Guided wave interaction in photonic integrated circuits

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Photonics

"… photonics includes the generation, emission, transmission, modulation, signal processing, switching, amplification, detection and sensing of light … closely related to optics. …"

www.wikipedia.org

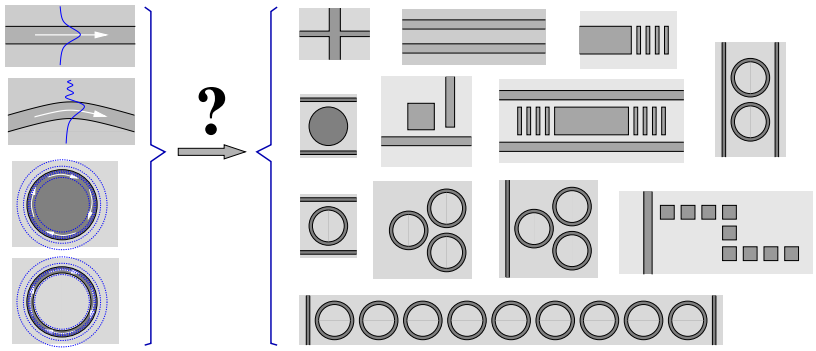
Basis: the Maxwell equations of classical electrodynamics (here).

Photonic integrated circuits

"A photonic integrated circuit (PIC) or integrated optical circuit … integrates multiple photonic functions … more compact … higher performance …"

www.wikipedia.org

Guided wave interaction in photonic integrated circuits



Outline

- Simulations in guided wave optics
  - Macroscopic Maxwell equations
  - Straight dielectric waveguides
  - Waveguide bends
  - Whispering gallery resonances
- Hybrid analytical / numerical coupled-mode modeling
  - Field ansatz
  - Amplitude discetization, 1-D FEM
  - Solution
- Examples
  - Coupled straight waveguides
  - Channel crossing
  - Corrugated waveguides
  - Ring resonators
  - Excitation of whispering gallery resonances

Macroscopic Maxwell equations

. . . for the optical electric and magnetic fields $\boldsymbol{E}$, $\boldsymbol{H}$ (SI)

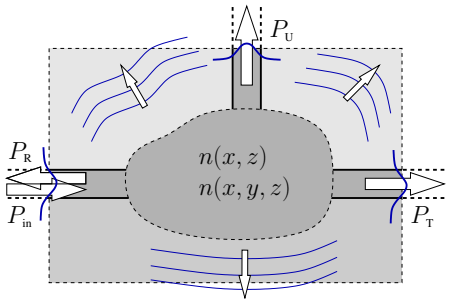
- frequency domain, $\sim \exp(\mathrm{i}\omega t)$,
- no free currents and charges, no sources, homogeneous equations,
- typical media:
  - nonmagnetic at optical frequencies,
  - linear, isotropic, lossless (transparent) dielectrica

↳ relative permittivity $\hat{\epsilon} = \epsilon 1, \quad \epsilon = n^2,$
refractive index $n(x, y, z; \omega) \in \mathbb{R}.$

$$\mathrm{curl}\, \boldsymbol{E} = -\mathrm{i}\omega\mu_0 \boldsymbol{H}, \quad \mathrm{curl}\, \boldsymbol{H} = \mathrm{i}\omega\epsilon_0 \hat{\epsilon} \boldsymbol{E},$$

given excitation frequency $\omega = k\mathrm{c} = 2\pi\mathrm{c}/\lambda$, scans over ω ⇝ spectral data.

Abstract scattering problem



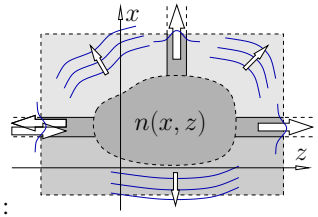
Typical parameters:

- vacuum wavelength
  $\lambda \in [400, 700]$ nm (visible light),
  $\lambda \approx 1.3\,\mu$m, $1.55\,\mu$m
  (optical fibers, attenuation min.),
- refractive indices $n \in [1, 3.4]$.

- Interesting domain: $(10\,\lambda \text{ — } 100\,\lambda)^d$, $d = 2, 3$ (2-D, 3-D).
- Details: $\approx \lambda/10$, $\approx \lambda/100$.
- Influx and outflux: guided & nonguided waves ⇜⇝ boundary conditions.
- Emphasis: device concepts, design.

2-D problems

$\partial_y \epsilon = 0$, $\epsilon(x, z) = n^2(x, z)$,
$\partial_y \boldsymbol{E} = 0$, $\partial_y \boldsymbol{H} = 0$;
equations split into two subsets:



TE, E_y, H_x, and H_z, principal component E_y:

$$\mathrm{i}\omega\mu_0 H_x = \partial_z E_y, \quad \mathrm{i}\omega\mu_0 H_z = -\partial_x E_y, \quad \mathrm{i}\omega\epsilon_0\epsilon E_y = \partial_z H_x - \partial_x H_z,$$

or

$$\partial_x^2 E_y, + \partial_z^2 E_y + k^2 \epsilon E_y = 0.$$

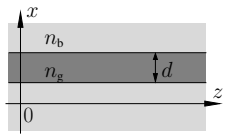
TM, H_y, E_x, and E_z, principal component H_y:

$$\mathrm{i}\omega\epsilon_0\epsilon E_x = -\partial_z H_y, \quad \mathrm{i}\omega\epsilon_0\epsilon E_z = \partial_x H_y, \quad -\mathrm{i}\omega\mu_0 H_y = \partial_z E_x - \partial_x E_z,$$

or

$$\partial_x \frac{1}{\epsilon} \partial_x H_y, + \partial_z \frac{1}{\epsilon} \partial_z H_y + k^2 H_y = 0.$$

Straight dielectric waveguides



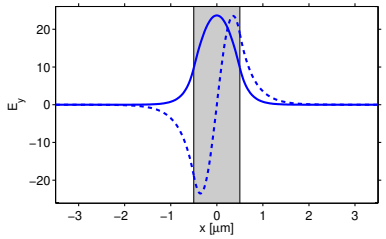
$n_g > n_b$

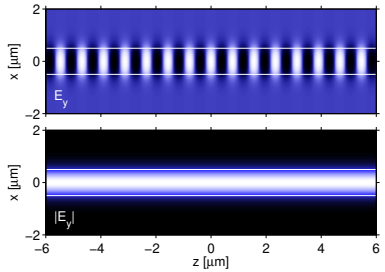
$\partial_z n = 0,$ $\omega = kc = 2\pi c/\lambda$ given,

$E_y(x,z) = \tilde{E}_y(x)\,e^{-i\beta z},$

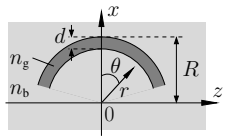
$\left(\partial_x^2 + k^2 n^2(x)\right)\tilde{E}_y = \beta^2 \tilde{E}_y \quad \rightsquigarrow \quad \left\{\beta, \tilde{E}_y\right\}.$

$n_b = 1.5,\ n_g = 2.0,\ d = 1.0\,\mu m,\ \lambda = 1.5\,\mu m,$
$\beta_0/k = 1.924,\ \beta_1/k = 1.697.$





Waveguide bends



$n_g > n_b$

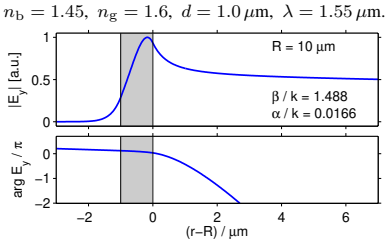
$\partial_\theta n = 0,$ $\omega = kc = 2\pi c/\lambda$ given,

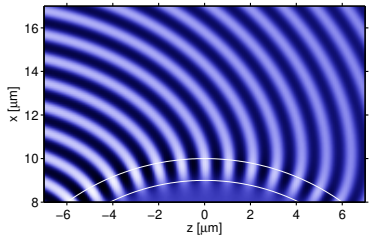
$E_y(r,\theta) = \tilde{E}_y(r)\,e^{-i\gamma R\theta},$ $\gamma = \beta - i\alpha \in \mathbb{C},$

$\dfrac{\partial^2 \tilde{E}_y}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \tilde{E}_y}{\partial r} + \left(k^2 n^2 - \dfrac{\gamma^2 R^2}{r^2}\right)\tilde{E}_y = 0$

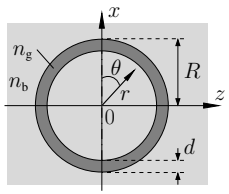
$\rightsquigarrow \quad \left\{\gamma, \tilde{E}_y\right\}.$

$n_b = 1.45,\ n_g = 1.6,\ d = 1.0\,\mu m,\ \lambda = 1.55\,\mu m.$





Whispering gallery resonances



$n_g > n_b$

$\partial_\theta n = 0,$ $\omega^c \in \mathbb{C}$ eigenvalue,

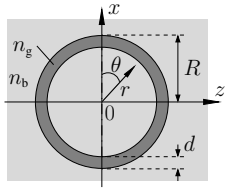
$E_y(r,\theta) = \tilde{E}_y(r)\,e^{-im\theta},$ $m \in \mathbb{Z},$

$\dfrac{\partial^2 \tilde{E}_y}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \tilde{E}_y}{\partial r} + \left(\left(\dfrac{\omega^c}{c}\right)^2 n^2 - \dfrac{m^2}{r^2}\right)\tilde{E}_y = 0$

$\rightsquigarrow \quad \left\{\omega^c, \tilde{E}_y\right\},\ \left\{\mathrm{WGM}(l,m)\right\}.$

