Coupled Mode Modeling in Integrated Optics: a Variational, Hybrid Analytical-Numerical Approach

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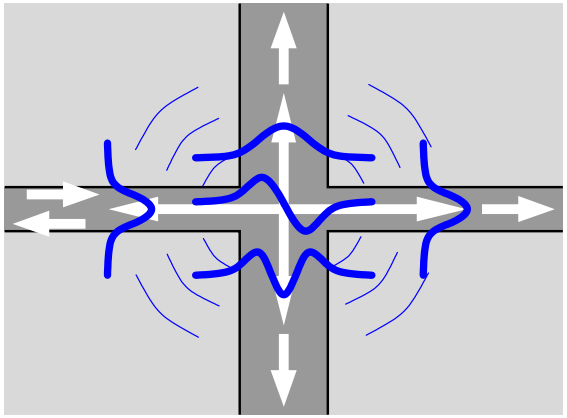
A waveguide crossing



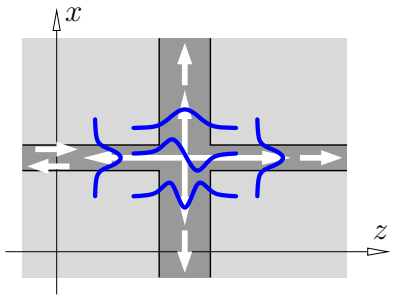
Coupled Mode Model ?

Outline

Hybrid analytical / numerical coupled-mode modeling

- CMT field ansatz
- Amplitude discretization, 1-D FEM
- Solution
  - Variational formulation
  - Galerkin procedure
- Examples
  - Straight waveguide
  - Two coupled cores
  - Waveguide crossing
  - Bragg grating & resonator
  - Chains of square resonators
  - Ringresonator circuits

Field ansatz



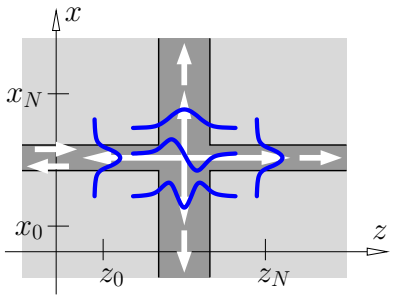
Basis elements (crossing):

- guided modes of the horizontal WG
  $$\boldsymbol{\psi}_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_m^{\mathrm{f,b}}(x)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{f,b}}z},$$
- guided modes of the vertical WG
  $$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix}\tilde{\boldsymbol{E}}\\ \tilde{\boldsymbol{H}}\end{pmatrix}_m^{\mathrm{u,d}}(z)\,\mathrm{e}^{\mp\mathrm{i}\beta_m^{\mathrm{u,d}}x}$$
- (and further terms).

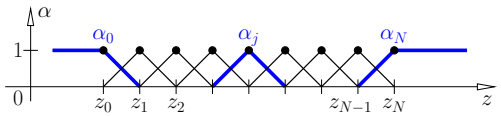
$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_m f_m(z)\boldsymbol{\psi}_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\boldsymbol{\psi}_m^{\mathrm{b}}(x,z) + \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x,z) \qquad f_m,\, b_m,\, u_m,\, d_m\text{: ?}$$

General: a reasonable superposition of known fields with amplitudes that are functions of suitable propagation coordinate(s).

Amplitude functions, discretization



1-D linear finite elements



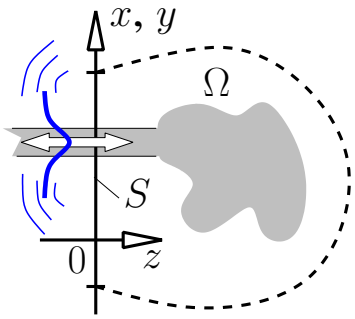
$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\,\alpha_j(z)\,,$$

$b_m(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\hookrightarrow \quad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \Big(\alpha_\cdot(\cdot)\boldsymbol{\psi}_\cdot(x,z) \Big) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

$k \in \{\text{waveguides, modes, elements}\}$, $\quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}$, $\qquad a_k$: ?

Abstract scattering problem



Ω: domain of interest,

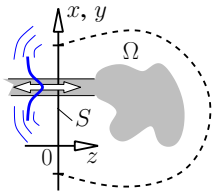
$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon \boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0 \boldsymbol{H} &= 0 \end{aligned}\right\} \text{ in } \Omega$$

for given frequency ω, permittivity $\epsilon = n^2$,

S: an exemplary port plane,
waveguides enter Ω through S.

Variational form including suitable boundary conditions ?

Boundary conditions



Ingredients:

- Complete set of normal modes on S,
  $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y)$ ↭ propagation along $\pm z$.
- Product on S: $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \iint_S (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{e}_z \,\mathrm{d}x\,\mathrm{d}y$.
- Modal orthogonality properties $\langle \tilde{\boldsymbol{E}}_l, \tilde{\boldsymbol{H}}_k \rangle = \delta_{lk} N_k$, $N_k = \langle \tilde{\boldsymbol{E}}_k, \tilde{\boldsymbol{H}}_k \rangle$.

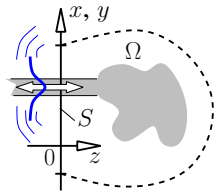
"Any" electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ on S can be expanded as

$$\boldsymbol{E} = \sum_m e_m \tilde{\boldsymbol{E}}_m, \quad e_m = \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle, \quad \boldsymbol{H} = \sum_m h_m \tilde{\boldsymbol{H}}_m, \quad h_m = \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle,$$

or $$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_m f_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix} + \sum_m b_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ -\tilde{\boldsymbol{H}}_m \end{pmatrix}, \qquad \begin{aligned} f_m &= (e_m + h_m)/2, \\ b_m &= (e_m - h_m)/2 \end{aligned}$$

(transverse components only).

