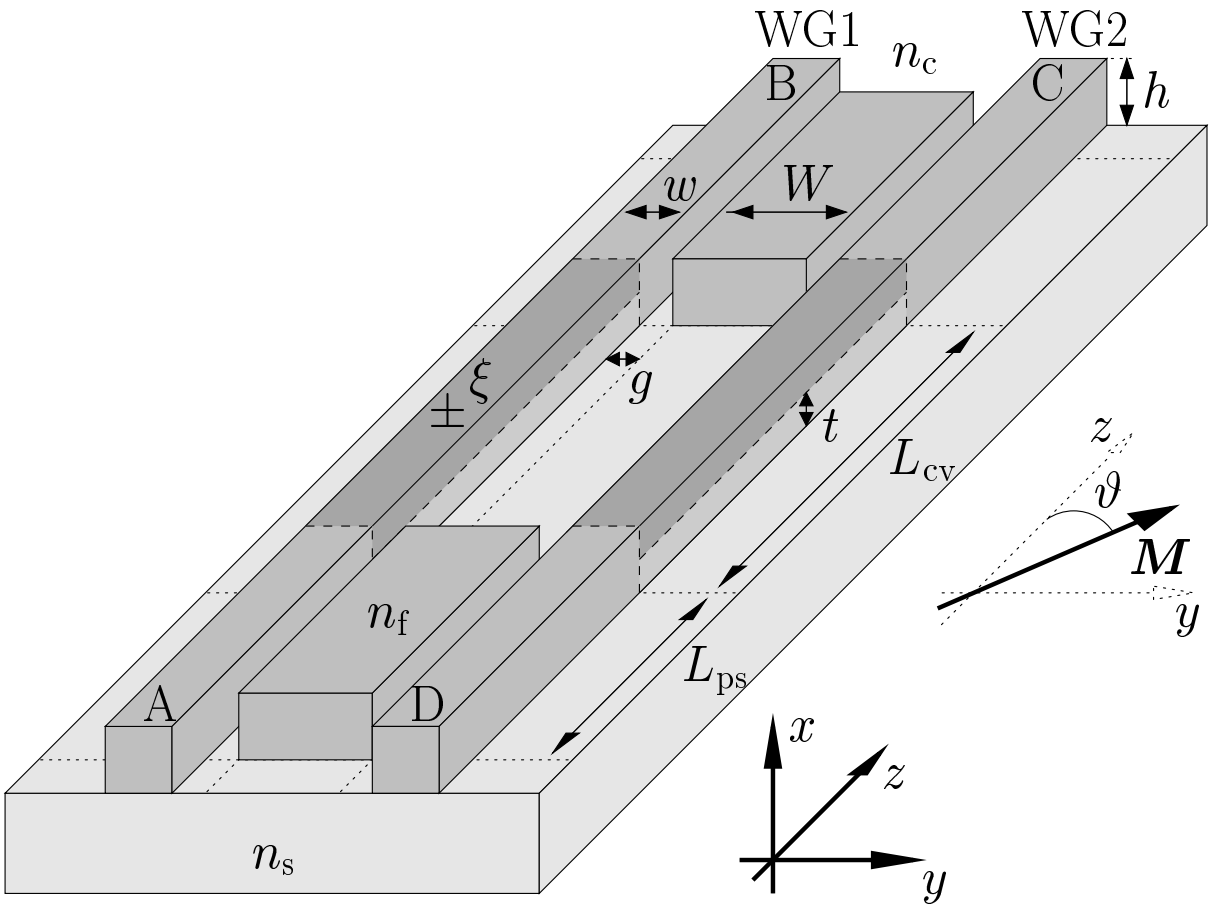
$n_{\rm s} = 1.95$,
$n_{\rm f} = 2.302$,
$n_{\rm c} = 1.0$,
$\xi = 0.005$,
$\lambda = 1.3\,\mu$m.