$Q = \mathrm{Re}\,\omega^c/(2\mathrm{Im}\,\omega^c),$ $\lambda_r = 2\pi c/\mathrm{Re}\,\omega^c,$ outgoing radiation, FWHM: $\Delta\lambda = \lambda_r/Q.$

Whispering gallery resonances



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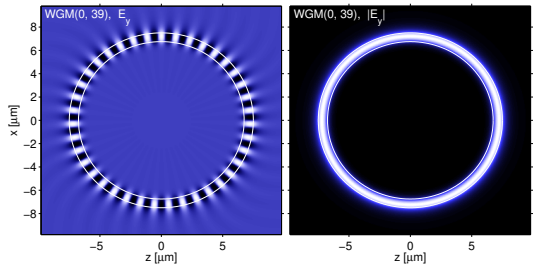
$\partial_\theta n = 0,$ $\omega^c \in \mathbb{C}$ eigenvalue,

$E_y(r,\theta) = \tilde{E}_y(r)\,e^{-im\theta},$ $m \in \mathbb{Z},$

$\dfrac{\partial^2 \tilde{E}_y}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \tilde{E}_y}{\partial r} + \left(\left(\dfrac{\omega^c}{c}\right)^2 n^2 - \dfrac{m^2}{r^2}\right)\tilde{E}_y = 0$

$\rightsquigarrow \quad \left\{\omega^c, \tilde{E}_y\right\},\ \left\{\mathrm{WGM}(l,m)\right\}.$

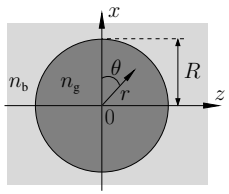


TE, $R = 7.5\,\mu m,\ d = 0.75\,\mu m,$
$n_g = 1.5,\ n_b = 1.0.$

WGM(0, 39):

$\lambda_r = 1.5637\,\mu m,$
$Q = 1.1 \cdot 10^5,$
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu m.$

Whispering gallery resonances



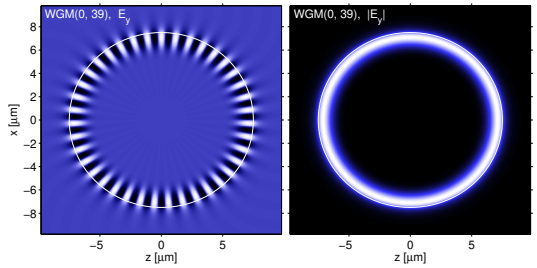
$n_\mathrm{g} > n_\mathrm{b}$

$\partial_\theta n = 0$, $\omega^\mathrm{c} \in \mathbb{C}$ eigenvalue,

$E_y(r,\theta) = \tilde{E}_y(r)\,\mathrm{e}^{-\mathrm{i}m\theta}$, $m \in \mathbb{Z}$,

$$\frac{\partial^2 \tilde{E}_y}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{E}_y}{\partial r} + \left(\left(\frac{\omega^\mathrm{c}}{\mathrm{c}}\right)^2 n^2 - \frac{m^2}{r^2}\right)\tilde{E}_y = 0$$

$$\rightsquigarrow \quad \left\{\omega^\mathrm{c}, \tilde{E}_y\right\}, \ \left\{\mathrm{WGM}(l,m)\right\}.$$

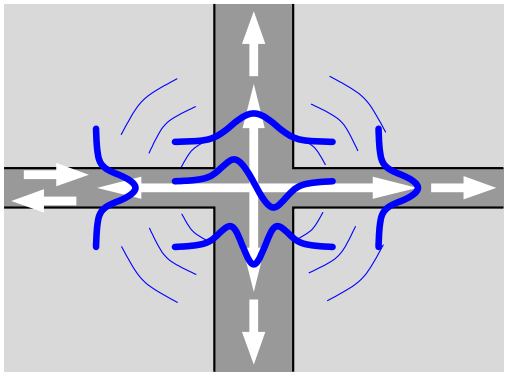


TE, $R = 7.5\,\mu$m,
$n_\mathrm{g} = 1.5$, $n_\mathrm{b} = 1.0$.

WGM(0, 39):

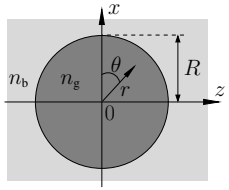
$\lambda_\mathrm{r} = 1.6025\,\mu$m,
$Q = 5.7 \cdot 10^5$,
$\Delta\lambda = 2.8 \cdot 10^{-6}\,\mu$m.

A waveguide crossing



Coupled Mode Model ?

Whispering gallery resonances



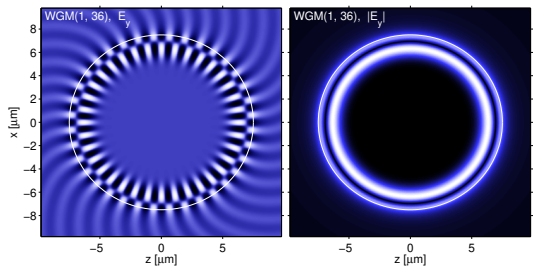
$n_\mathrm{g} > n_\mathrm{b}$

$\partial_\theta n = 0$, $\omega^\mathrm{c} \in \mathbb{C}$ eigenvalue,

$E_y(r,\theta) = \tilde{E}_y(r)\,\mathrm{e}^{-\mathrm{i}m\theta}$, $m \in \mathbb{Z}$,

$$\frac{\partial^2 \tilde{E}_y}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{E}_y}{\partial r} + \left(\left(\frac{\omega^\mathrm{c}}{\mathrm{c}}\right)^2 n^2 - \frac{m^2}{r^2}\right)\tilde{E}_y = 0$$

$$\rightsquigarrow \quad \left\{\omega^\mathrm{c}, \tilde{E}_y\right\}, \ \left\{\mathrm{WGM}(l,m)\right\}.$$

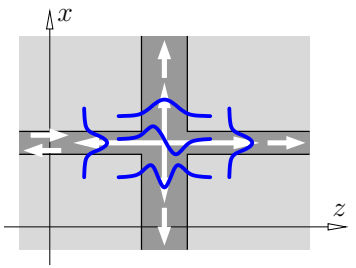


TE, $R = 7.5\,\mu$m,
$n_\mathrm{g} = 1.5$, $n_\mathrm{b} = 1.0$.

WGM(1, 36):

$\lambda_\mathrm{r} = 1.5367\,\mu$m,
$Q = 2.2 \cdot 10^4$,
$\Delta\lambda = 7.0 \cdot 10^{-4}\,\mu$m.

Field ansatz



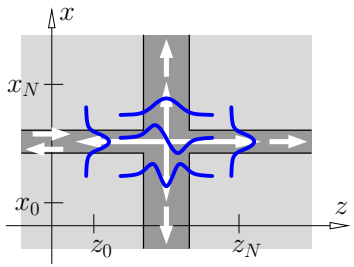
Basis elements (crossing):

- guided modes of the horizontal WG
  $\boldsymbol{\psi}_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_m^{\mathrm{f,b}}(x)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{f,b}} z}$,
- guided modes of the vertical WG
  $\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_m^{\mathrm{u,d}}(z)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{u,d}} x}$
- (and further terms).

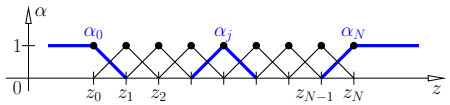
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_m f_m(z)\boldsymbol{\psi}_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\boldsymbol{\psi}_m^{\mathrm{b}}(x,z) + \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x,z) \qquad f_m,\, b_m,\, u_m,\, d_m\text{: ?}$$

General: a reasonable superposition of known fields with amplitudes that are functions of suitable propagation coordinate(s).

Amplitude functions, discretization



1-D linear finite elements



$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\,\alpha_j(z)\,,$$

$b_m(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_\cdot(\cdot)\psi_\cdot\right)(x,z) =: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z),$$

$k \in \{\text{waveguides, modes, elements}\}$, $\quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}$, $\qquad a_k$: ?

Galerkin procedure

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0\\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0\end{aligned} \quad\Big|\quad \cdot\begin{pmatrix}\boldsymbol{F}\\ \boldsymbol{G}\end{pmatrix}^*, \qquad \iint_{\text{comp. domain}}$$

$$\iint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{G}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^*\cdot\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^*\cdot\boldsymbol{H}\,.$$

Galerkin procedure, continued

- Insert $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E}_k,\boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z$.

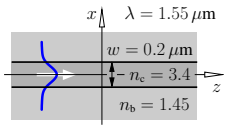
$$\sum_{k\in\{\mathrm{u,g}\}} K_{lk}a_k = 0\,,\ l \in \{\mathrm{u}\}\,,$$

$$\begin{pmatrix}\mathsf{K}_{\mathrm{u\,u}} & \mathsf{K}_{\mathrm{u\,g}}\end{pmatrix}\begin{pmatrix}\boldsymbol{a}_{\mathrm{u}}\\ \boldsymbol{a}_{\mathrm{g}}\end{pmatrix} = 0\,, \qquad \text{or} \qquad \mathsf{K}_{\mathrm{u\,u}}\boldsymbol{a}_{\mathrm{u}} = -\mathsf{K}_{\mathrm{u\,g}}\boldsymbol{a}_{\mathrm{g}}\,.$$

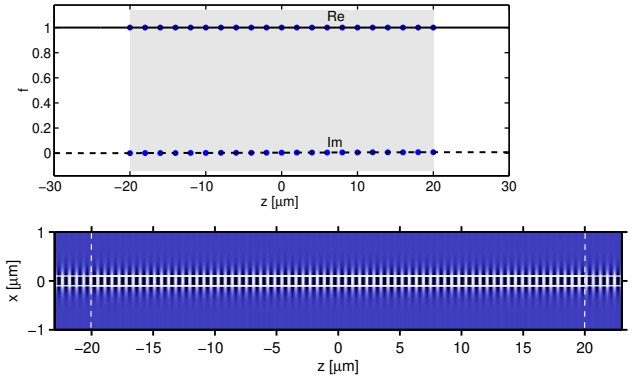
Further issues

… plenty.