Transparent influx boundary conditions (TIBCs)



... on S for inhomogeneous exterior, incoming waveguides:

$$\begin{aligned} \boldsymbol{E} &= \sum_m 2F_m \tilde{\boldsymbol{E}}_m - \sum_m \frac{1}{N_m}\langle \tilde{\boldsymbol{E}}_m, \boldsymbol{H} \rangle \tilde{\boldsymbol{E}}_m\,, \\ \boldsymbol{H} &= \sum_m 2F_m \tilde{\boldsymbol{H}}_m - \sum_m \frac{1}{N_m}\langle \boldsymbol{E}, \tilde{\boldsymbol{H}}_m \rangle \tilde{\boldsymbol{H}}_m\,; \end{aligned}$$

F_m: influx, given coefficients of incoming waves; $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}_{\text{inc}} = \sum_m F_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix}$.

↪ For a general field of the form $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_m f_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ \tilde{\boldsymbol{H}}_m \end{pmatrix} + \sum_m b_m \begin{pmatrix} \tilde{\boldsymbol{E}}_m \\ -\tilde{\boldsymbol{H}}_m \end{pmatrix}$

the TIBCs require $f_m = F_m$, while b_m can be arbitrary.

Frequency domain Maxwell equations, variational form

Consider the functional

$$\mathcal{L}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

(cf. e.g. C. Vassallo. *Optical Waveguide Concepts*. Elsevier, Amsterdam, 1991).

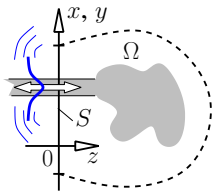
First variation:

$$\begin{aligned}\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; & \iiint_\Omega \left\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E})\right. \\ & \left. \quad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ - \; & \iint_{\partial\Omega}\left\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\right\}\mathrm{d}A\,.\end{aligned}$$

Stationarity $\delta\mathcal{L}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega$.

Variational form of the scattering problem

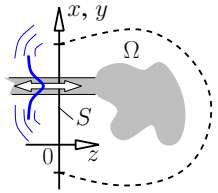


. . . based on the functional:

$$\begin{aligned}\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = & \iiint_\Omega \left\{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ & - \sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\right\} \\ & + \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\right\}\end{aligned}$$

Variational form of the scattering problem, first variation

$$\begin{aligned}\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) \;=\; & \iiint_\Omega \left\{2\delta\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E})\right. \\ & \left. \quad + 2\delta\boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H})\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \\ + \; & \left\langle\boldsymbol{E} - \sum_m 2F_m\tilde{\boldsymbol{E}}_m + \sum_m \frac{1}{N_m}\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle\tilde{\boldsymbol{E}}_m,\,\delta\boldsymbol{H}\right\rangle \\ - \; & \left\langle\delta\boldsymbol{E},\,\boldsymbol{H} - \sum_m 2F_m\tilde{\boldsymbol{H}}_m + \sum_m \frac{1}{N_m}\langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\tilde{\boldsymbol{H}}_m\right\rangle \\ - \; & \iint_{\partial\Omega\setminus S}\left\{(\boldsymbol{n}\times\boldsymbol{E})\cdot\delta\boldsymbol{H} + (\boldsymbol{n}\times\boldsymbol{H})\cdot\delta\boldsymbol{E}\right\}\mathrm{d}A\,.\end{aligned}$$



Stationarity $\delta\mathcal{F}(\boldsymbol{E},\boldsymbol{H};\delta\boldsymbol{E},\delta\boldsymbol{H}) = 0$ for arbitrary $\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$ implies

- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy the Maxwell equations in Ω,
- that $\boldsymbol{E}$, $\boldsymbol{H}$ satisfy TIBCs on S,
- and that transverse components of $\boldsymbol{E}$ and $\boldsymbol{H}$ vanish on $\partial\Omega\setminus S$.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\;(\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k)\;} \mathcal{F}_\mathrm{r}(\boldsymbol{a})$$

Restricted functional:

$$\mathcal{F}_\mathrm{r}(\boldsymbol{a}) = \sum_{l,k} a_l F_{lk} a_k + \sum_l R_l a_l + \sum_{l,k} a_l B_{lk} a_k\,,$$

$$\begin{aligned}F_{lk} = & \iiint_\Omega \left\{\boldsymbol{E}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}_k) + \boldsymbol{H}_l\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}_k)\right. \\ & \left. \quad - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}_l\cdot\boldsymbol{E}_k + \mathrm{i}\omega\mu_0\boldsymbol{H}_l\cdot\boldsymbol{H}_k\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,,\end{aligned}$$

$$R_l = -\sum_m 2F_m\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle - \langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\right\},$$

$$B_{lk} = \sum_m \frac{1}{2N_m}\left\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_l\rangle\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}_k\rangle - \langle\boldsymbol{E}_l,\tilde{\boldsymbol{H}}_m\rangle\langle\boldsymbol{E}_k,\tilde{\boldsymbol{H}}_m\rangle\right\},$$

$+$ contributions R, B from other port planes.

Variational HCMT scheme

$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \xrightarrow{\quad (\boldsymbol{E},\boldsymbol{H}) = \sum_k a_k(\boldsymbol{E}_k,\boldsymbol{H}_k) \quad} \mathcal{F}_{\mathrm{r}}(\boldsymbol{a})$$

Restricted functional:

$\mathcal{F}_{\mathrm{r}}(\boldsymbol{a}) = \boldsymbol{a}\cdot\mathsf{M}\boldsymbol{a} + \boldsymbol{R}\cdot\boldsymbol{a}.$

↳ Require $\delta\mathcal{F}_{\mathrm{r}} = \delta\boldsymbol{a}\cdot\left(\left(\mathsf{M}+\mathsf{M}^{\mathsf{T}}\right)\boldsymbol{a}+\boldsymbol{R}\right) = 0$ for all $\delta\boldsymbol{a}$,

↳ $\left(\mathsf{M}+\mathsf{M}^{\mathsf{T}}\right)\boldsymbol{a}+\boldsymbol{R} = 0$,

↳ $\boldsymbol{a}$,

↳ $f_m,\ b_m,\ u_m,\ d_m,\ \boldsymbol{E},\ \boldsymbol{H}$.

Further issues

$\ldots$ plenty.