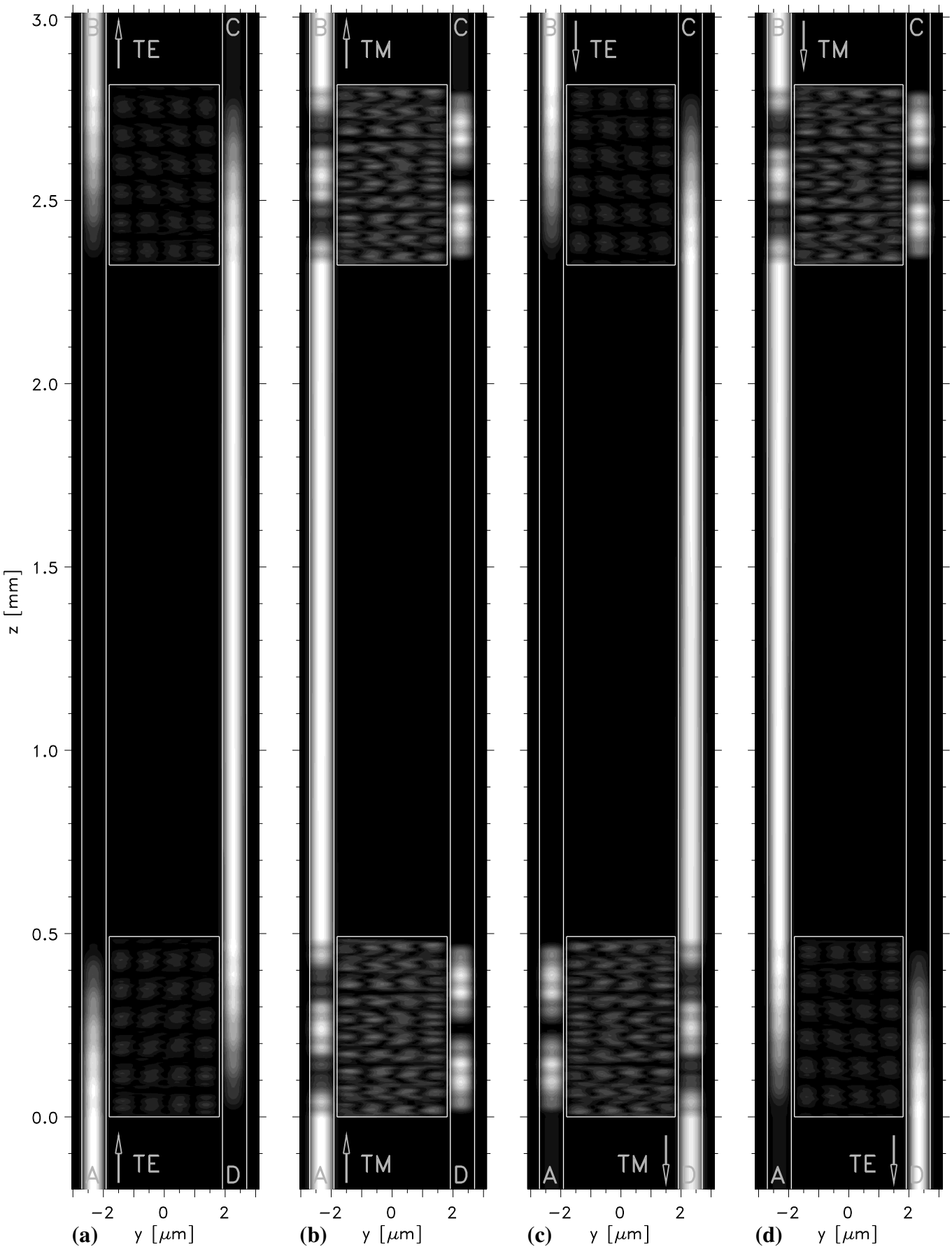


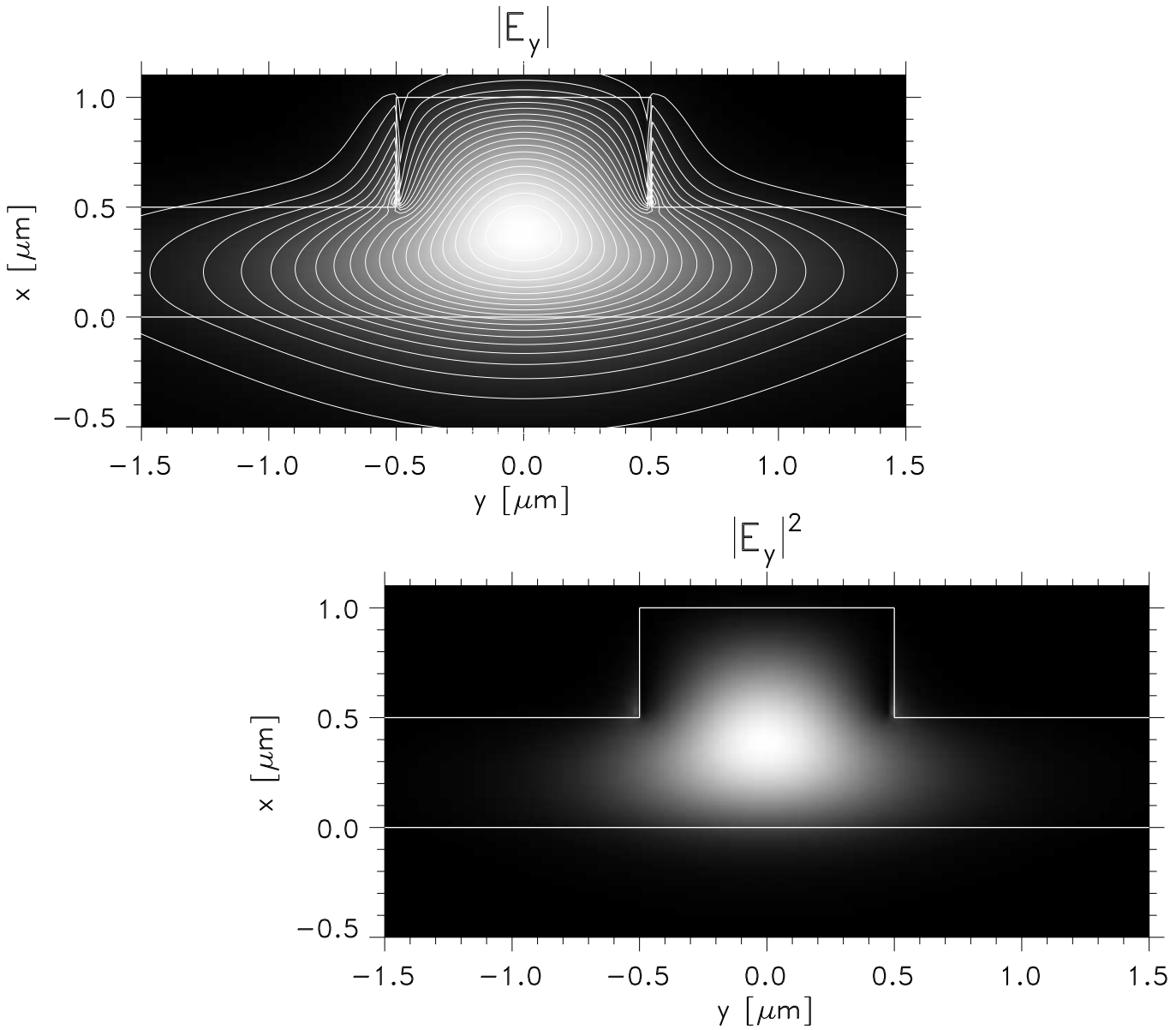
W	w	g	h	λ	L	n_s	n_f	n_c
$4.96\,\mu$m	$1.21\,\mu$m	$0.11\,\mu$m	$0.70\,\mu$m	$1.30\,\mu$m	$850\,\mu$m	1.95	2.2	1.0





h/μm	w/μm	t/μm	$\vartheta/\,^\circ$	g/μm	W/μm	L_{ps}/μm	L_{cv}/μm	n_s	n_f	n_c	$\pm\xi$	λ/μm
0.634	0.798	0.181	63	0.103	3.616	492	1832	1.95	2.302	1.0	0.005	1.3


3.0
2.5
2.0
1.5
1.0
0.5
0.0
z [mm]
B
C
A
D
TE
TM
−2
0
2
y [μm]
(a)
(b)
(c)
(d)



- Guided modes for piecewise constant refractive index profiles
- Wave-matching method for numerical mode analysis
- Examples for semivectorial and fully vectorial simulations
- Perturbation theory for magnetooptic waveguides
- Nonreciprocal interferometry
- Unidirectional polarization conversion
- Concept for a polarization independent circulator