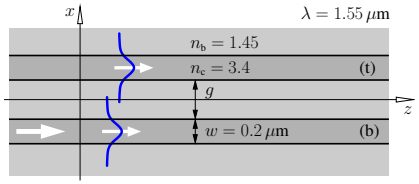
Straight waveguide



Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\ \mu$m, $\Delta z = 2\ \mu$m,
computational domain $z \in [< -20, > 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.



Two coupled parallel cores, amplitudes

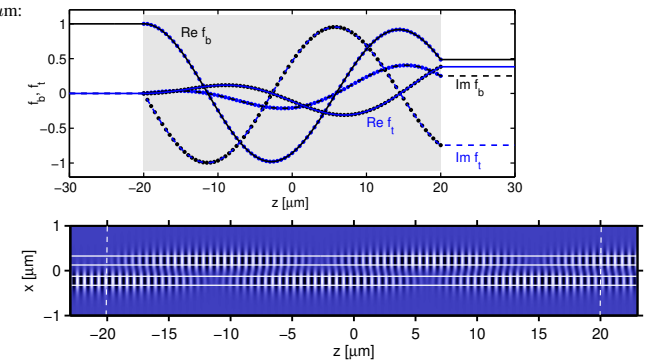


Basis elements: forward propagating
fundamental TE modes of the separate cores,
input amplitude $f_\mathrm{b} = 1$,

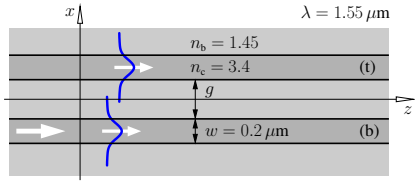
FEM discretization:
$z \in [-20, 20]\ \mu$m, $\Delta z = 0.5\ \mu$m,

computational domain:
$z \in [-20, 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.

$g = 0.25\ \mu$m:



Two coupled parallel cores, modal power

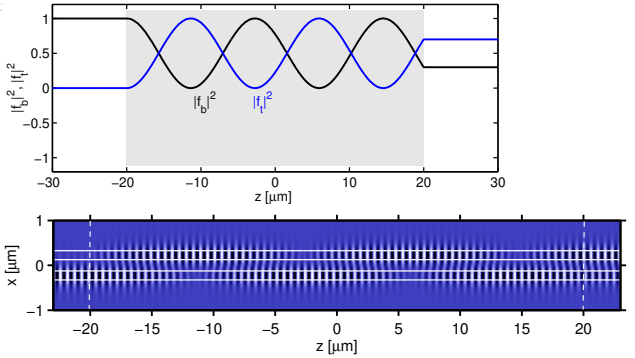


Basis elements: forward propagating
fundamental TE modes of the separate cores,
input amplitude $f_\mathrm{b} = 1$,

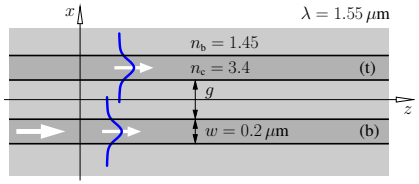
FEM discretization:
$z \in [-20, 20]\ \mu$m, $\Delta z = 0.5\ \mu$m,

computational domain:
$z \in [-20, 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.

$g = 0.25\ \mu$m



Two coupled parallel cores, coupling length

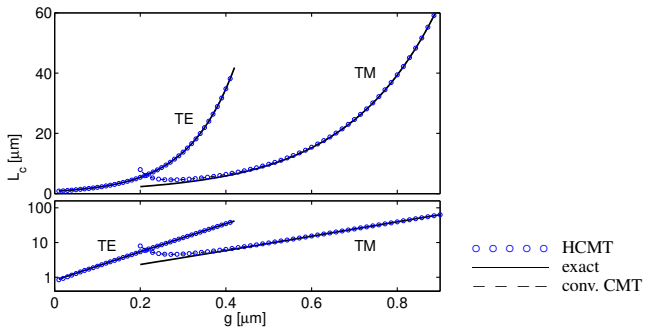


Basis elements: polarized forward propagating
fundamental modes of the separate cores,
input amplitude $f_\mathrm{b} = 1$,

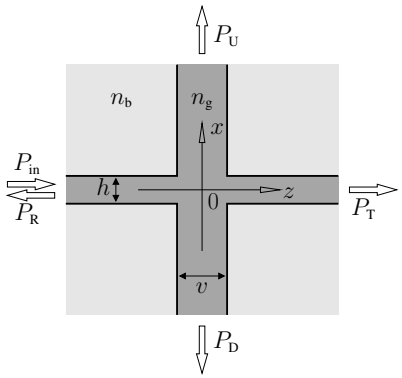
FEM discretization (TE):
$z \in [-20, 20]\ \mu$m, $\Delta z = 0.5\ \mu$m,

computational domain (TE):
$z \in [-20, 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.

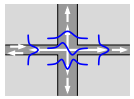
Coupling length:



Waveguide crossing



$n_g = 3.4,\ n_b = 1.45,\ \lambda = 1.55\,\mu$m,
$h = 0.2\,\mu$m, v variable, TE polarization.



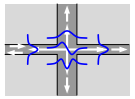
Basis elements:
guided modes of the horizontal and vertical cores (directional variants).

FEM discretization:
$z \in [v/2 - 1.5\,\mu\text{m}, v/2 + 1.5\,\mu\text{m}]$, $\Delta x = 0.025\,\mu$m,
$x \in [w/2 - 1.5\,\mu\text{m}, w/2 + 1.5\,\mu\text{m}]$, $\Delta z = 0.025\,\mu$m.

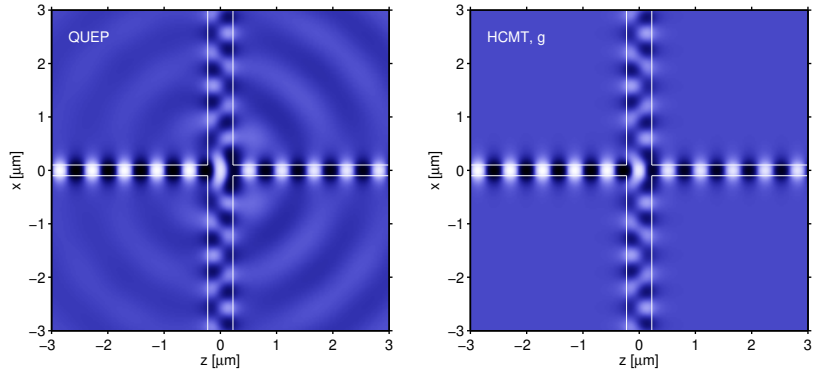
Computational window:
$z \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$, $x \in [-4\,\mu\text{m}, 4\,\mu\text{m}]$.

Waveguide crossing, fields (I)

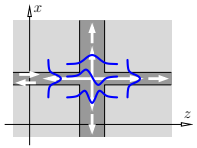
$v = 0.45\,\mu$m:



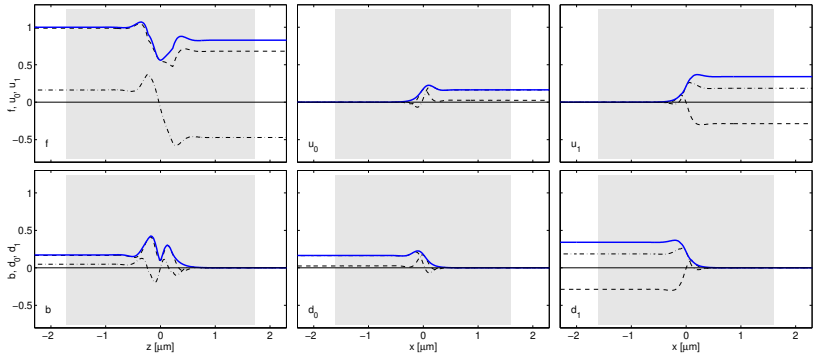
reference



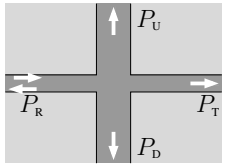
Waveguide crossing, amplitude functions



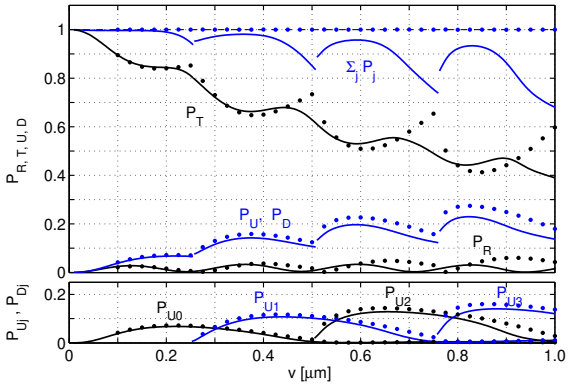
$v = 0.45\,\mu$m:



Waveguide crossing, power transfer (I)



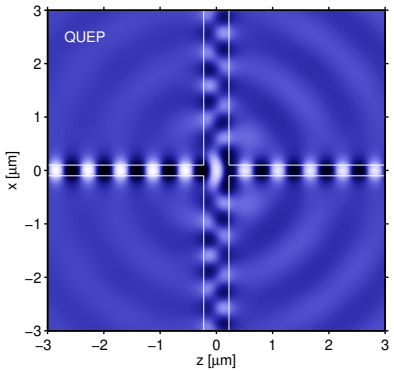
QUEP, reference
HCMT



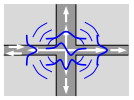
Waveguide crossing, fields (II)

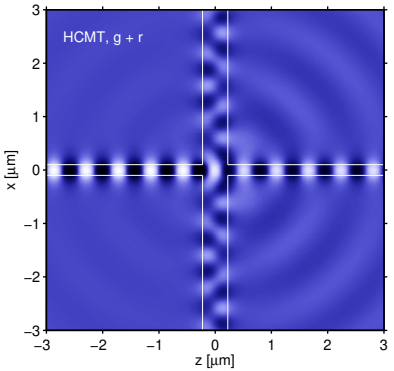
$v = 0.45\,\mu$m:

reference

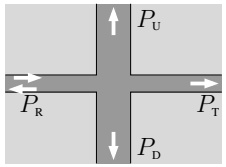


HCMT basis fields:
guided modes
+ 4 Gaussian beams,
outgoing along the diagonals.



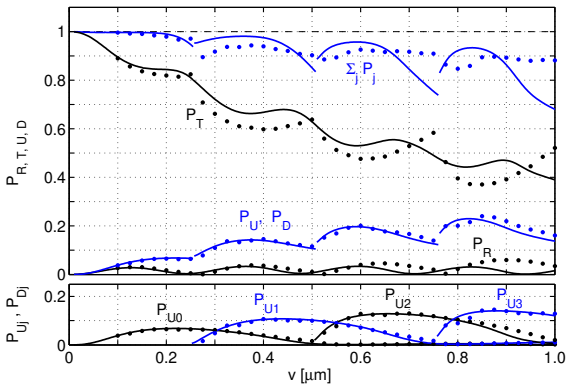


Waveguide crossing, power transfer (II)

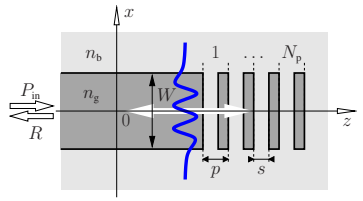


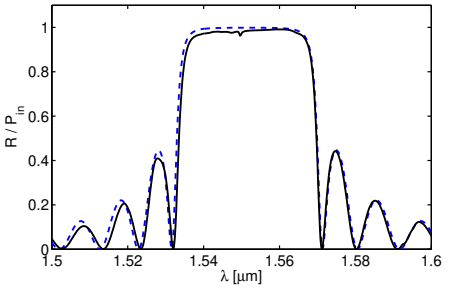
QUEP, reference

HCMT,
incl. templates
for radiated fields



Waveguide Bragg reflector

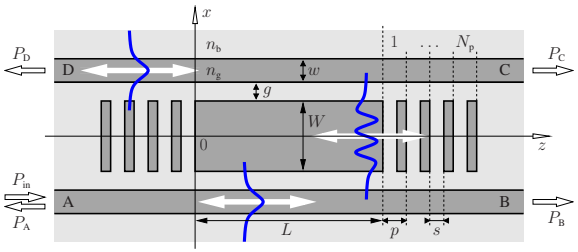




TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

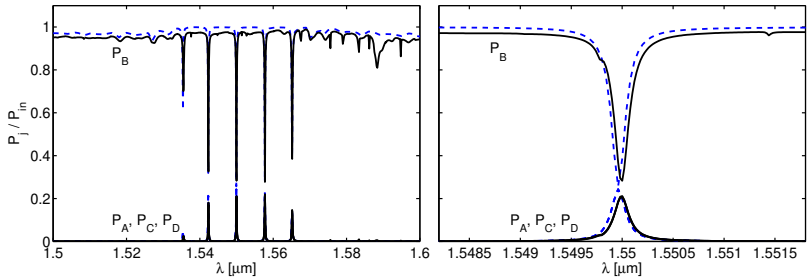
BEP, reference,
HCMT.

Grating-assisted rectangular resonator

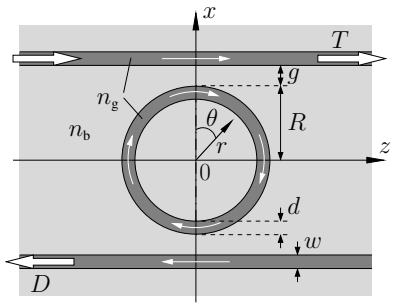


$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

BEP, reference,
HCMT.

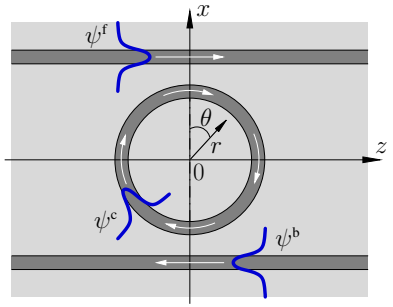


Ringresonator



TE, $R = 7.5\,\mu$m, $w = 0.6\,\mu$m, $d = 0.75\,\mu$m, $g = 0.3\,\mu$m, $n_g = 1.5$, $n_b = 1.0$, $\lambda \approx 1.55\,\mu$m.

Ringresonator, field template



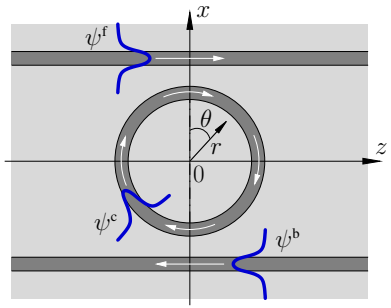
Basis elements:

- bus WGs:
  $$\boldsymbol{\psi}^{f,b}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{f,b}(x)\,e^{\mp i\beta z},$$
- cavity:
  $$\boldsymbol{\psi}^{c}(r,\theta) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}^{c}(r)\,e^{\mp i\gamma R\theta},$$
  $$\gamma R \;\to\; \text{floor}(\text{Re}\gamma R + 1/2),$$
- further terms:
  bidirectional propagation, higher order modes, other channels, etc..

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{f}(x,z) + b(z)\,\boldsymbol{\psi}^{b}(x,z) + c(\theta)\,\boldsymbol{\psi}^{c}(r,\theta),$$

$r = r(x,z),\ \theta = \theta(x,z).$ $\qquad f, b, c$: ?

Ringresonator, HCMT procedure



1-D FEM discretization:

$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$,

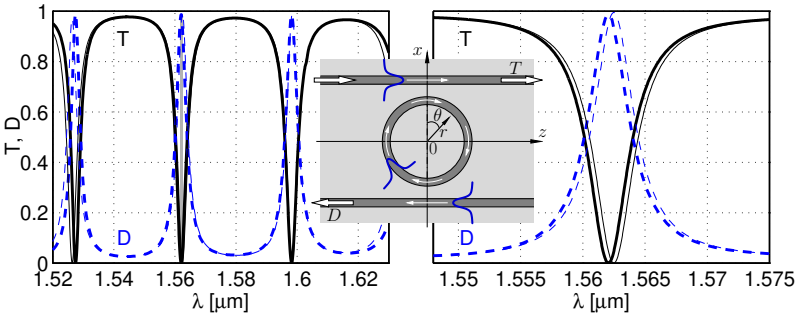
$c(\theta) \to \{c_j\}$, identify nodes 0 and N_θ,
$r \to r(x,z),\ \theta \to \theta(x,z)$.

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_\cdot(\cdot)\boldsymbol{\psi}_\cdot\right)(x,z) =: \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}(x,z),$$

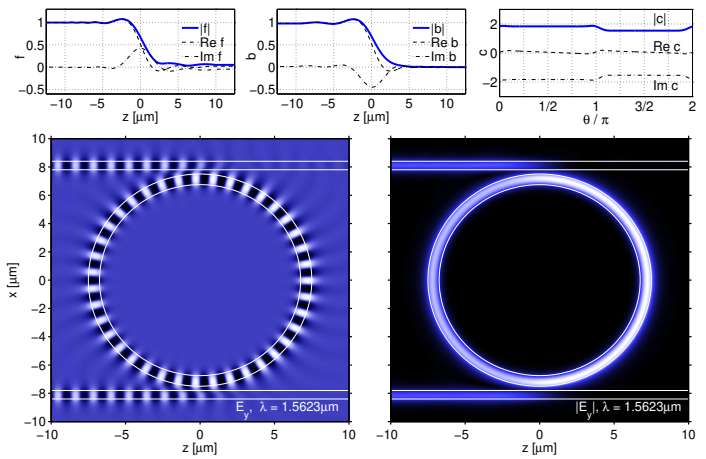
$k \in \{\text{channels, modes, elements}\}$, $\quad a_k \in \{f_j, b_j, c_j\}$.

HCMT solution as before.

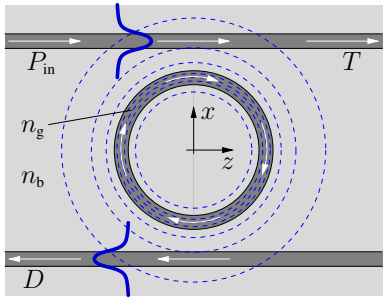
Single ring filter, spectral response



Single ring filter, resonance

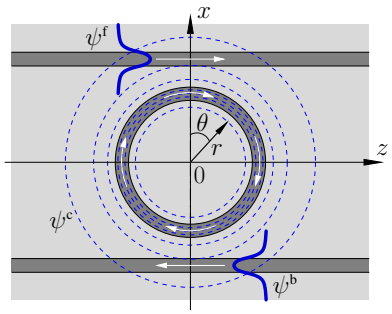


Excitation of whispering gallery resonances



$n_{\mathrm{g}} > n_{\mathrm{b}}$, $\quad \left\{ \omega_j^{\mathrm{c}},\ \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\mathrm{c}}(x,z) \right\}$, $\quad P_{\mathrm{in}}(\omega)$ given: $T(\omega)$, $D(\omega)$ = ?

Ringresonator, field template

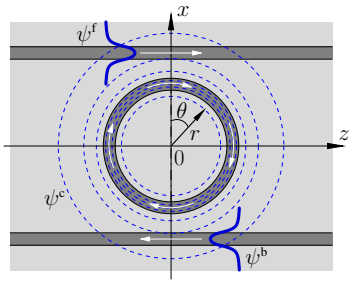


- Frequency ω given, $\sim \exp(\mathrm{i}\omega t)$.
- Bus channels:
  $\boldsymbol{\psi}^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i}\beta z}.$
- Cavity, WGMs:
  $\boldsymbol{\psi}_j^{\mathrm{c}}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_j^{\mathrm{c}}(r)\, \mathrm{e}^{-\mathrm{i}m_j\theta},$
  $m_j \in \mathbb{Z}.$
- Further terms:
  bidirectional propagation, higher order modes, other channels, etc. .

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\mathrm{c}}(r,\theta),$$

$r = r(x,z),\ \theta = \theta(x,z).$ $\qquad f, b, c_j$: ?

Ringresonator, HCMT procedure



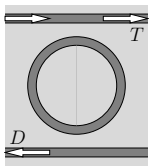
Channels: 1-D FEM discretization,
$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$.

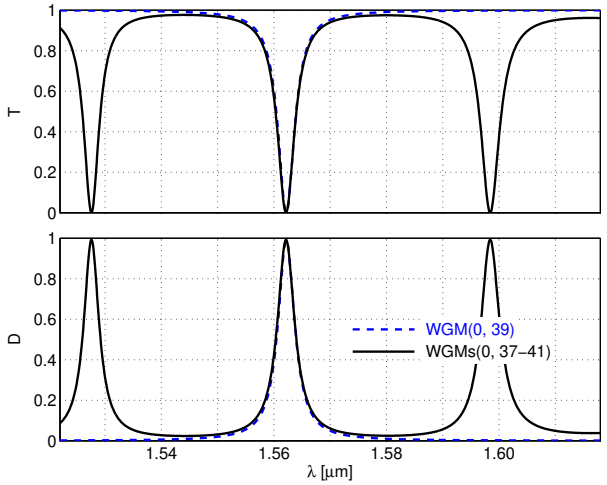
$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_j f_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{f}}\right)(x,z) + \sum_j b_j \left(\alpha_j \boldsymbol{\psi}_j^{\mathrm{b}}\right)(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j'^{\mathrm{c}}(x,z)$$

$$=: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z), \qquad a_k \in \{f_j, b_j, c_j\}.$$

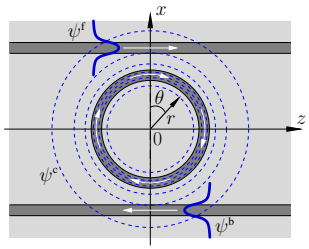
$\hookrightarrow$ HCMT solution as before.

Single ring filter, spectral response

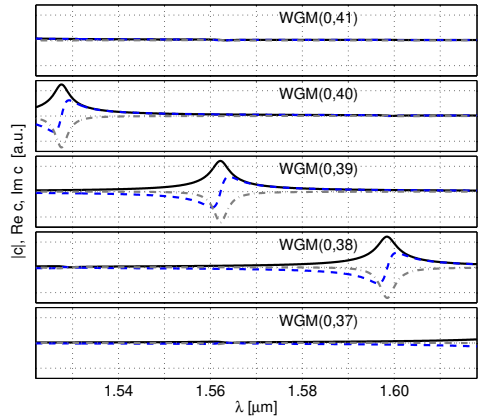




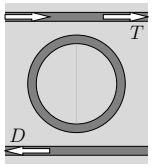
Single ring filter, WGM amplitudes

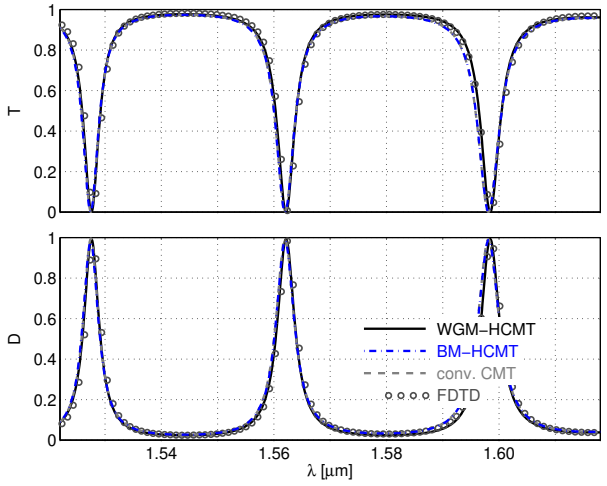


$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + \sum_j c_j\,\boldsymbol{\psi}_j^{\mathrm{c}}(x,z)$$

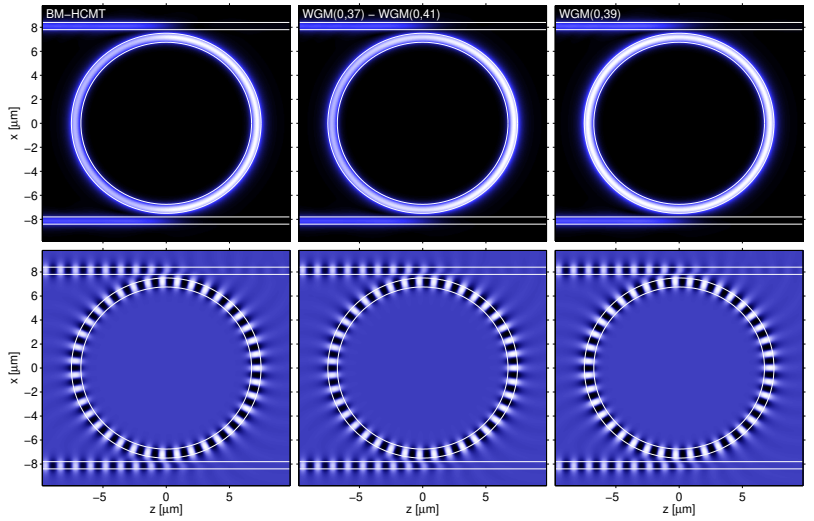


Single ring filter, benchmark

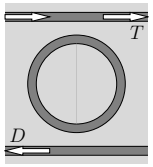


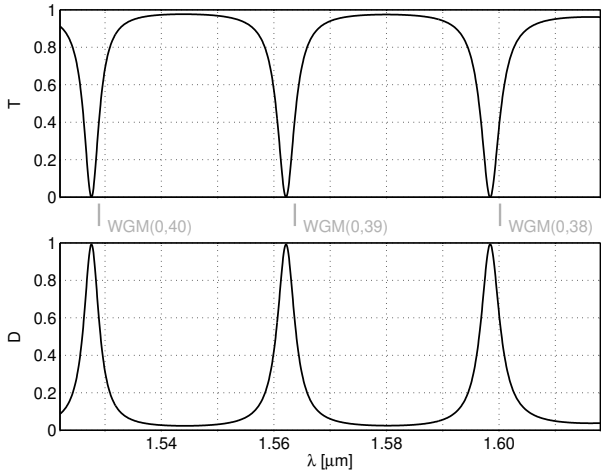


Single ring filter, transmission resonance



Single ring filter, resonance positions I





Supermodes

Look for $\omega^{\mathrm{s}} \in \mathbb{C}$ where the system

$$\left\{ \begin{array}{r} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \right. \quad \& \text{ boundary conditions: “outgoing waves”} \Bigg\}$$

permits nontrivial solutions $\boldsymbol{E}, \boldsymbol{H}$.

$$\left. \begin{array}{r} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega^{\mathrm{s}}\epsilon_0\epsilon\boldsymbol{E} = 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega^{\mathrm{s}}\mu_0\boldsymbol{H} = 0 \end{array} \right| \qquad \cdot \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{pmatrix}^*, \qquad \iint_{\text{comp. domain}}$$

$\Longleftrightarrow$

$$\iint \mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F}, \boldsymbol{G},$$

where $\mathcal{A}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{F}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{G}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E})$,

$\mathcal{B}(\boldsymbol{F}, \boldsymbol{G}; \boldsymbol{E}, \boldsymbol{H}) = \mathrm{i}\epsilon_0\epsilon\boldsymbol{F}^* \cdot \boldsymbol{E} + \mathrm{i}\mu_0\boldsymbol{G}^* \cdot \boldsymbol{H}$.

HCMT supermode analysis

- Insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- require

$$\iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z - \omega^{\mathrm{s}} \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } l\,,$$

- compute $A_{lk} = \iint \mathcal{A}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,,$

$$B_{lk} = \iint \mathcal{B}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,.$$

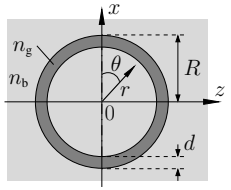
$\sum_k A_{lk}a_k - \omega^{\mathrm{s}}B_{lk}a_k = 0$ for all l, or $\mathsf{A}\boldsymbol{a} = \omega^{\mathrm{s}}\mathsf{B}\boldsymbol{a}$.

$\hookrightarrow \left\{ \omega,\ \lambda_{\mathrm{r}},\ Q,\ \Delta\lambda;\ \boldsymbol{E},\ \boldsymbol{H} \right\}^{\mathrm{s}}$.

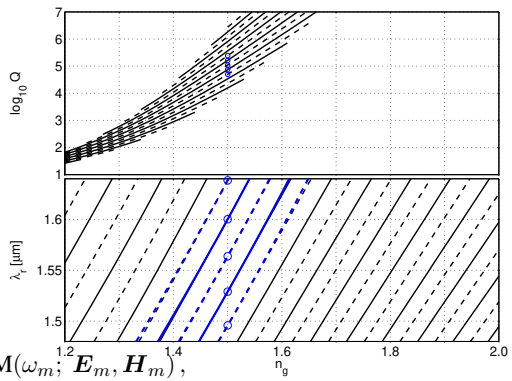
Further issues

… plenty.

WGMs, small uniform perturbations



TE, $R = 7.5\,\mu\text{m}$, $d = 0.75\,\mu\text{m}$
$n_b = 1.0$.

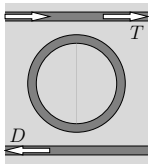


$\epsilon_m \quad \leftrightsquigarrow \quad \text{WGM}(\omega_m;\ \boldsymbol{E}_m, \boldsymbol{H}_m)\,,$

$\epsilon = \epsilon_m + \Delta\epsilon \quad \leftrightsquigarrow \quad \text{WGM}(\omega_m + \Delta\omega;\ \approx \boldsymbol{E}_m, \approx \boldsymbol{H}_m)\,,$

$$\Delta\omega = -\frac{\omega_m \epsilon_0 \iint \Delta\epsilon |\boldsymbol{E}_m|^2 \,\mathrm{d}x\,\mathrm{d}z}{\iint \left(\epsilon_m \epsilon_0 |\boldsymbol{E}_m|^2 + \mu_0 |\boldsymbol{H}_m|^2\right) \mathrm{d}x\,\mathrm{d}z}\,.$$

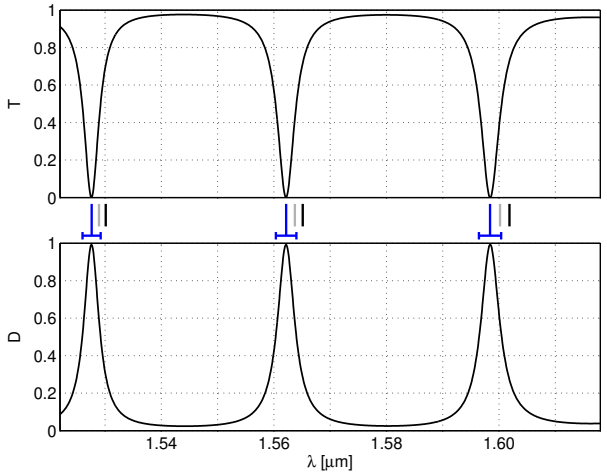
Single ring filter, resonance positions II



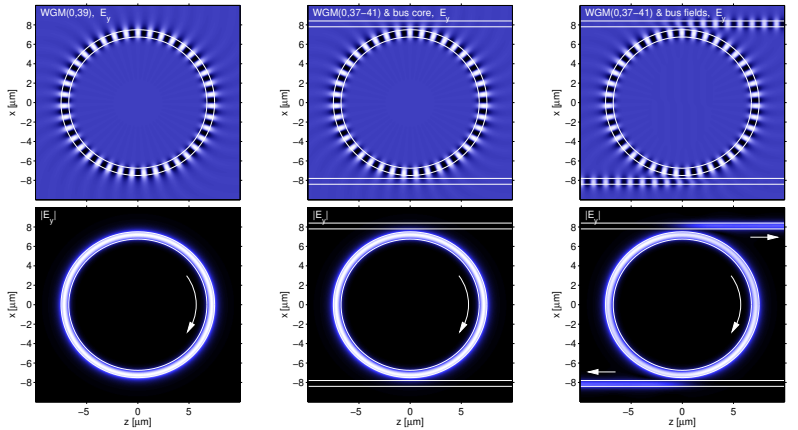
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Single ring filter, unidirectional supermodes

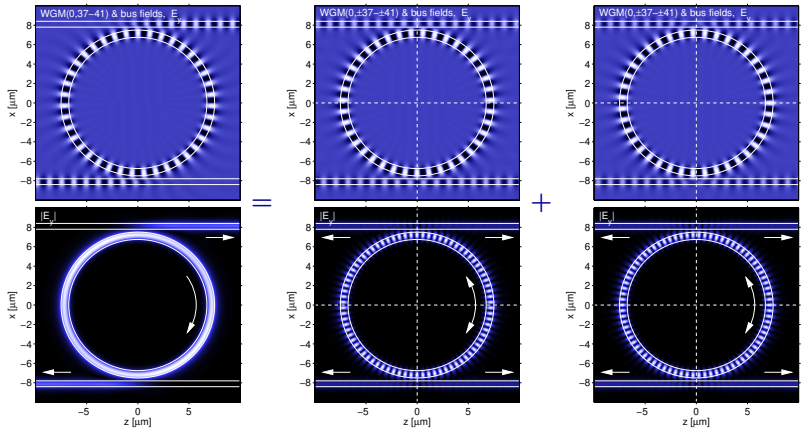


$\lambda_r = 1.5637\,\mu\text{m}$,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu\text{m}$.

$\lambda_r = 1.5651\,\mu\text{m}$,
$Q = 1.1 \cdot 10^5$,
$\Delta\lambda = 1.4 \cdot 10^{-5}\,\mu\text{m}$.

$\lambda_r = 1.5622\,\mu\text{m}$,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu\text{m}$.

Single ring filter, bidirectional supermodes

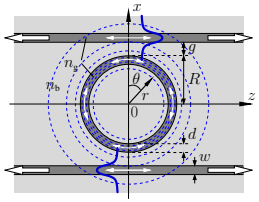


$\lambda_r = 1.56219\,\mu\text{m}$,
$Q = 4.3 \cdot 10^2$,
$\Delta\lambda = 3.7 \cdot 10^{-3}\,\mu\text{m}$.

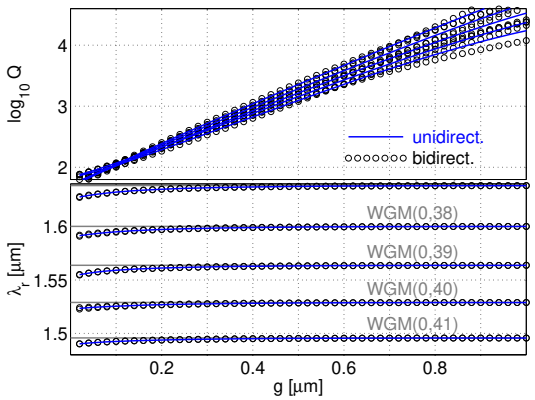
$\lambda_r = 1.56223\,\mu\text{m}$,
$Q = 4.4 \cdot 10^2$,
$\Delta\lambda = 3.5 \cdot 10^{-3}\,\mu\text{m}$.

$\lambda_r = 1.56215\,\mu\text{m}$,
$Q = 4.0 \cdot 10^2$,
$\Delta\lambda = 3.9 \cdot 10^{-3}\,\mu\text{m}$.

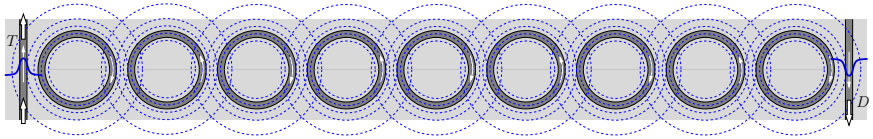
Single ring filter, supermodes vs. gap



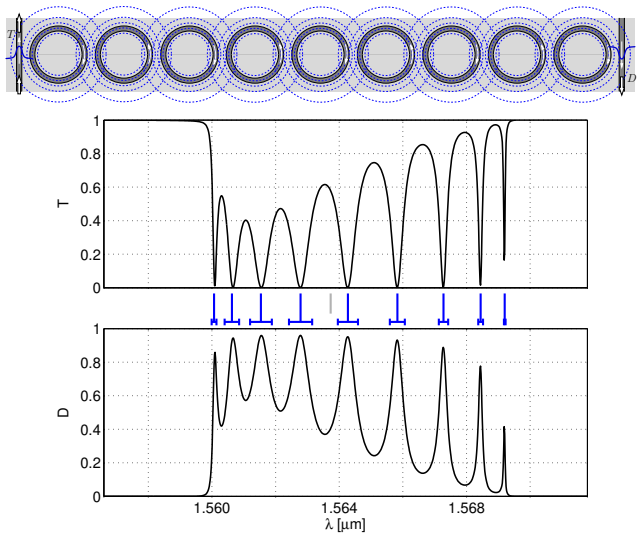
TE,
$R = 7.5\,\mu$m, $d = 0.75\,\mu$m,
$w = 0.6\,\mu$m,
$n_g = 1.5$, $n_b = 1.0$.



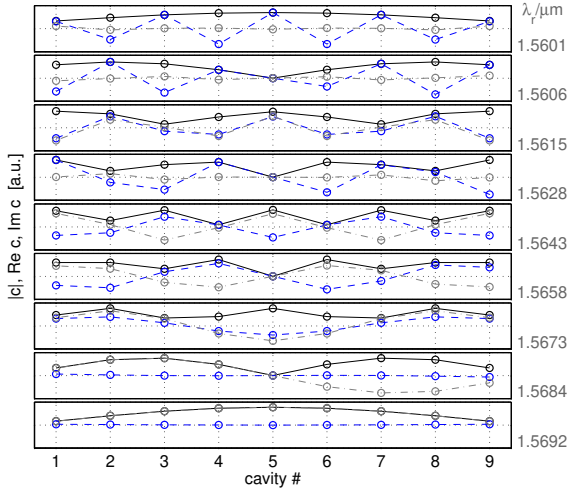
CROW



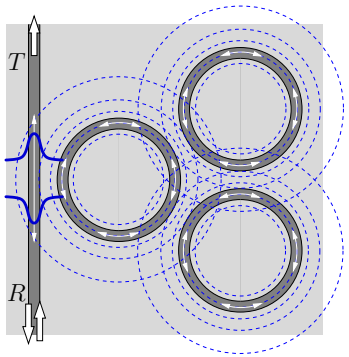
CROW, spectral response I



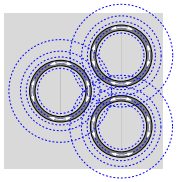
CROW, supermode pattern



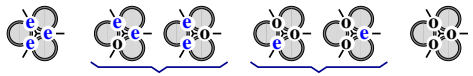
Three-ring molecule

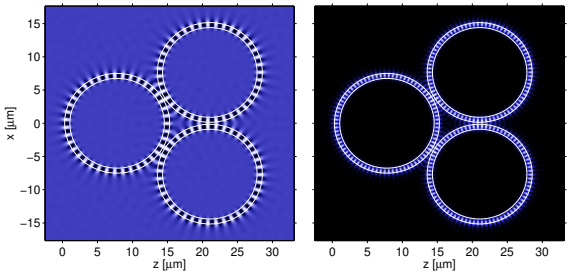


Three-ring molecule, supermodes



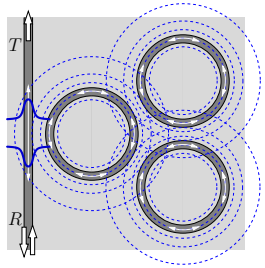
Template: $3 \times \mathrm{WGM}(0, \pm 39)$ ⇝ 6 supermodes.

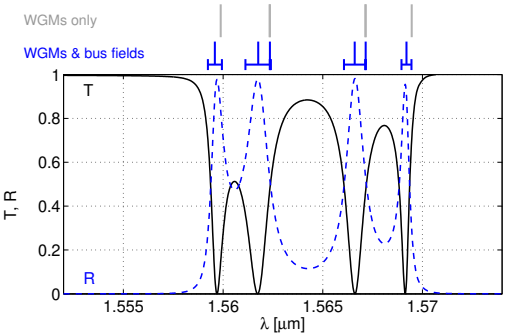
Symmetries: 



$\lambda_\mathrm{r} = 1.56946\,\mu\mathrm{m}$,
$Q = 1.3 \cdot 10^5$,
$\Delta\lambda = 1.1 \cdot 10^{-5}\,\mu\mathrm{m}$.

Three-ring molecule, excitation

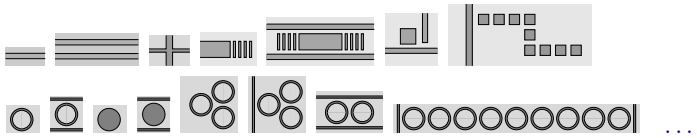




Concluding remarks

Hybrid analytical / numerical Coupled Mode Theory, HCMT:

- an ab-initio, quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- configurations with localized resonances: demonstrated,
- extension to 3-D (todo): numerical basis fields, still moderate effort,
- very close to common ways of reasoning in integrated optics,
- reasonably versatile:



. . .

— *supplementary material* —

Fast evaluation of spectral properties

Time consuming: evaluation of modal "overlaps" K_{lk} in K:

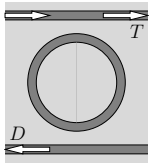
$$K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}z\,.$$

All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

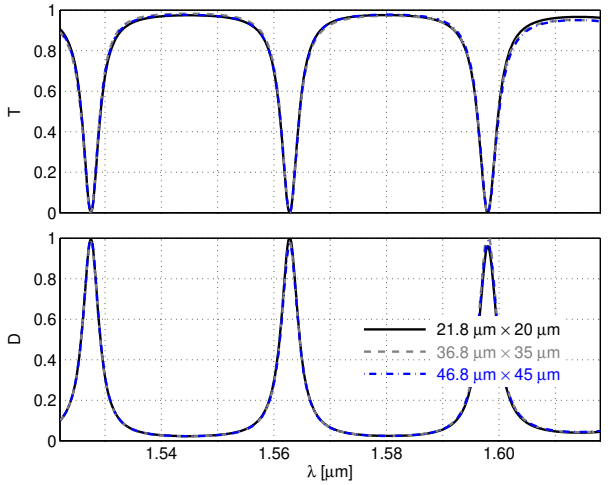
⇝ Interpolate $\mathsf{K}(\lambda)$:

- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\quad \lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b,\ \lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b,$
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}\,(\mathsf{K}_1 - \mathsf{K}_0)$,
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

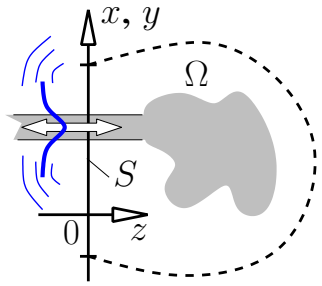
Computational window



$R = 7.5\,\mu\mathrm{m}$



Abstract scattering problem



Ω: domain of interest,

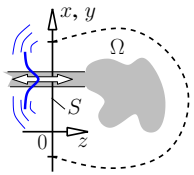
$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right\} \text{ in } \Omega$$

for given frequency ω, permittivity $\epsilon = n^2$,

S: an exemplary port plane,
waveguides enter Ω through S.

Variational form including suitable boundary conditions ?

Boundary conditions



Ingredients:

- Complete set of normal modes on S, $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y)$ ◂⇝ propagation along $\pm z$.
- Product on S: $\langle \boldsymbol{A}, \boldsymbol{B}\rangle = \iint_S (\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{e}_z \,\mathrm{d}x\,\mathrm{d}y$.
- Modal orthogonality properties $\langle \tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k\rangle = \delta_{lk} N_k$, $N_k = \langle \tilde{\boldsymbol{E}}_k, \tilde{\boldsymbol{H}}_k\rangle$.

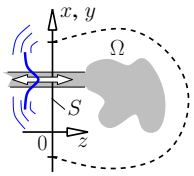
"Any" electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ on S can be expanded as

$$\boldsymbol{E} = \sum_m e_m \tilde{\boldsymbol{E}}_m,\quad e_m = \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle,\quad \boldsymbol{H} = \sum_m h_m \tilde{\boldsymbol{H}}_m,\quad h_m = \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle,$$

or $$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_m f_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix} + \sum_m b_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ -\tilde{\boldsymbol{H}}_m\end{pmatrix}, \qquad \begin{aligned} f_m &= (e_m + h_m)/2,\\ b_m &= (e_m - h_m)/2\end{aligned}$$

(transverse components only).

Transparent influx boundary conditions (TIBCs)



... on S for inhomogeneous exterior, incoming waveguides:

$$\begin{aligned}\boldsymbol{E} &= \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H}\rangle \tilde{\boldsymbol{E}}_m\,,\\ \boldsymbol{H} &= \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m\rangle \tilde{\boldsymbol{H}}_m\,;\end{aligned}$$

F_m: influx, given coefficients of incoming waves; $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}_{\text{inc}} = \sum_m F_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix}$.

↳ For a general field of the form $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_m f_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ \tilde{\boldsymbol{H}}_m\end{pmatrix} + \sum_m b_m \begin{pmatrix}\tilde{\boldsymbol{E}}_m\\ -\tilde{\boldsymbol{H}}_m\end{pmatrix}$

the TIBCs require $f_m = F_m$, while b_m can be arbitrary.

Frequency domain Maxwell equations, variational form

Consider the functional

$$\mathcal{L}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

(cf. e.g. C. Vassallo. *Optical Waveguide Concepts*. Elsevier, Amsterdam, 1991).

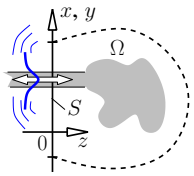
First variation:

$$\begin{aligned}\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; &\iiint_\Omega \big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E})\\ &\quad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\\ &- \iint_{\partial\Omega}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\,\mathrm{d}A\,.\end{aligned}$$

Stationarity $\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega$.

Variational form of the scattering problem

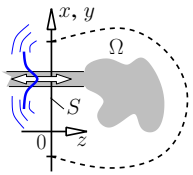


... based on the functional:

$$\begin{aligned}\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = &\iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\\ &- \sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\right\}\\ &+ \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\right\}\end{aligned}$$

Variational form of the scattering problem, first variation

$$\begin{aligned}\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; & \iiint_\Omega \big\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}-\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}) \\ & \quad +2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H})\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ & + \Big\langle \boldsymbol{E}-\sum_m 2F_m\tilde{\boldsymbol{E}}_m+\sum_m\frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\;\delta\boldsymbol{H}\Big\rangle \\ & - \Big\langle \delta\boldsymbol{E},\;\boldsymbol{H}-\sum_m 2F_m\tilde{\boldsymbol{H}}_m+\sum_m\frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\Big\rangle \\ & - \iint_{\partial\Omega\setminus S}\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H}+(\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\}\,\mathrm{d}A\,.\end{aligned}$$



Stationarity $\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H})=0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω,
- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy TIBCs on S,
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega\setminus S$.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\;(\boldsymbol{E},\boldsymbol{H})=\sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)\;} \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \boldsymbol{a}\cdot\mathsf{M}\boldsymbol{a}+\boldsymbol{R}\cdot\boldsymbol{a}$.

↪ Require $\delta\mathcal{F}_\mathrm{r}=\delta\boldsymbol{a}\cdot\left(\left(\mathsf{M}+\mathsf{M}^\mathsf{T}\right)\boldsymbol{a}+\boldsymbol{R}\right)=0$ for all $\delta\boldsymbol{a}$,

↪ $\left(\mathsf{M}+\mathsf{M}^\mathsf{T}\right)\boldsymbol{a}+\boldsymbol{R}=0$,

↪ $\boldsymbol{a}$,

↪ $f_m,\ b_m,\ u_m,\ d_m,\ \ \boldsymbol{E},\ \boldsymbol{H}$.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\;(\boldsymbol{E},\boldsymbol{H})=\sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)\;} \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \sum_{l,k} a_l F_{lk} a_k + \sum_l R_l a_l + \sum_{l,k} a_l B_{lk} a_k\,,$$

$$\begin{aligned}F_{lk} = & \iiint_\Omega \big\{\boldsymbol{E}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}_k)+\boldsymbol{H}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}_k) \\ & \quad -\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}_l\cdot\boldsymbol{E}_k+\mathrm{i}\omega\mu_0\boldsymbol{H}_l\cdot\boldsymbol{H}_k\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,,\end{aligned}$$

$$R_l = -\sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle-\langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\right\},$$

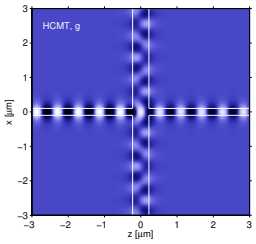
$$B_{lk} = \sum_m\frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_k\rangle-\langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\langle\boldsymbol{E}_k,\tilde{\boldsymbol{H}}_m\rangle\right\},$$

+ contributions R, B from other port planes.

Comments

HCMT scheme based on the variational form of the guided wave scattering problem:

- Expansions at the TIBC ports reduce to single terms due to modal orthogonality.
- Bidirectional basis fields are required for all channels in the field templates.



Alternative functional:

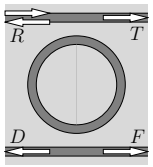
$$\begin{aligned}\mathcal{C}(\boldsymbol{E},\boldsymbol{H}) = & \iiint_\Omega \big\{\boldsymbol{E}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H})-\boldsymbol{H}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) \\ & \quad -\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^*\cdot\boldsymbol{E}+\mathrm{i}\omega\mu_0\boldsymbol{H}^*\cdot\boldsymbol{H}\big\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.\end{aligned}$$

?

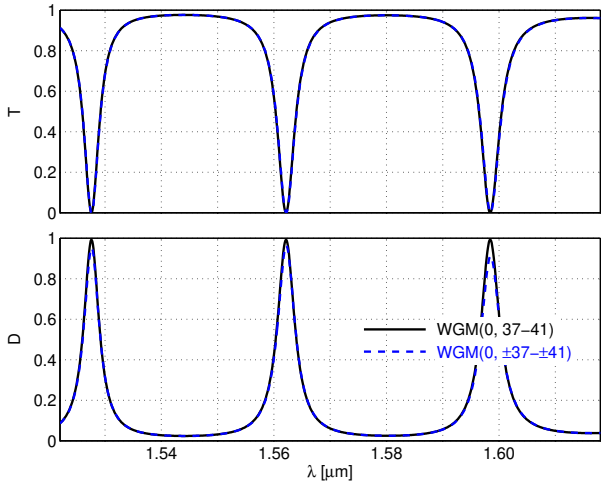
Extend $\mathcal{C}$ by boundary integrals such that

- the boundary terms in $\delta\mathcal{C}$ cancel ↭ the Galerkin scheme could be viewed as a variational restriction of $\mathcal{C}$.
- TIBCs are satisfied as natural boundary conditions if $\mathcal{C}$ becomes stationary ↭ variational scheme with complex conjugate fields.

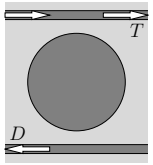
Single ring filter, transmission, bidirectional template



$R < 10^{-4}$,
$F < 10^{-4}$.



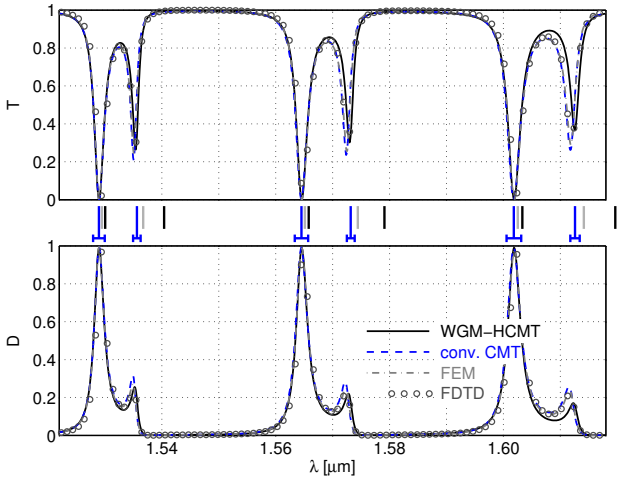
Micro-disk resonator, spectral response



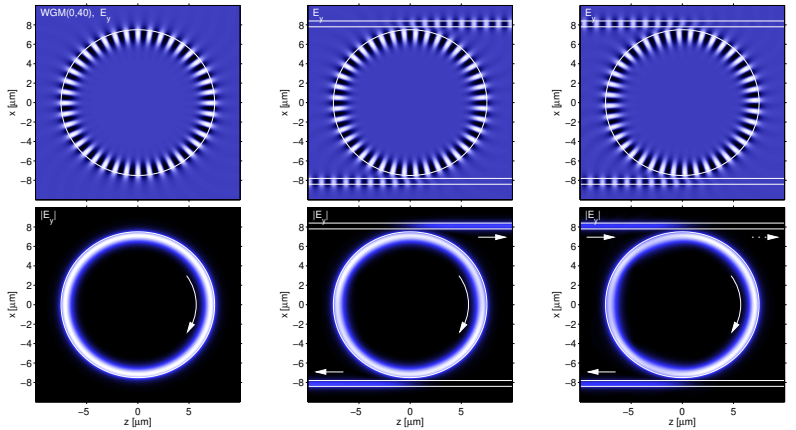
WGMs only

WGMs
& bus cores

WGMs
& bus fields



Micro-disk, resonant fields (0)

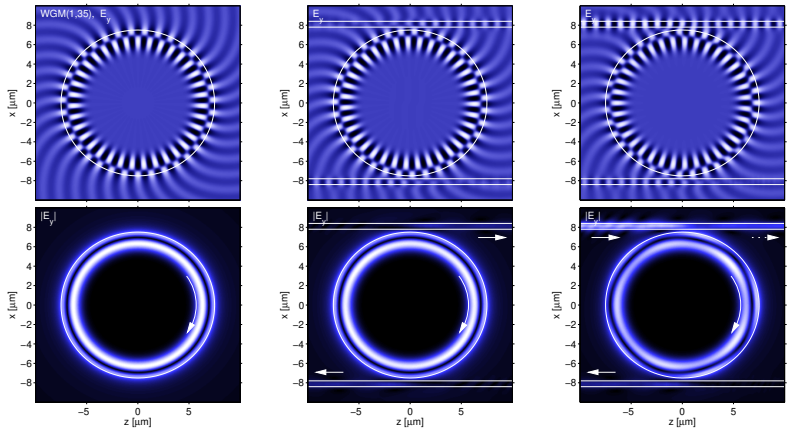


$\lambda_r = 1.56514\,\mu$m,
$Q = 8.2 \cdot 10^5$,
$\Delta\lambda = 1.9 \cdot 10^{-6}\,\mu$m.

$\lambda_r = 1.56454\,\mu$m,
$Q = 6.7 \cdot 10^2$,
$\Delta\lambda = 2.3 \cdot 10^{-3}\,\mu$m.

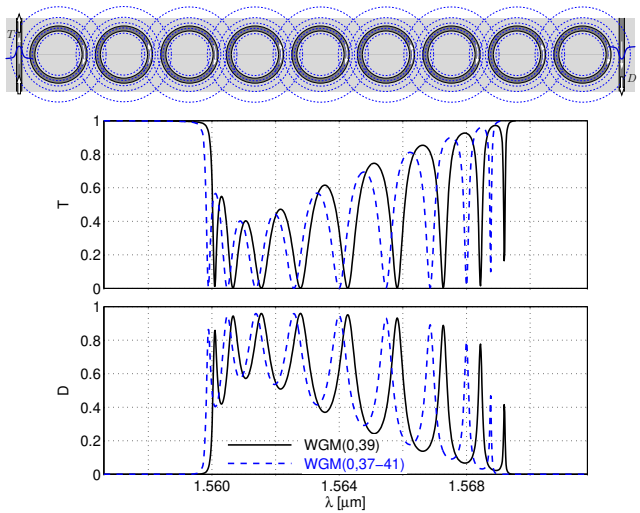
$\lambda_r = 1.56456\,\mu$m.

Micro-disk, resonant fields (1)



$\lambda_r = 1.57444\,\mu$m,
$Q = 1.6 \cdot 10^3$,
$\Delta\lambda = 9.2 \cdot 10^{-4}\,\mu$m.

$\lambda_r = 1.57320\,\mu$m,
$Q = 1.1 \cdot 10^3$,
$\Delta\lambda = 1.4 \cdot 10^{-3}\,\mu$m.

$\lambda_r = 1.57306\,\mu$m.

CROW, spectral response II