Galerkin procedure

$$\begin{array}{l}\boldsymbol{\nabla}\times\boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} = 0\\ -\boldsymbol{\nabla}\times\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} = 0\end{array} \quad\Big|\quad \cdot\begin{pmatrix}\boldsymbol{F}\\ \boldsymbol{G}\end{pmatrix}^{*}, \qquad \iiint_{\text{comp. domain}}$$

$$\rightsquigarrow \quad \iiint \mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{F},\ \boldsymbol{G},$$

where

$$\mathcal{K}(\boldsymbol{F},\boldsymbol{G};\boldsymbol{E},\boldsymbol{H}) = \boldsymbol{F}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{G}^*\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{F}^*\cdot\boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{G}^*\cdot\boldsymbol{H}\,.$$

Galerkin procedure, continued

- insert $\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix} = \sum_k a_k \begin{pmatrix}\boldsymbol{E}_k\\ \boldsymbol{H}_k\end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iiint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E},\boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0$ for $l\in\{\mathrm{u}\}$,
- compute $K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l,\boldsymbol{H}_l;\boldsymbol{E}_k,\boldsymbol{H}_k)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

$$\sum_{k\in\{\mathrm{u},\mathrm{g}\}} K_{lk}a_k = 0\,,\ l\in\{\mathrm{u}\}\,, \qquad \begin{pmatrix}K_{\mathrm{u}\,\mathrm{u}} & K_{\mathrm{u}\,\mathrm{g}}\end{pmatrix}\begin{pmatrix}\boldsymbol{a}_{\mathrm{u}}\\ \boldsymbol{a}_{\mathrm{g}}\end{pmatrix} = 0\,, \quad \text{or} \quad K_{\mathrm{u}\,\mathrm{u}}\boldsymbol{a}_{\mathrm{u}} = -K_{\mathrm{u}\,\mathrm{g}}\boldsymbol{a}_{\mathrm{g}}\,.$$

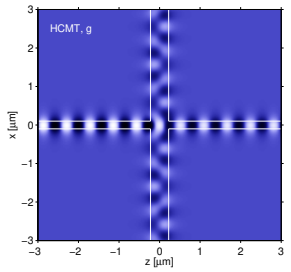
Further issues

... plenty.

Comments

HCMT scheme based on the variational form of the guided wave scattering problem:

- Expansions at the TIBC ports reduce to single terms due to modal orthogonality.
- Bidirectional basis fields are required for all channels in the field templates.



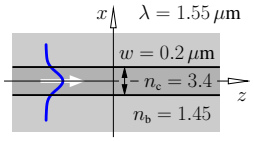
Alternative functional:

$$\mathcal{C}(\boldsymbol{E},\boldsymbol{H}) = \iiint_{\Omega} \big\{ \boldsymbol{E}^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) - \boldsymbol{H}^{*}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^{*}\cdot\boldsymbol{E} + \mathrm{i}\omega\mu_0\boldsymbol{H}^{*}\cdot\boldsymbol{H}\big\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

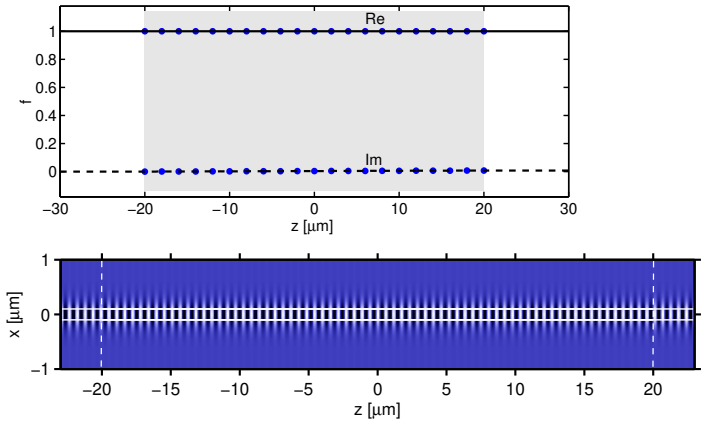
Extend $\mathcal{C}$ by boundary integrals such that

- the boundary terms in $\delta\mathcal{C}$ cancel ⟷ the Galerkin scheme could be viewed as a variational restriction of $\mathcal{C}$.
- TIBCs are satisfied as natural boundary conditions if $\mathcal{C}$ becomes stationary ⟷ variational scheme with complex conjugate fields.

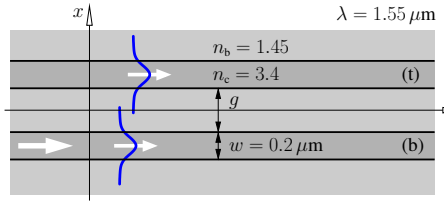
Straight waveguide



Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\ \mu$m, $\Delta z = 2\,\mu$m,
computational domain $z \in [< -20, > 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.



Two coupled parallel cores, amplitudes

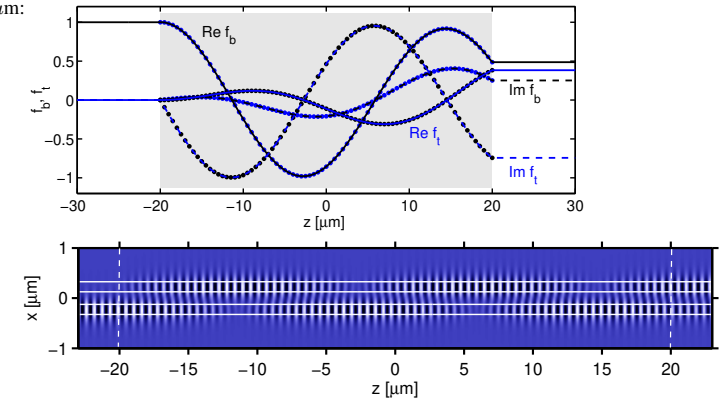


Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

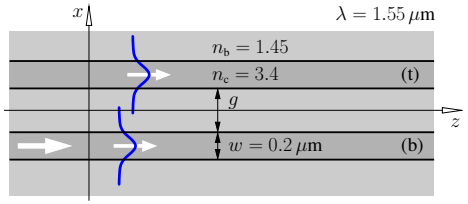
FEM discretization:
$z \in [-20, 20]\ \mu$m, $\Delta z = 0.5\,\mu$m,

computational domain:
$z \in [-20, 20]\ \mu$m, $x \in [-3.0, 3.0]\ \mu$m.

$g = 0.25\,\mu$m:



Two coupled parallel cores, modal power

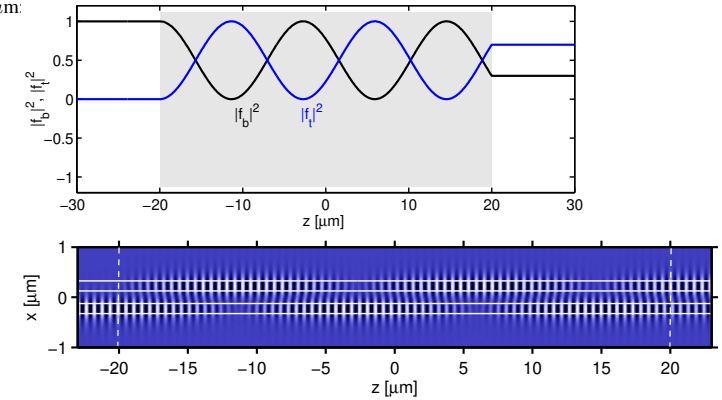


Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_\mathrm{b} = 1$,

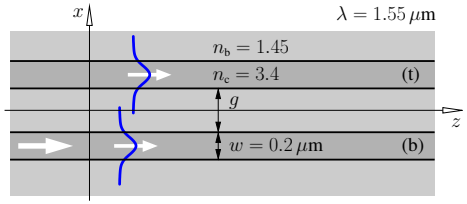
FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

$g = 0.25\,\mu$m:



Two coupled parallel cores, coupling length

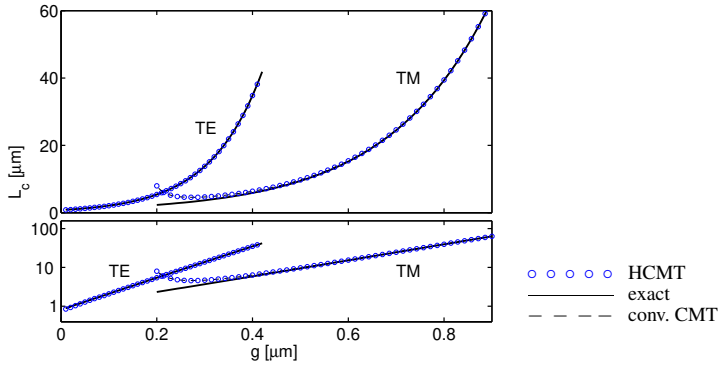


Basis elements: polarized forward propagating fundamental modes of the separate cores, input amplitude $f_\mathrm{b} = 1$,

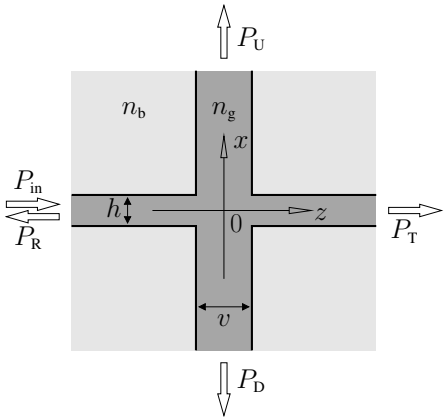
FEM discretization (TE):
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain (TE):
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

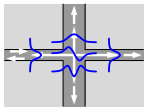
Coupling length:



Waveguide crossing



$n_\mathrm{g} = 3.4$, $n_\mathrm{b} = 1.45$, $\lambda = 1.55\,\mu$m,
$h = 0.2\,\mu$m, v variable, TE polarization.



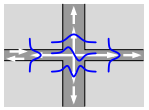
Basis elements:
guided modes of the horizontal and vertical cores (directional variants).

FEM discretization:
$z \in [v/2 - 1.5\,\mu\mathrm{m}, v/2 + 1.5\,\mu\mathrm{m}]$, $\Delta x = 0.025\,\mu$m,
$x \in [w/2 - 1.5\,\mu\mathrm{m}, w/2 + 1.5\,\mu\mathrm{m}]$, $\Delta z = 0.025\,\mu$m.

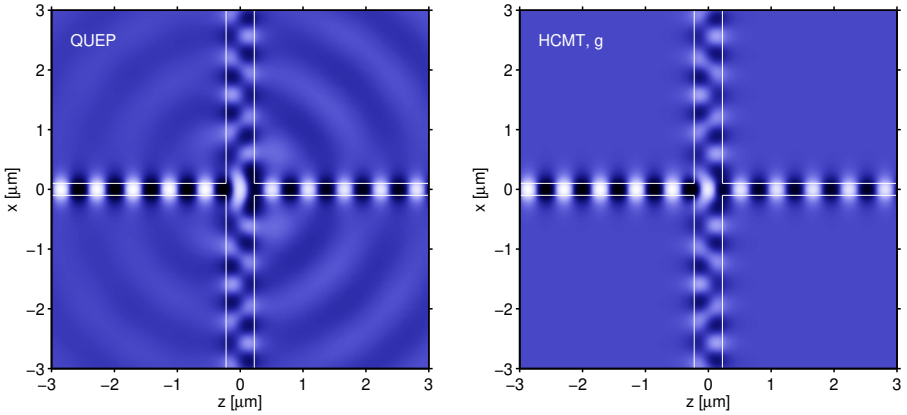
Computational window:
$z \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$, $x \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$.

Waveguide crossing, fields (I)

$v = 0.45\,\mu$m:

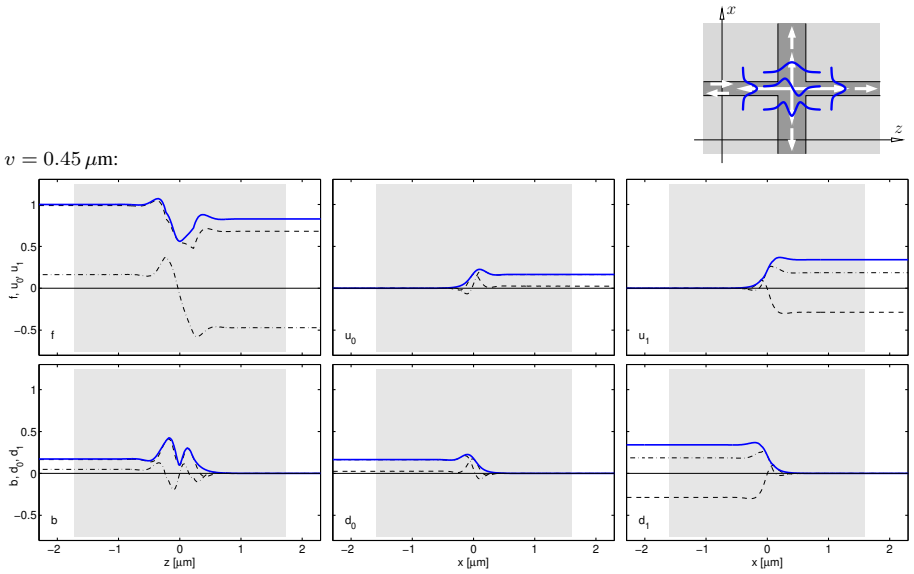


reference

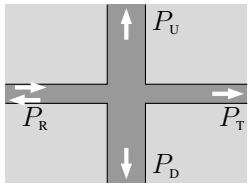


Waveguide crossing, amplitude functions

$v = 0.45\,\mu$m:

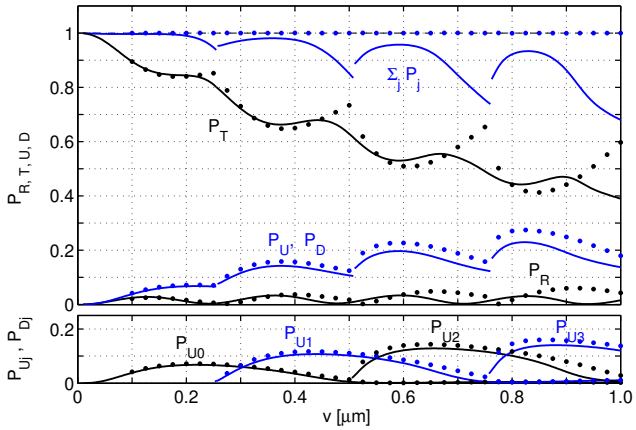


Waveguide crossing, power transfer (I)



QUEP, reference

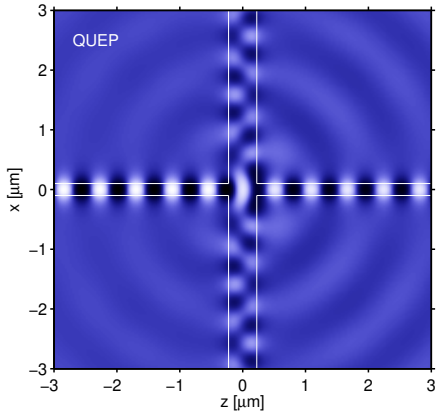
HCMT



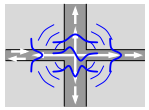
Waveguide crossing, fields (II)

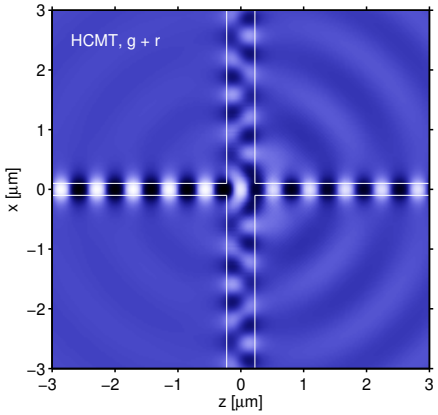
$v = 0.45\,\mu$m:

reference

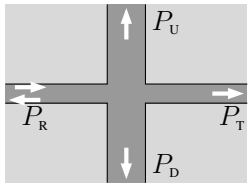


HCMT basis fields:
guided modes
+ 4 Gaussian beams,
outgoing along the diagonals.



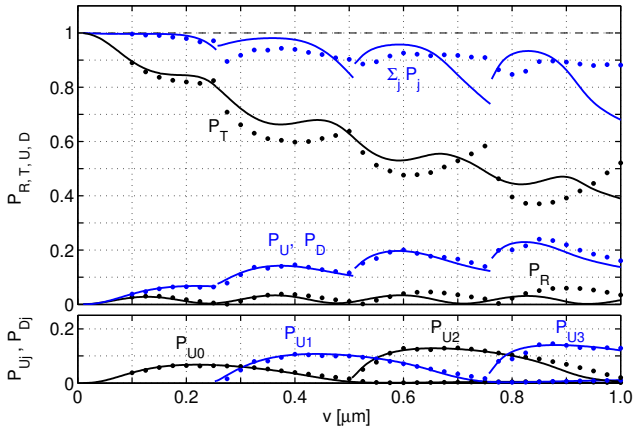


Waveguide crossing, power transfer (II)

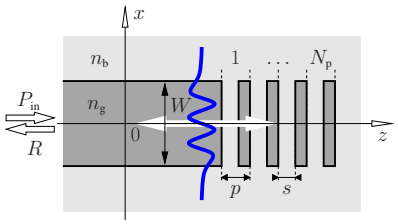


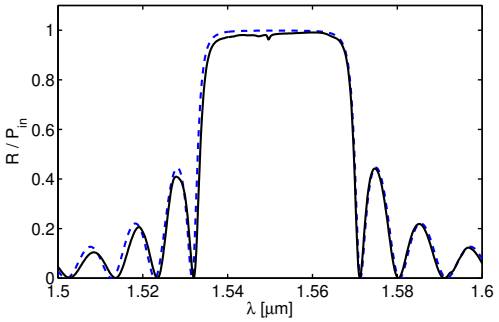
QUEP, reference

HCMT,
incl. templates
for radiated fields



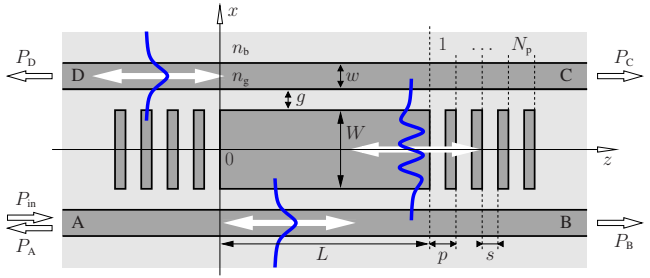
Waveguide Bragg reflector



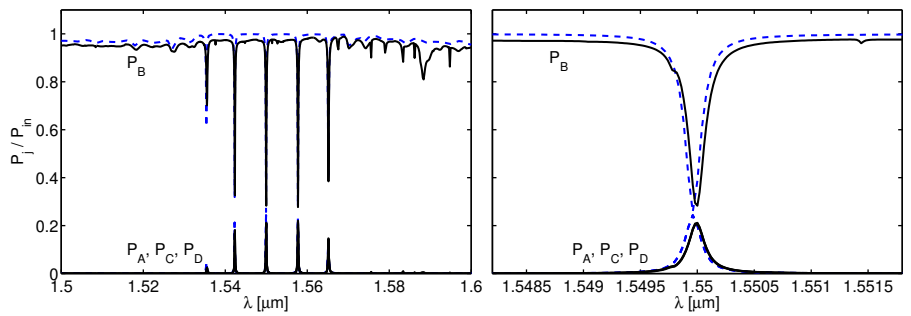


TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

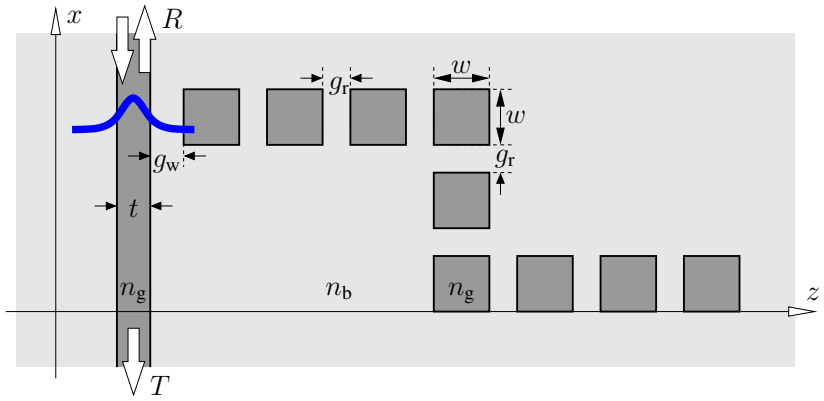
Grating-assisted rectangular resonator



$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

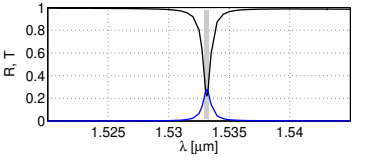


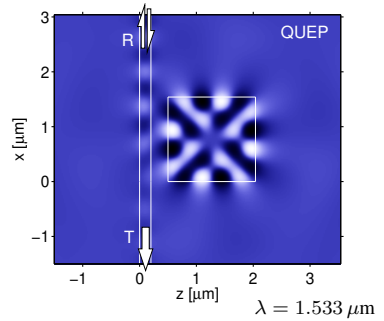
Chains of square microcavities



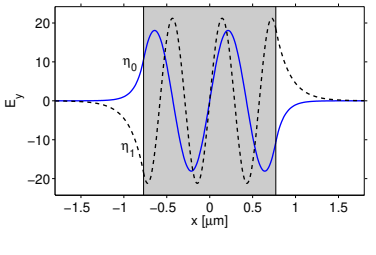
TE, $n_b = 1.0$, $n_g = 3.2$, $w = 1.54\,\mu$m, $g_r = 0.39\,\mu$m, $g_w = 0.3\,\mu$m, $t = 0.2\,\mu$m; $\lambda_0 = 1.532\,\mu$m.

A single resonator

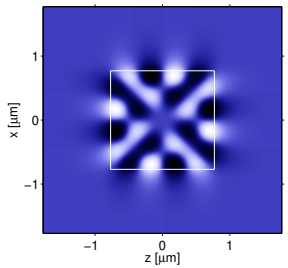




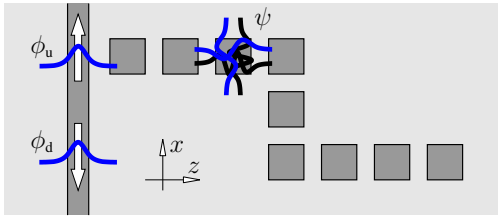
$\lambda = 1.533\,\mu$m



$$\psi(x,z) = \eta_0(x)\,\eta_1(z) - \eta_1(x)\,\eta_0(z)$$



Resonator chain, HCMT model



$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = u(x)\,\boldsymbol{\phi}_{\mathrm{u}}(x,z) + d(x)\,\boldsymbol{\phi}_{\mathrm{d}}(x,z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x,z)$$

$r_0 - r_8$, u, d: ?

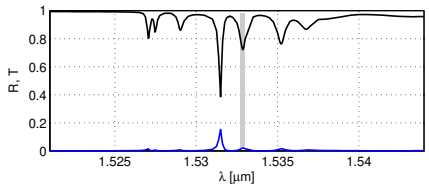
↳ 1-D FEM discretization $u \to \{u_l\}$ and $d \to \{d_l\}$:

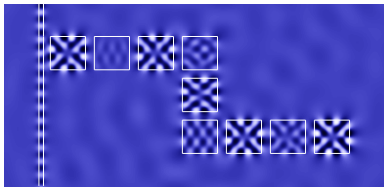
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k\,\boldsymbol{\chi}_k(x,z), \qquad a_k \in \{u_l, d_l, r_j\},\ a_k\text{: ?}$$

↳ HCMT procedure as before.

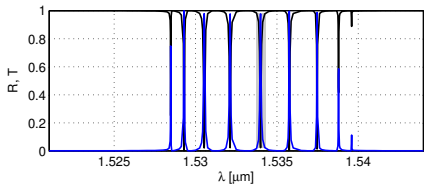
Resonator chain, spectral results

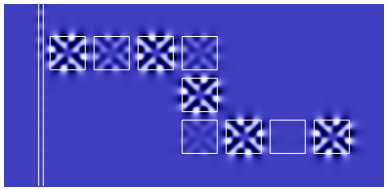
QUEP (reference)



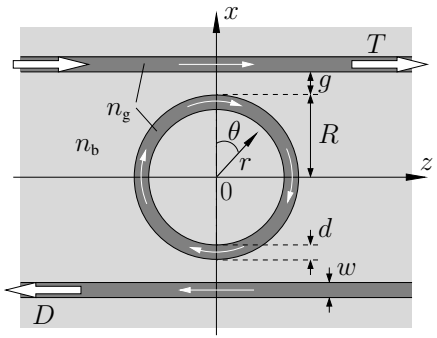


HCMT



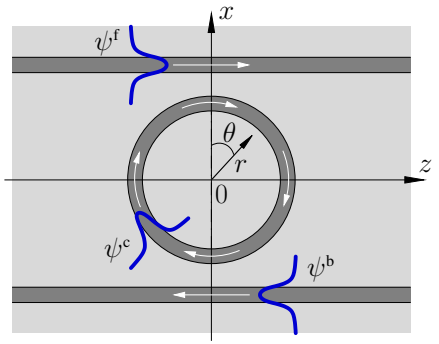


Ringresonator circuits



TE, $R = 7.5\,\mu$m, $w = 0.6\,\mu$m, $d = 0.75\,\mu$m, $g = 0.3\,\mu$m, $n_g = 1.5$, $n_b = 1.0$, $\lambda \approx 1.55\,\mu$m.

Ringresonator, field template



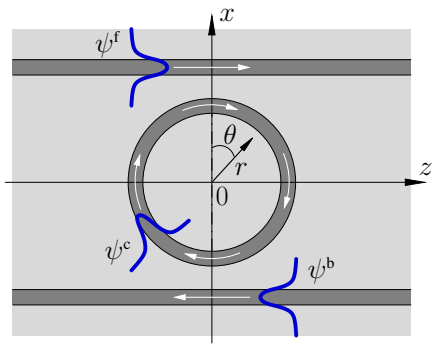
Basis elements:

- bus WGs:
  $$\boldsymbol{\psi}^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{f,b}}(x)\,\mathrm{e}^{\mp \mathrm{i}\beta z},$$
- cavity:
  $$\boldsymbol{\psi}^{\mathrm{c}}(r,\theta) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}^{\mathrm{c}}(r)\,\mathrm{e}^{\mp \mathrm{i}\gamma R\theta},$$
  $$\gamma R \to \mathrm{floor}(\mathrm{Re}\gamma R + 1/2),$$
- further terms:
  bidirectional propagation, higher order modes, other channels, etc..

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = f(z)\,\boldsymbol{\psi}^{\mathrm{f}}(x,z) + b(z)\,\boldsymbol{\psi}^{\mathrm{b}}(x,z) + c(\theta)\,\boldsymbol{\psi}^{\mathrm{c}}(r,\theta),$$

$r = r(x,z)$, $\theta = \theta(x,z)$. $\qquad f, b, c$: ?

Ringresonator, HCMT procedure



1-D FEM discretization:

$f(z) \to \{f_j\}$,
$b(z) \to \{b_j\}$,

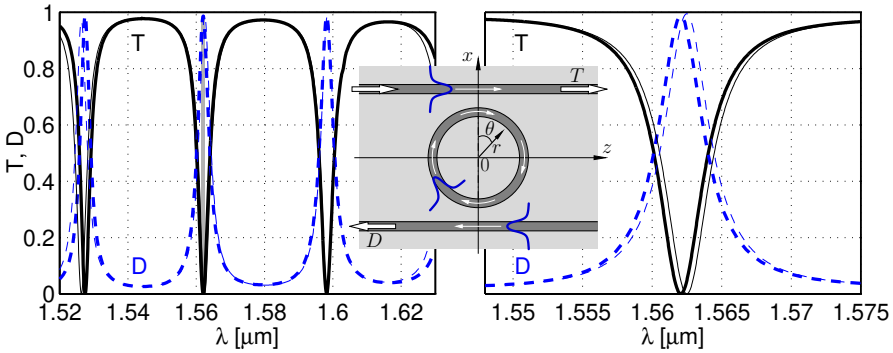
$c(\theta) \to \{c_j\}$, identify nodes 0 and N_θ,
$r \to r(x,z),\ \theta \to \theta(x,z)$.

$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_k a_k \left(\alpha_\cdot(\cdot)\boldsymbol{\psi}_\cdot(x,z) \right) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x,z),$$

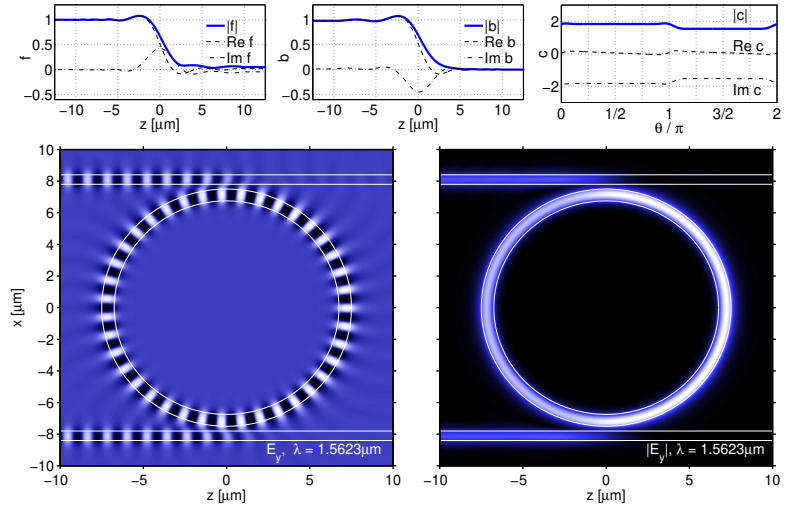
$k \in \{\text{channels, modes, elements}\}, \quad a_k \in \{f_j, b_j, c_j\}.$

HCMT solution as before.

Single ring filter, spectral response



Single ring filter, resonance



Fast evaluation of spectral properties

Time consuming: evaluation of modal "overlaps" K_{lk} in K:

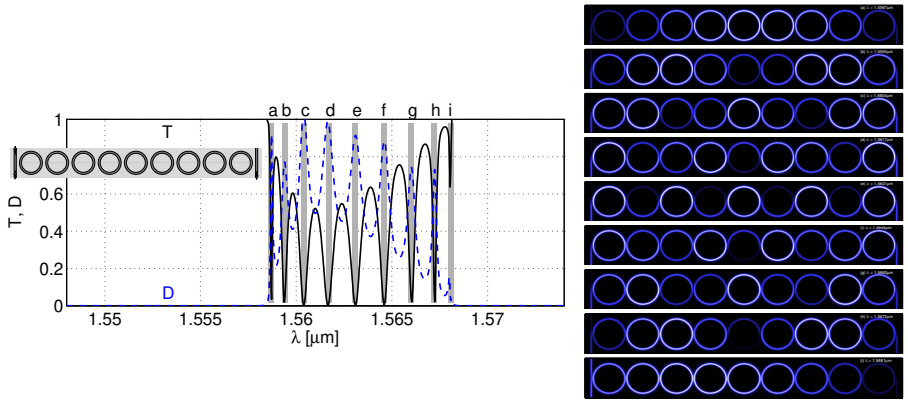
$$K_{lk} = \iiint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\,.$$

All properties of the modal basis fields change but slowly with λ;
rapid spectral variations are due to the *solution* of the linear system involving K.

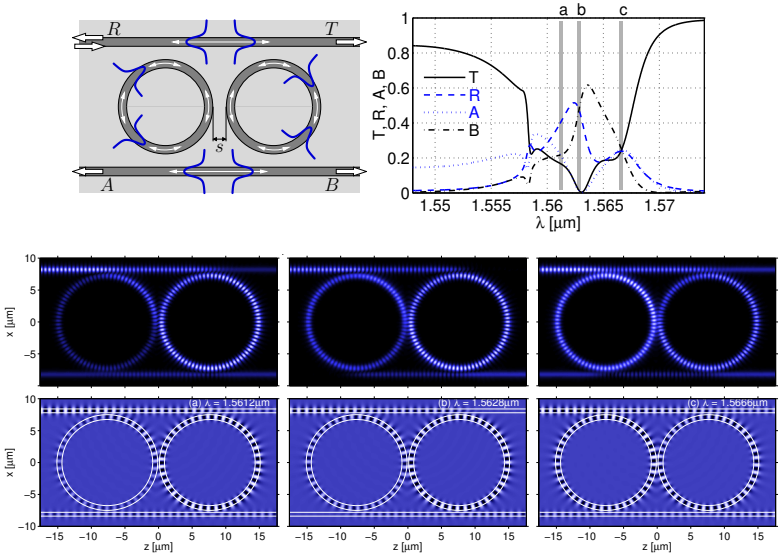
Interpolate $\mathsf{K}(\lambda)$:

- Interval of interest $\lambda \in [\lambda_a, \lambda_b]$, $\lambda_0 := \frac{3}{4}\lambda_a + \frac{1}{4}\lambda_b,\ \lambda_1 := \frac{1}{4}\lambda_a + \frac{3}{4}\lambda_b,$
- compute only $\mathsf{K}_0 = \mathsf{K}(\lambda_0)$ and $\mathsf{K}_1 = \mathsf{K}(\lambda_1)$ directly,
- interpolate $\mathsf{K}_\mathrm{i}(\lambda) = \mathsf{K}_0 + \dfrac{\lambda - \lambda_0}{\lambda_1 - \lambda_0}\,(\mathsf{K}_1 - \mathsf{K}_0)$,
- solve for $\boldsymbol{a}(\lambda)$ with $\mathsf{K}_\mathrm{i}(\lambda)$.

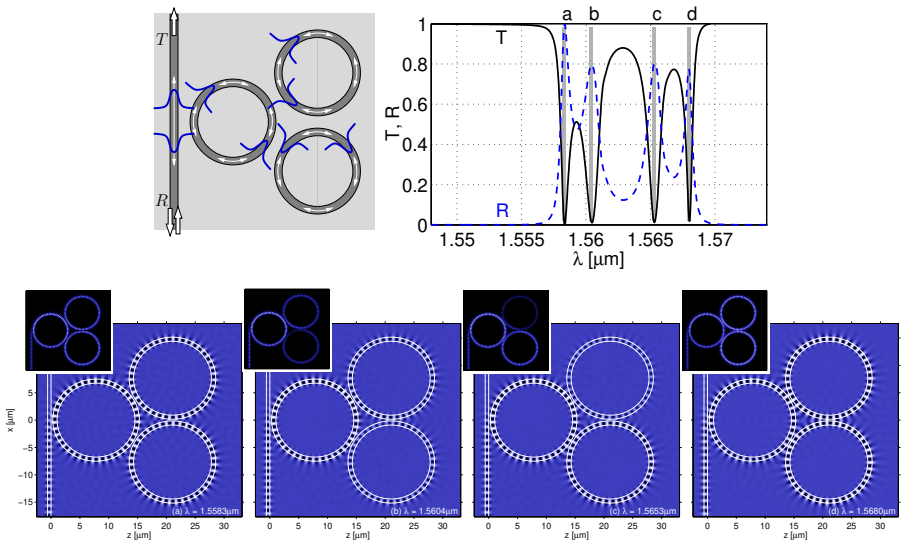
Coupled resonator optical waveguides, CROWs



Rings coupled in parallel



R3 resonator



Hybrid analytical / numerical coupled mode modeling

HCMT:

- an ab-initio, quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- extension to 3-D (todo): numerical basis fields, still moderate effort,
- reasonably versatile: