Hybrid analytical / numerical coupled-mode modeling of guided wave devices

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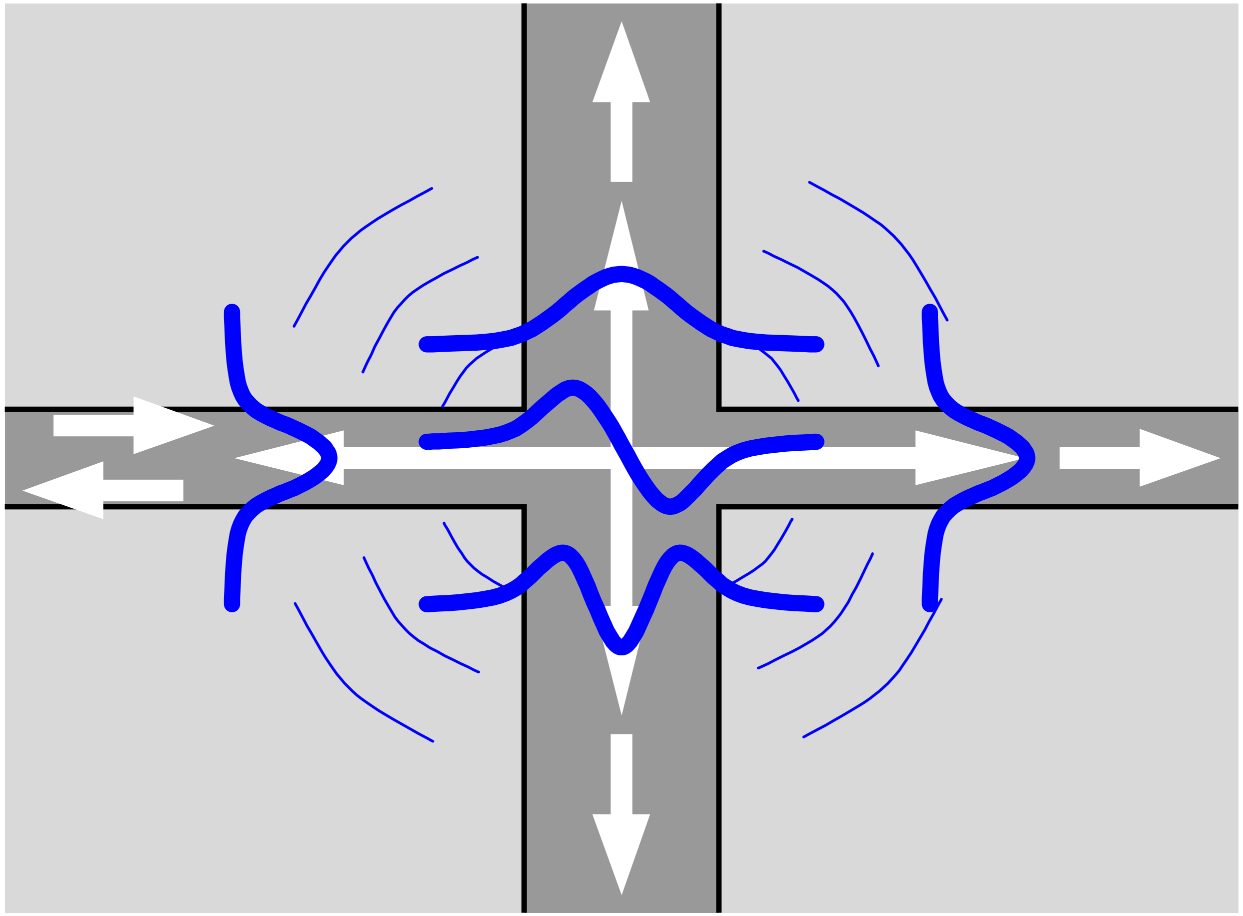
A waveguide crossing

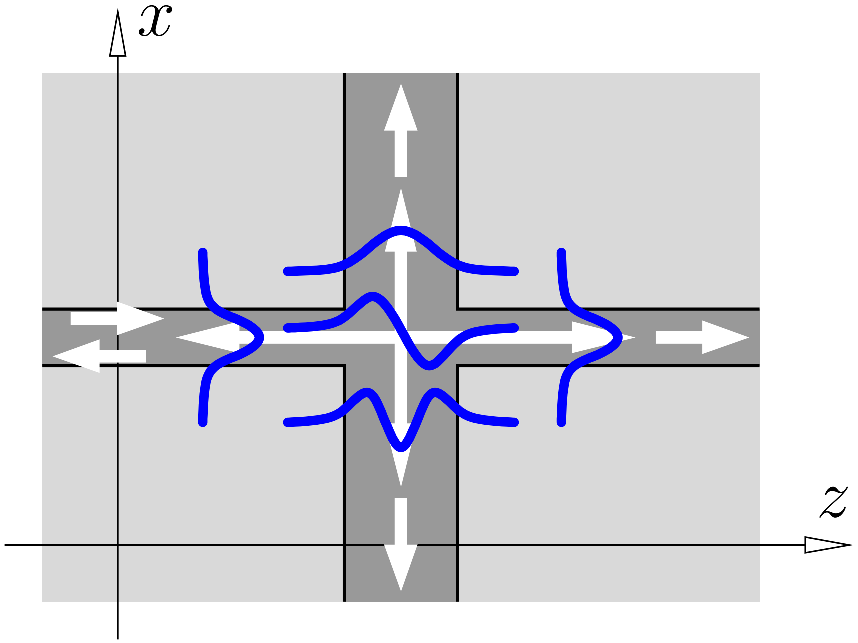


Outline

Hybrid analytical / numerical coupled-mode modeling

- An example problem
- CMT field ansatz
- Amplitude discretization, 1-D FEM
- Galerkin procedure
- Numerical results

Field ansatz



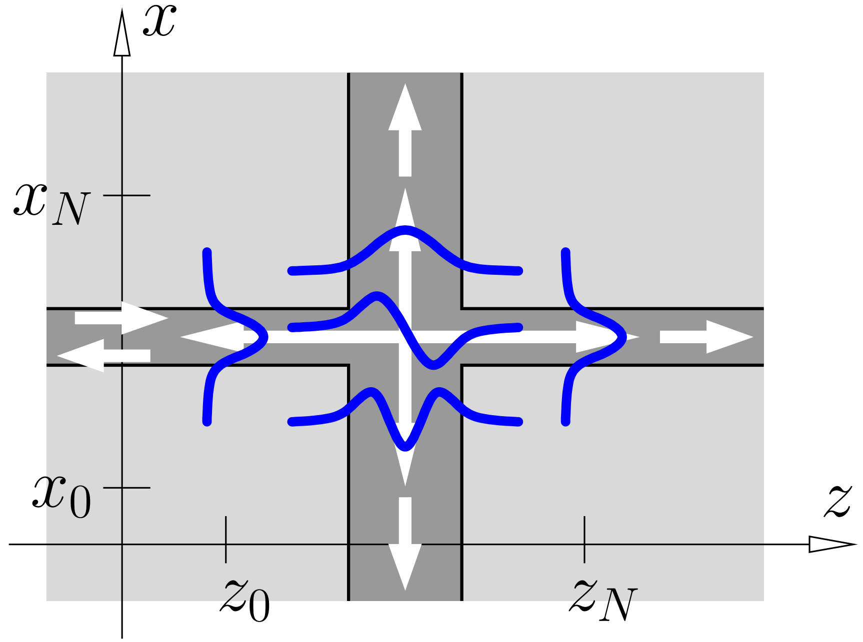
Basis elements (crossing):

- guided modes of the horizontal WG
$$\boldsymbol{\psi}_m^{\mathrm{f,b}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{f,b}}(x)\, \mathrm{e}^{\mp \mathrm{i} \beta_m^{\mathrm{f,b}} z},$$
- guided modes of the vertical WG
$$\boldsymbol{\psi}_m^{\mathrm{u,d}}(x,z) = \begin{pmatrix} \tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{H}} \end{pmatrix}_m^{\mathrm{u,d}}(z)\, \mathrm{e}^{\mp \mathrm{i} \beta_m^{\mathrm{u,d}} x}$$
- (and further terms).

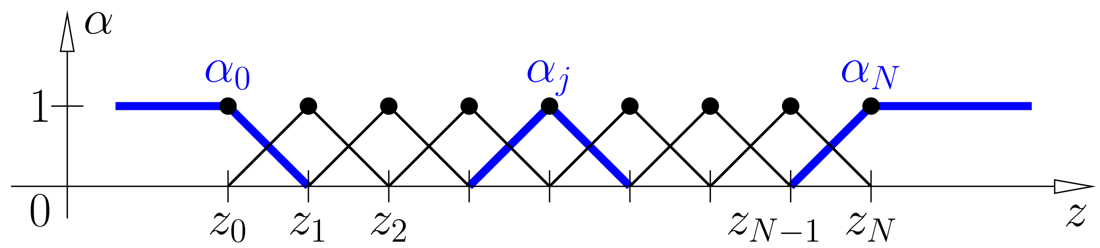
$$\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x,z) = \sum_m f_m(z)\boldsymbol{\psi}_m^{\mathrm{f}}(x,z) + \sum_m b_m(z)\boldsymbol{\psi}_m^{\mathrm{b}}(x,z)$$
$$+ \sum_m u_m(x)\boldsymbol{\psi}_m^{\mathrm{u}}(x,z) + \sum_m d_m(x)\boldsymbol{\psi}_m^{\mathrm{d}}(x,z) \qquad f_m,\, b_m,\, u_m,\, d_m\text{: } ?$$

General: a reasonable superposition of known fields with amplitudes that are functions of suitable propagation coordinate(s).

Amplitude functions, discretization



1-D linear finite elements



$$f_m(z) = \sum_{j=0}^{N} f_{m,j}\, \alpha_j(z)\,,$$

$b_m(z)$, $u_m(x)$, $d_m(x)$ analogous.

$$\hookrightarrow \qquad \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}(x, z) = \sum_k a_k \Big(\alpha_{\cdot}(\cdot)\, \boldsymbol{\psi}_{\cdot}^{\cdot}(x, z) \Big) =: \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}(x, z),$$

$k \in \{$waveguides, modes, elements$\}$, $\quad a_k \in \{f_{m,j}, b_{m,j}, u_{m,j}, d_{m,j}\}$.

Galerkin procedure

2-D:

$$\left.\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{H} - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} &= 0 \\ -\boldsymbol{\nabla} \times \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H} &= 0 \end{aligned}\right| \qquad \cdot \begin{pmatrix} \boldsymbol{E}_\mathrm{t} \\ \boldsymbol{H}_\mathrm{t} \end{pmatrix}^*, \quad \iint_{\text{comp. domain}}$$

$$\rightsquigarrow \qquad \iint \mathcal{K}(\boldsymbol{E}_\mathrm{t}, \boldsymbol{H}_\mathrm{t}; \boldsymbol{E}, \boldsymbol{H})\,\mathrm{d}x\,\mathrm{d}z = 0 \quad \text{for all } \boldsymbol{E}_\mathrm{t},\ \boldsymbol{H}_\mathrm{t},$$

where

$$\mathcal{K}(\boldsymbol{E}_\mathrm{t}, \boldsymbol{H}_\mathrm{t}; \boldsymbol{E}, \boldsymbol{H}) = \boldsymbol{E}_\mathrm{t}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}) - \boldsymbol{H}_\mathrm{t}^* \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}_\mathrm{t}^* \cdot \boldsymbol{E} - \mathrm{i}\omega\mu_0\boldsymbol{H}_\mathrm{t}^* \cdot \boldsymbol{H}\,.$$

Galerkin procedure, continued

- insert $\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} = \sum_k a_k \begin{pmatrix} \boldsymbol{E}_k \\ \boldsymbol{H}_k \end{pmatrix}$,
- select $\{\mathrm{u}\}$: indices of unknown coefficients,
  $\{\mathrm{g}\}$: given values related to prescribed influx,
- require $\iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}, \boldsymbol{H})\, \mathrm{d}x\, \mathrm{d}z = 0$ for $l \in \{\mathrm{u}\}$,
- compute $K_{lk} = \iint \mathcal{K}(\boldsymbol{E}_l, \boldsymbol{H}_l; \boldsymbol{E}_k, \boldsymbol{H}_k)\, \mathrm{d}x\, \mathrm{d}z$.

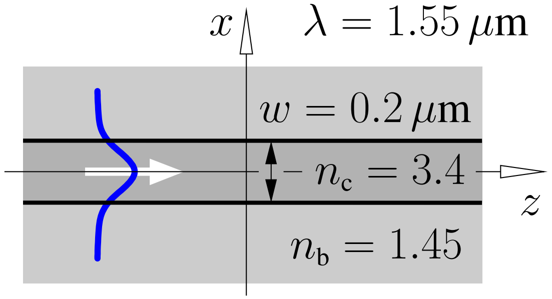
$$\sum_{k \in \{\mathrm{u,g}\}} K_{lk} a_k = 0\,, \;\; l \in \{\mathrm{u}\}\,,$$

$$\begin{pmatrix} K_{\mathrm{u\,u}} & K_{\mathrm{u\,g}} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_\mathrm{u} \\ \boldsymbol{a}_\mathrm{g} \end{pmatrix} = 0\,, \qquad \text{or} \qquad K_{\mathrm{u\,u}} \boldsymbol{a}_\mathrm{u} = -K_{\mathrm{u\,g}} \boldsymbol{a}_\mathrm{g}\,.$$

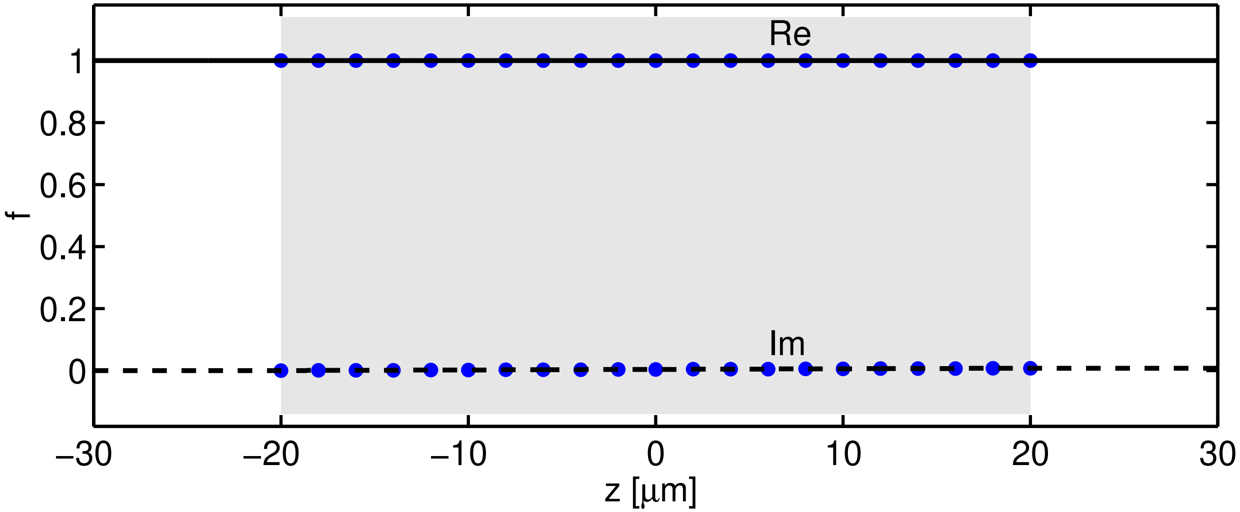
Further issues

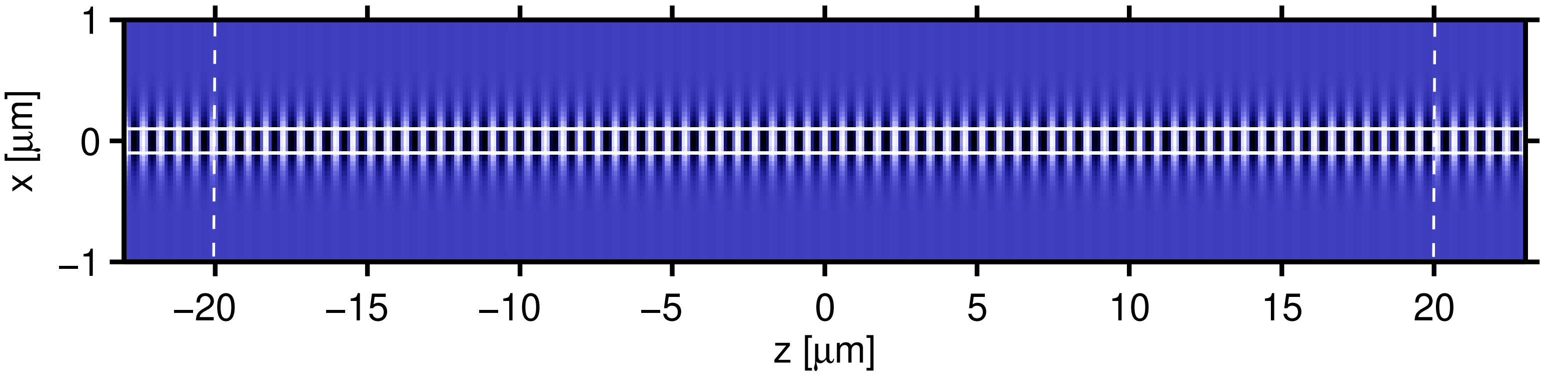
… plenty.

Straight waveguide

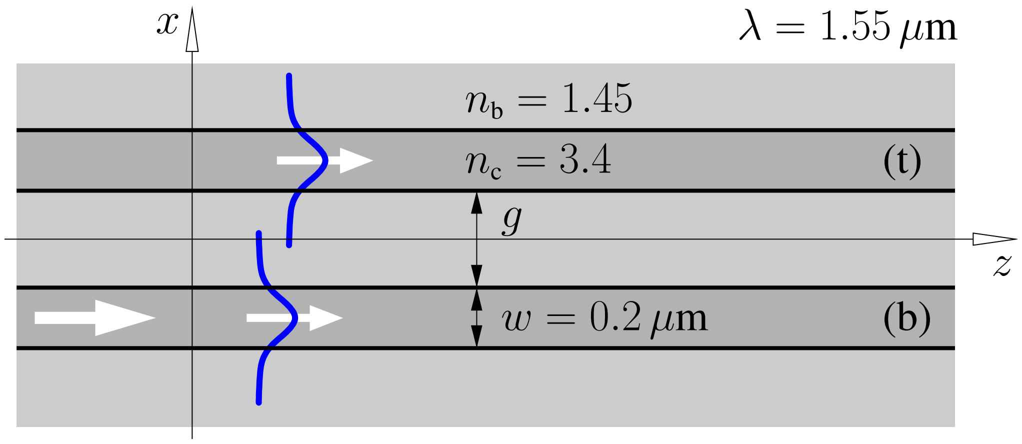


Basis element: fundamental forward propagating TE mode,
input amplitude $f_0 = 1$,
FEM discretization in $z \in [-20, 20]\,\mu$m, $\Delta z = 2\,\mu$m,
computational domain $z \in [< -20, > 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.





Two coupled parallel cores, amplitudes

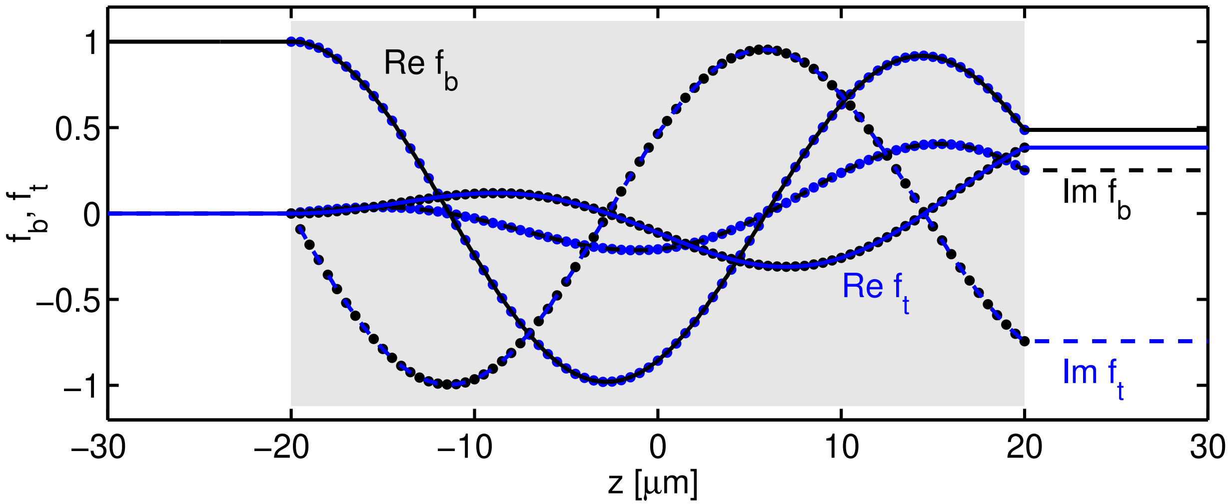


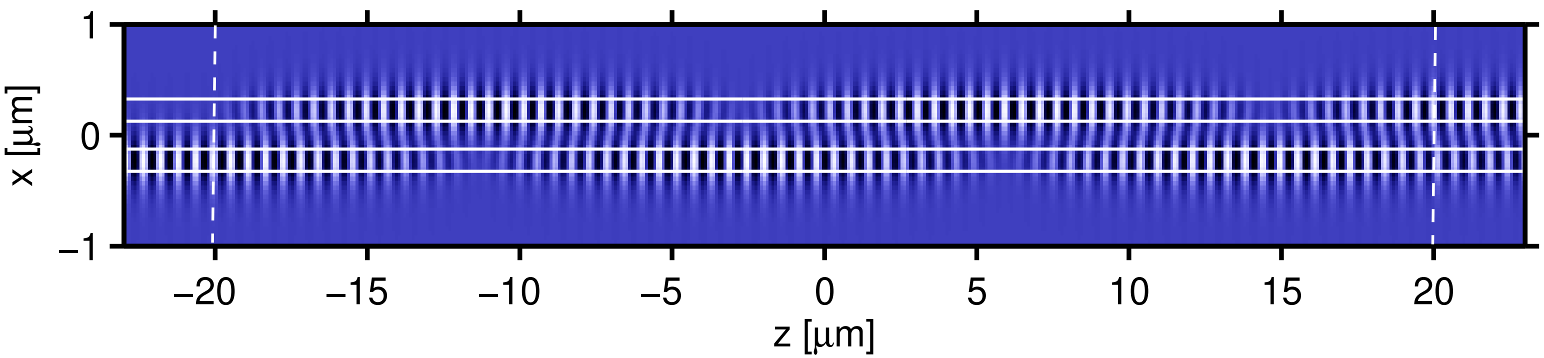
Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_\mathrm{b} = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

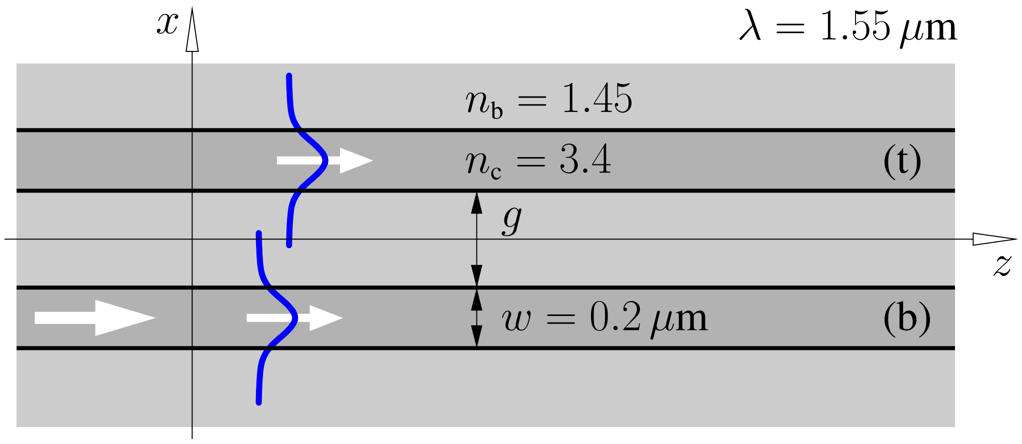
computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

$g = 0.25\,\mu$m:





Two coupled parallel cores, modal power

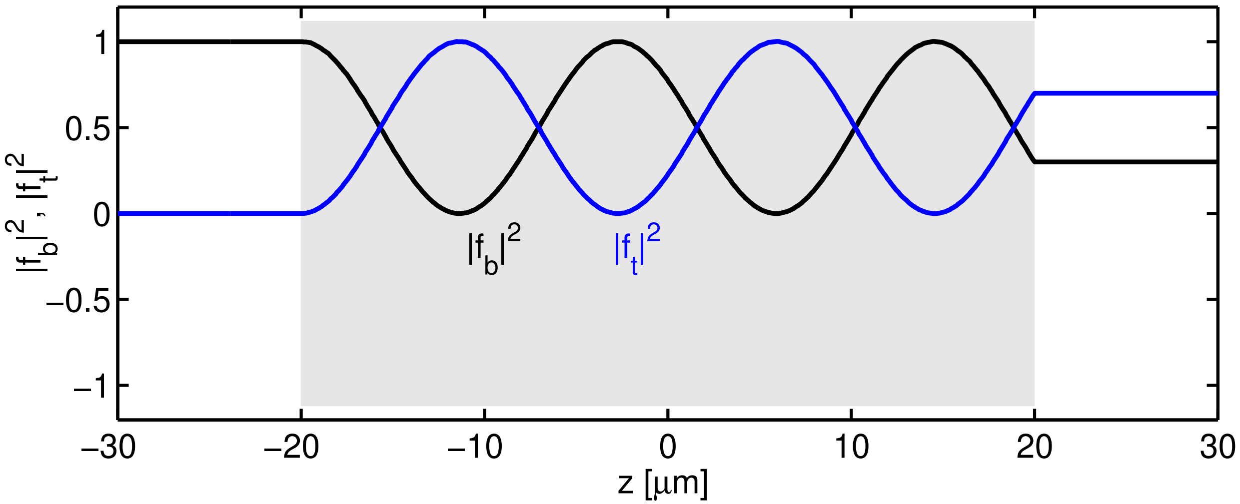


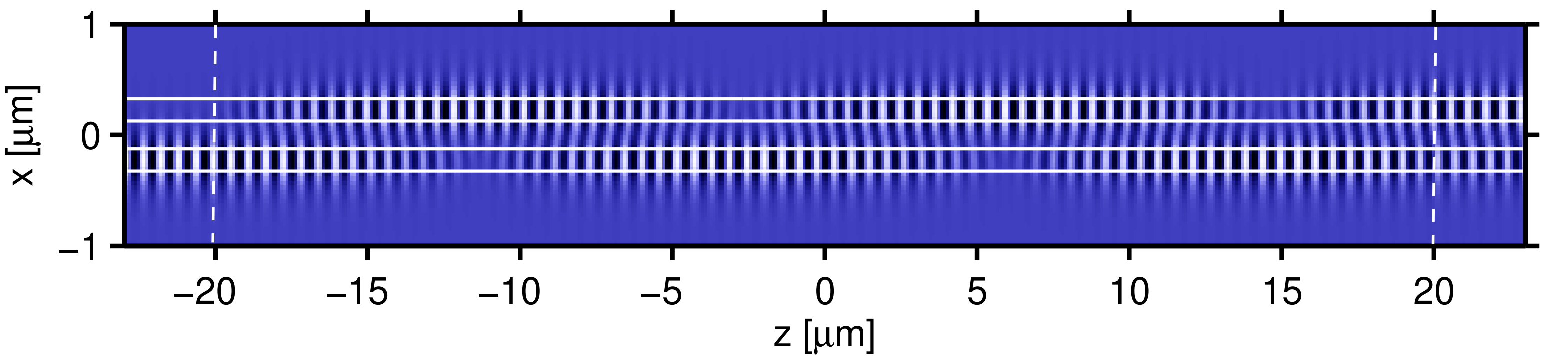
Basis elements: forward propagating fundamental TE modes of the separate cores, input amplitude $f_b = 1$,

FEM discretization:
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

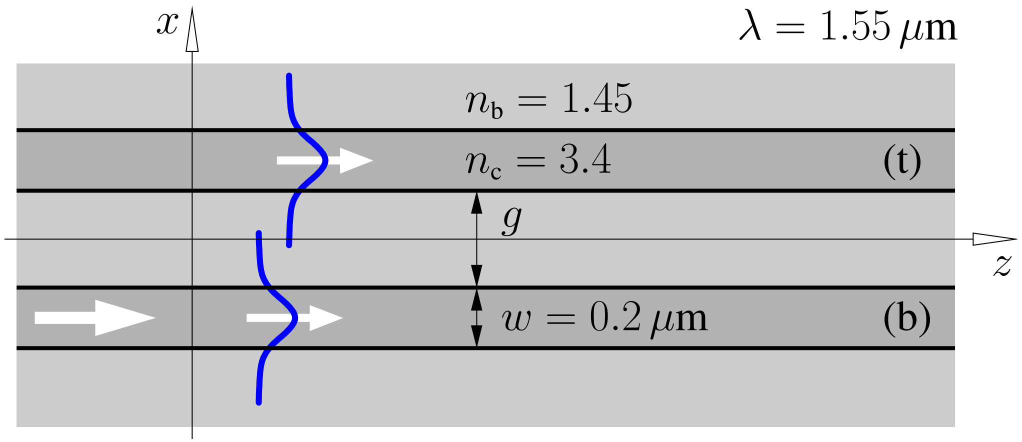
computational domain:
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

$g = 0.25\,\mu$m:





Two coupled parallel cores, coupling length

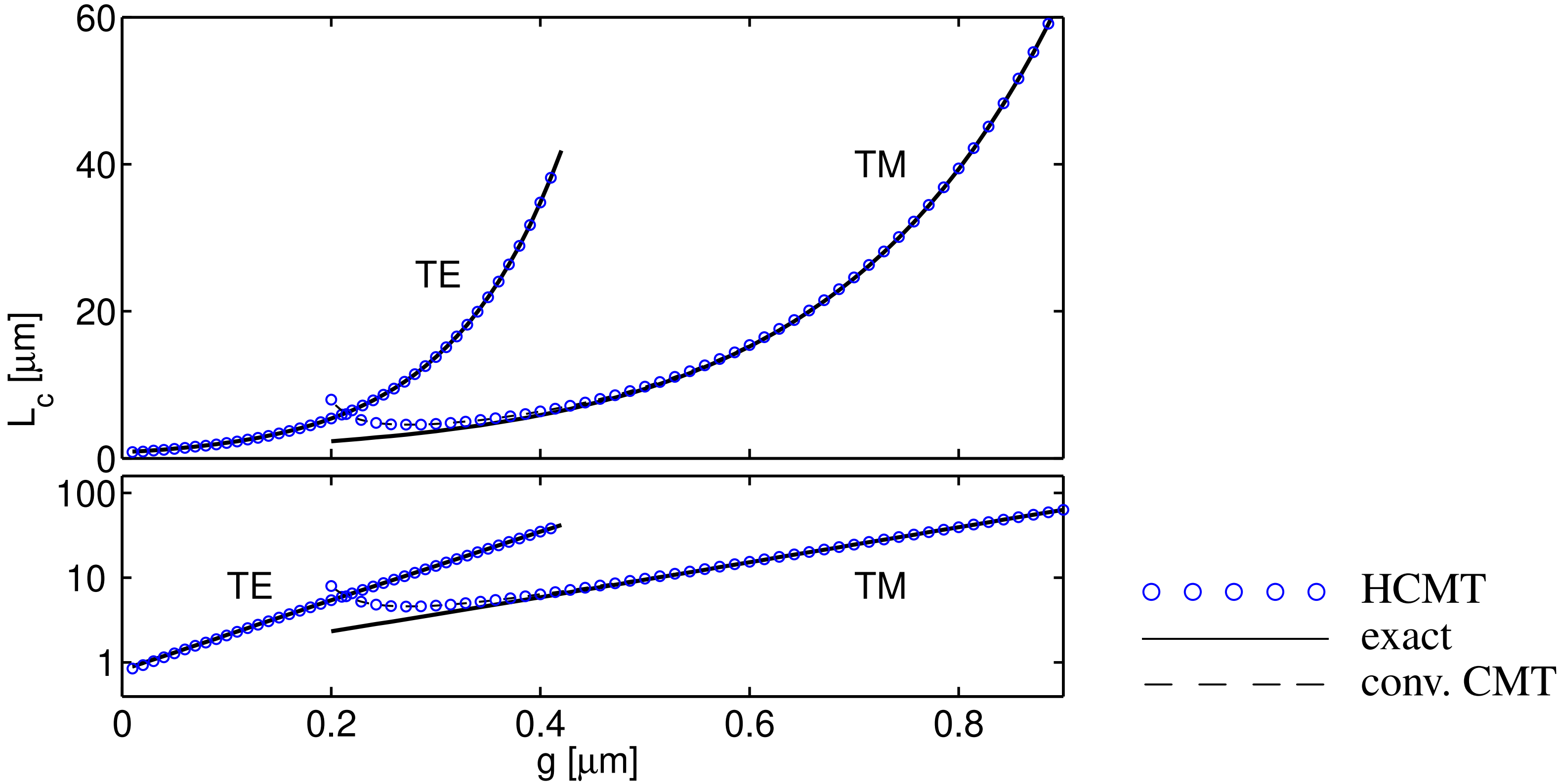


Basis elements: polarized forward propagating fundamental modes of the separate cores, input amplitude $f_\mathrm{b} = 1$,

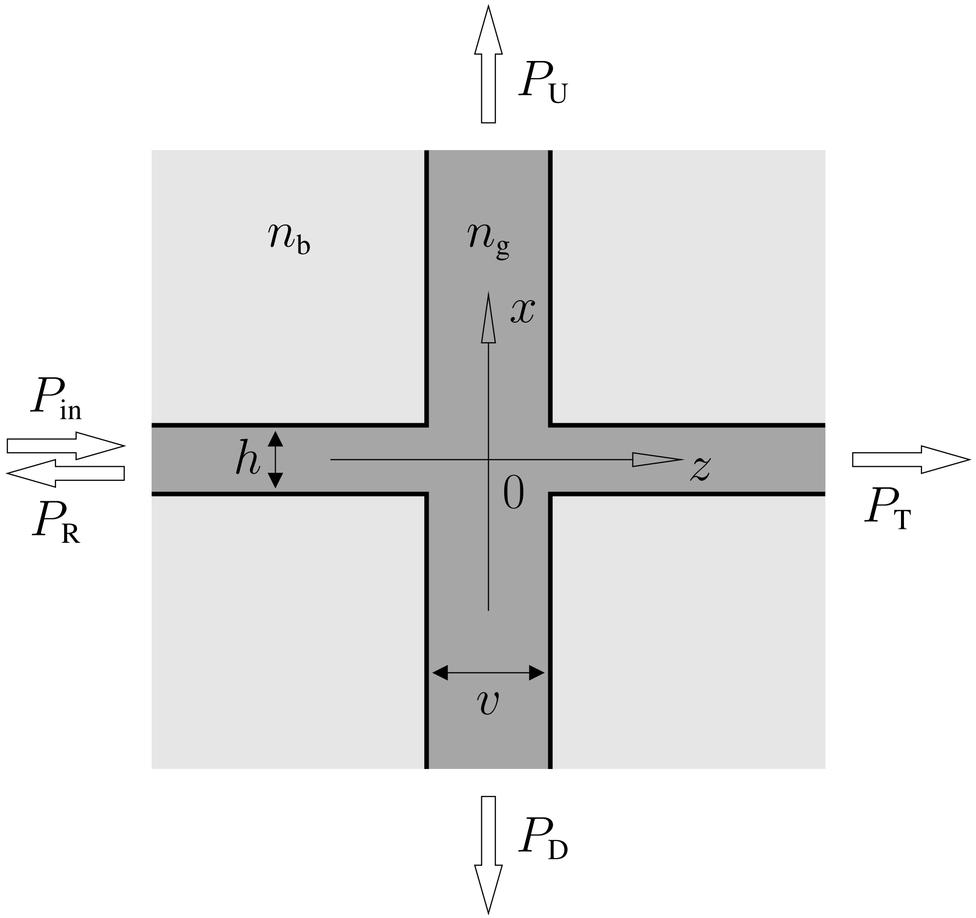
FEM discretization (TE):
$z \in [-20, 20]\,\mu$m, $\Delta z = 0.5\,\mu$m,

computational domain (TE):
$z \in [-20, 20]\,\mu$m, $x \in [-3.0, 3.0]\,\mu$m.

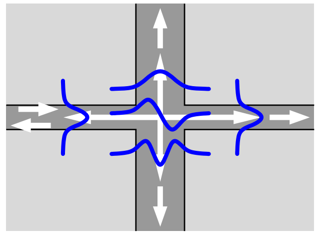
Coupling length:



The waveguide crossing



$n_g = 3.4$, $n_b = 1.45$, $\lambda = 1.55\,\mu$m,
$h = 0.2\,\mu$m, v variable, TE polarization.



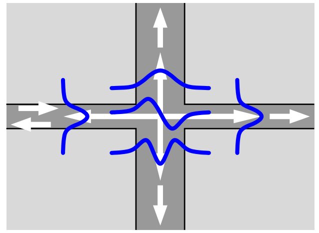
Basis elements:
guided modes of the horizontal
and vertical cores
(directional variants).

FEM discretization:
$z \in [v/2 - 1.5\,\mu\mathrm{m}, v/2 + 1.5\,\mu\mathrm{m}]$, $\Delta x = 0.025\,\mu$m,
$x \in [w/2 - 1.5\,\mu\mathrm{m}, w/2 + 1.5\,\mu\mathrm{m}]$, $\Delta z = 0.025\,\mu$m.

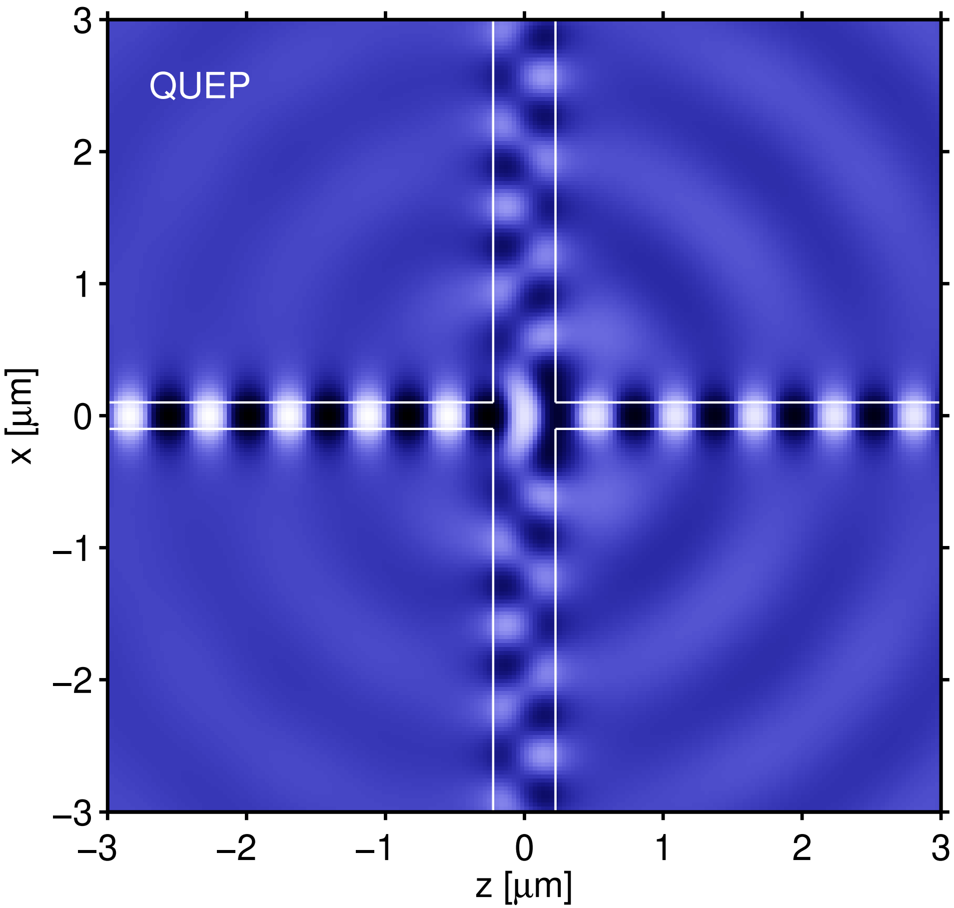
Computational window:
$z \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$, $x \in [-4\,\mu\mathrm{m}, 4\,\mu\mathrm{m}]$.

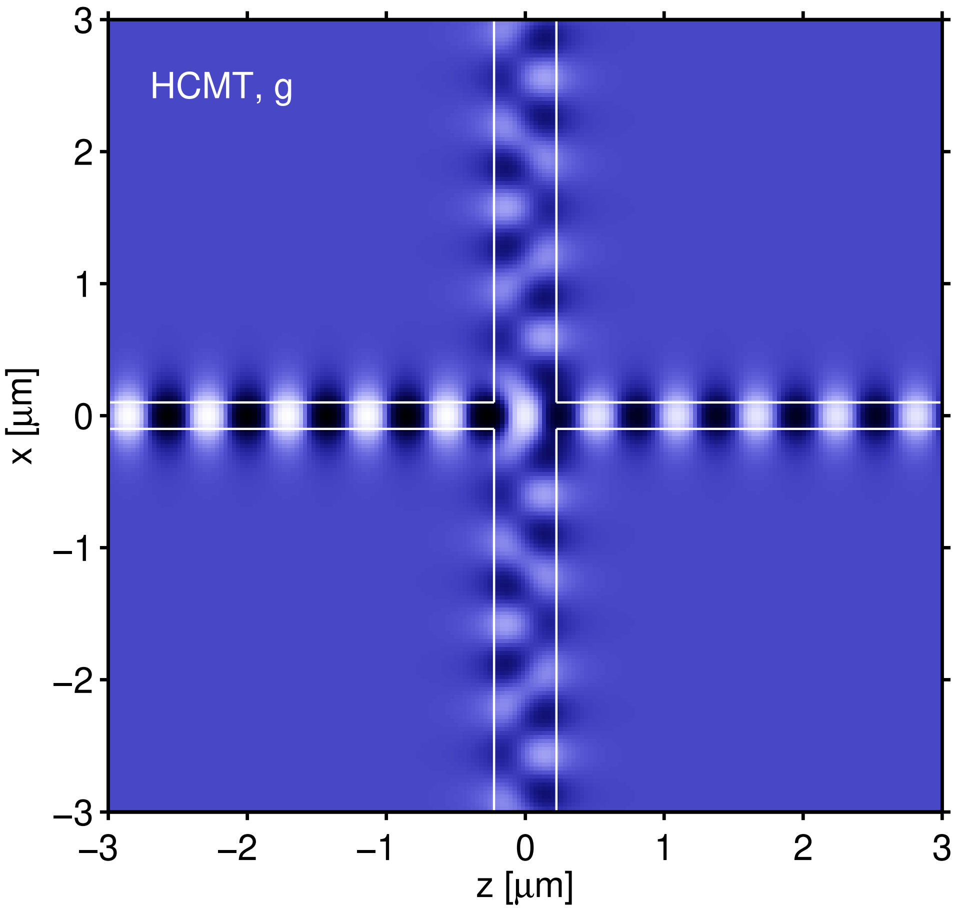
Waveguide crossing, fields (I)

$v = 0.45\ \mu$m:

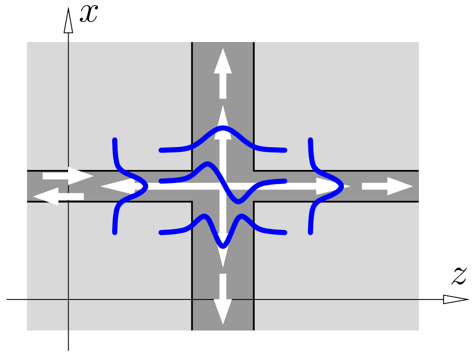


reference

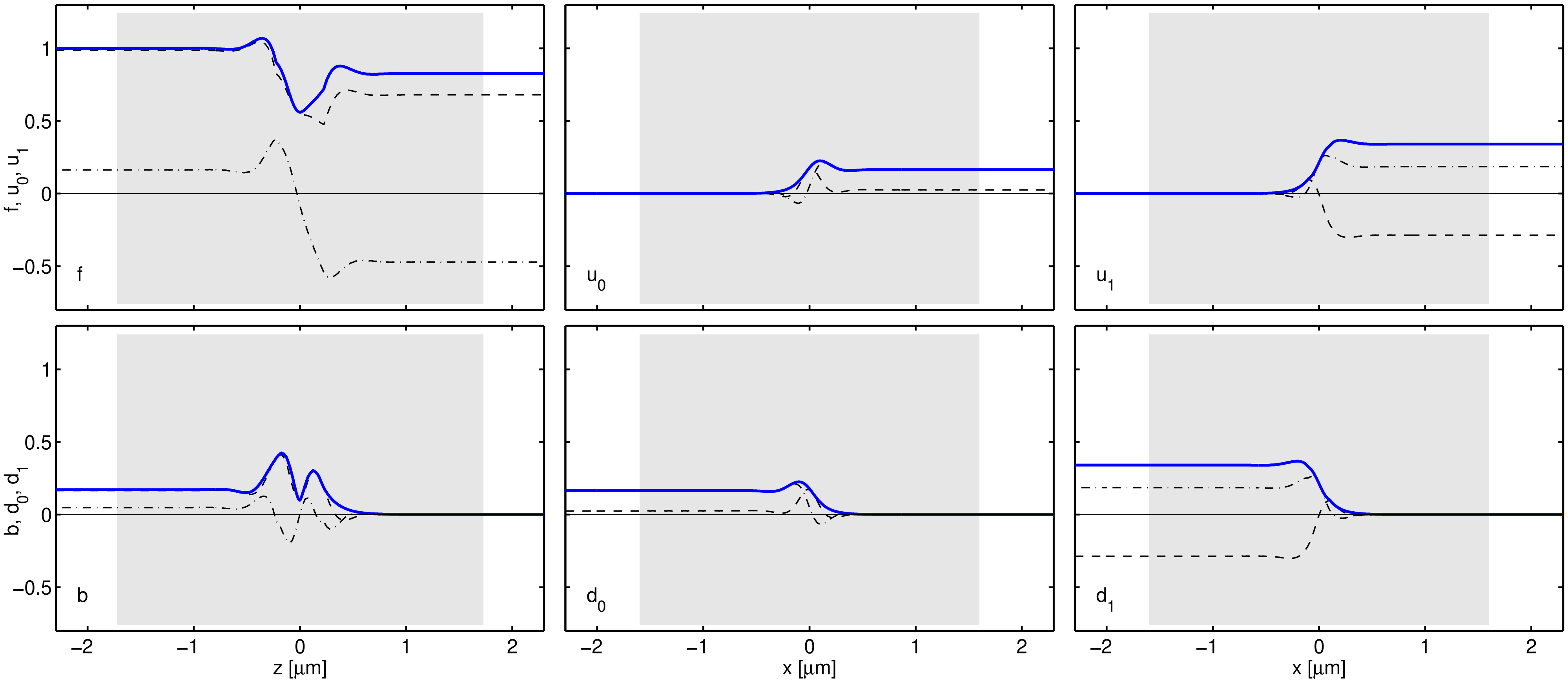




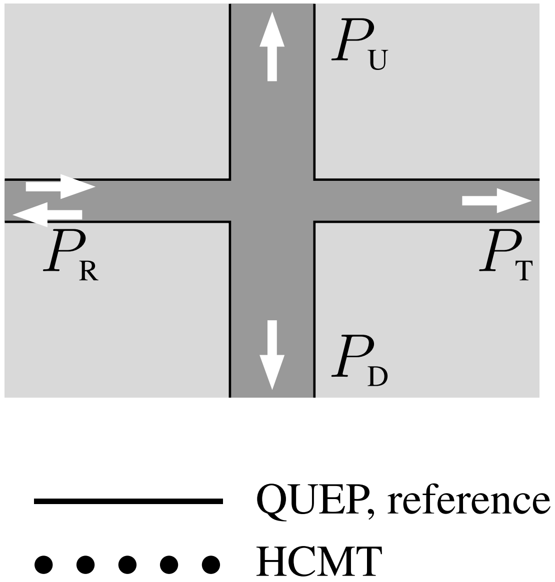
Waveguide crossing, amplitude functions

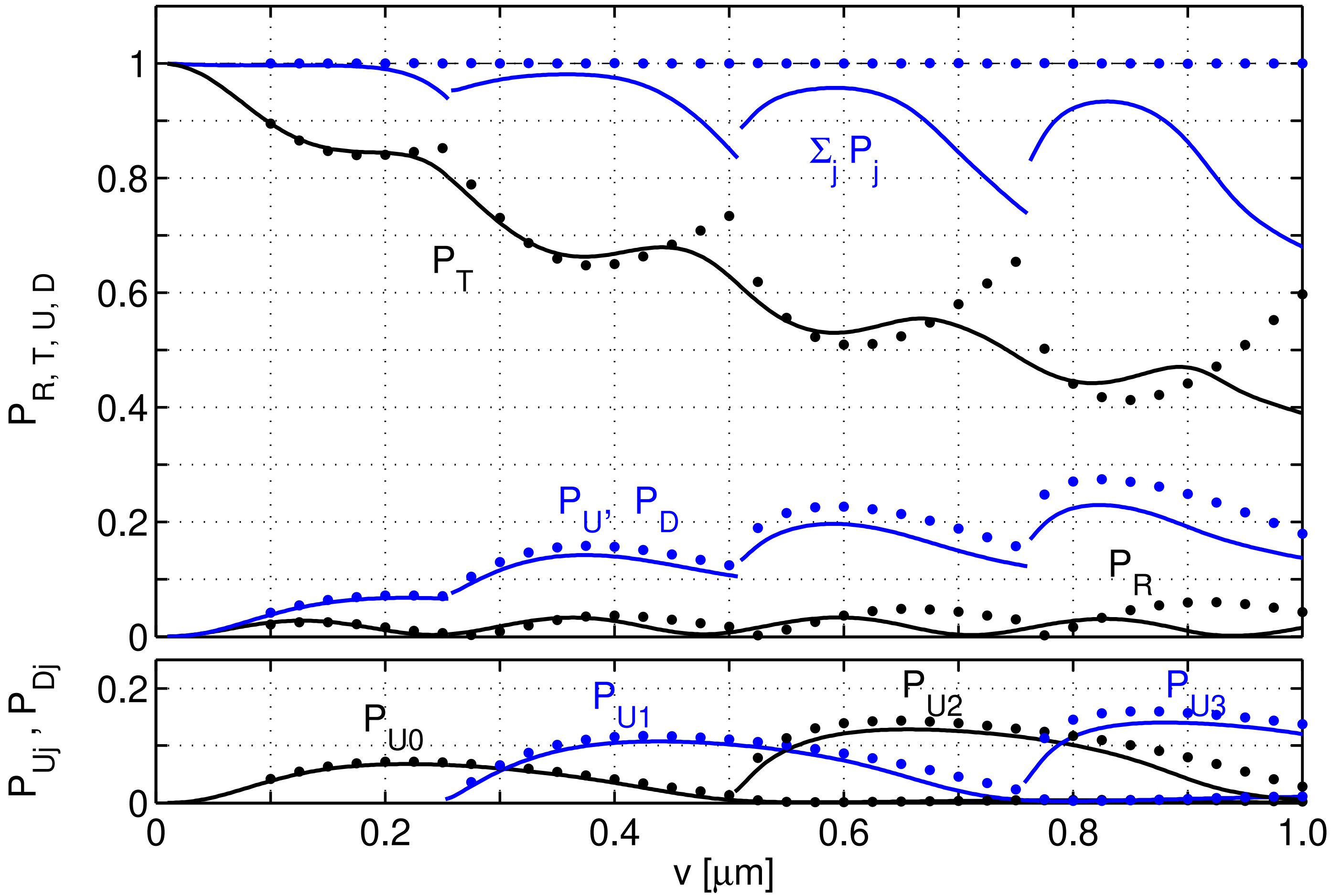


$v = 0.45\,\mu$m:



Waveguide crossing, power transfer (I)

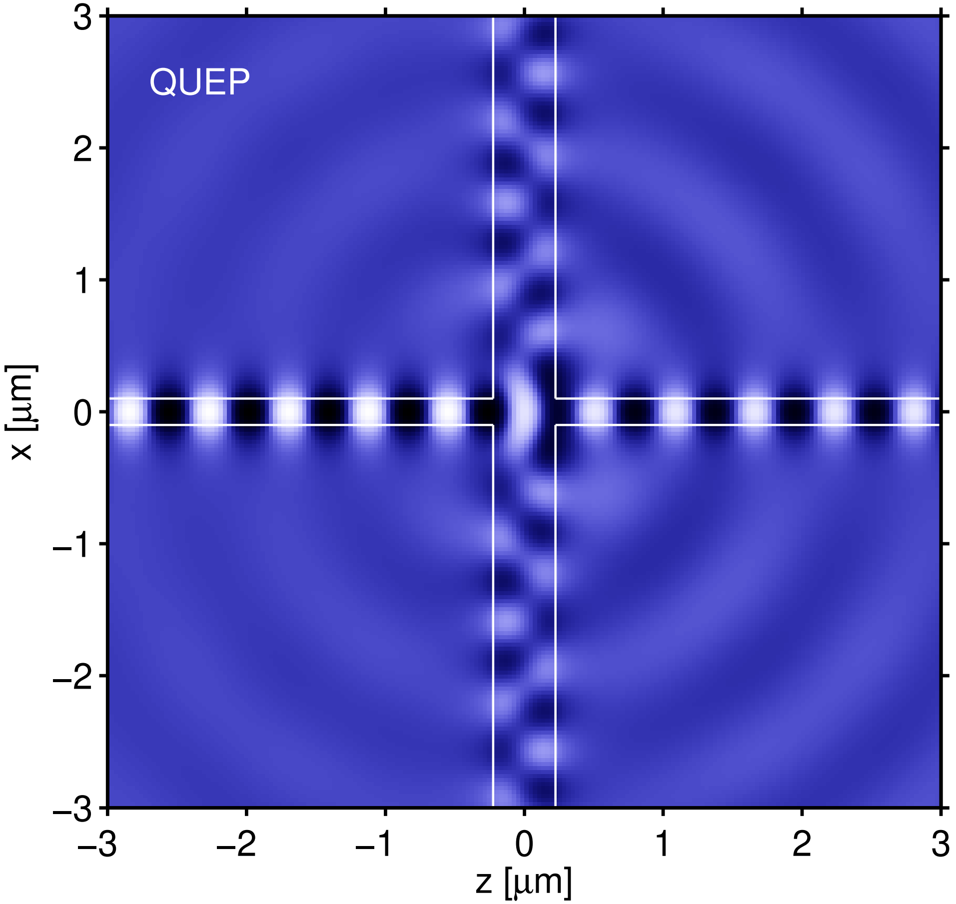




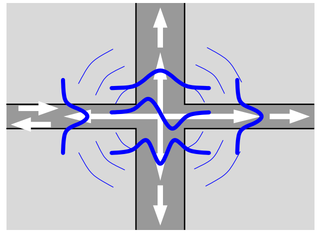
Waveguide crossing, fields (II)

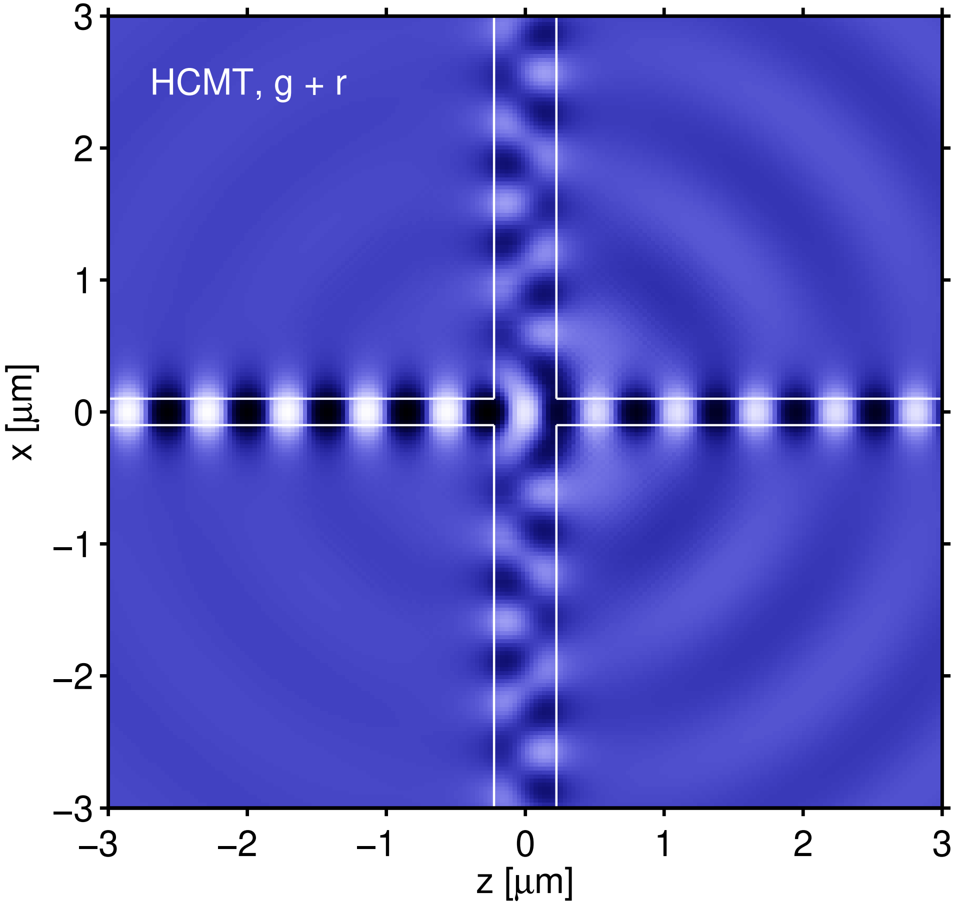
$v = 0.45\ \mu$m:

reference

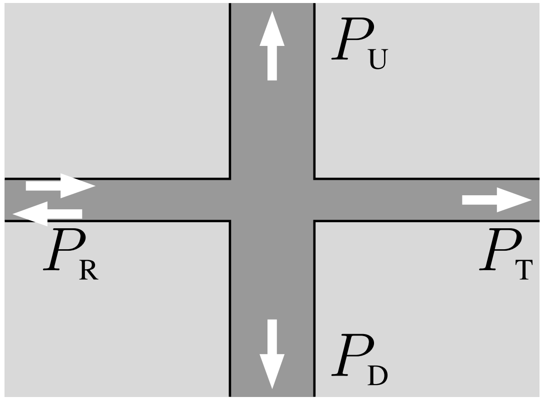


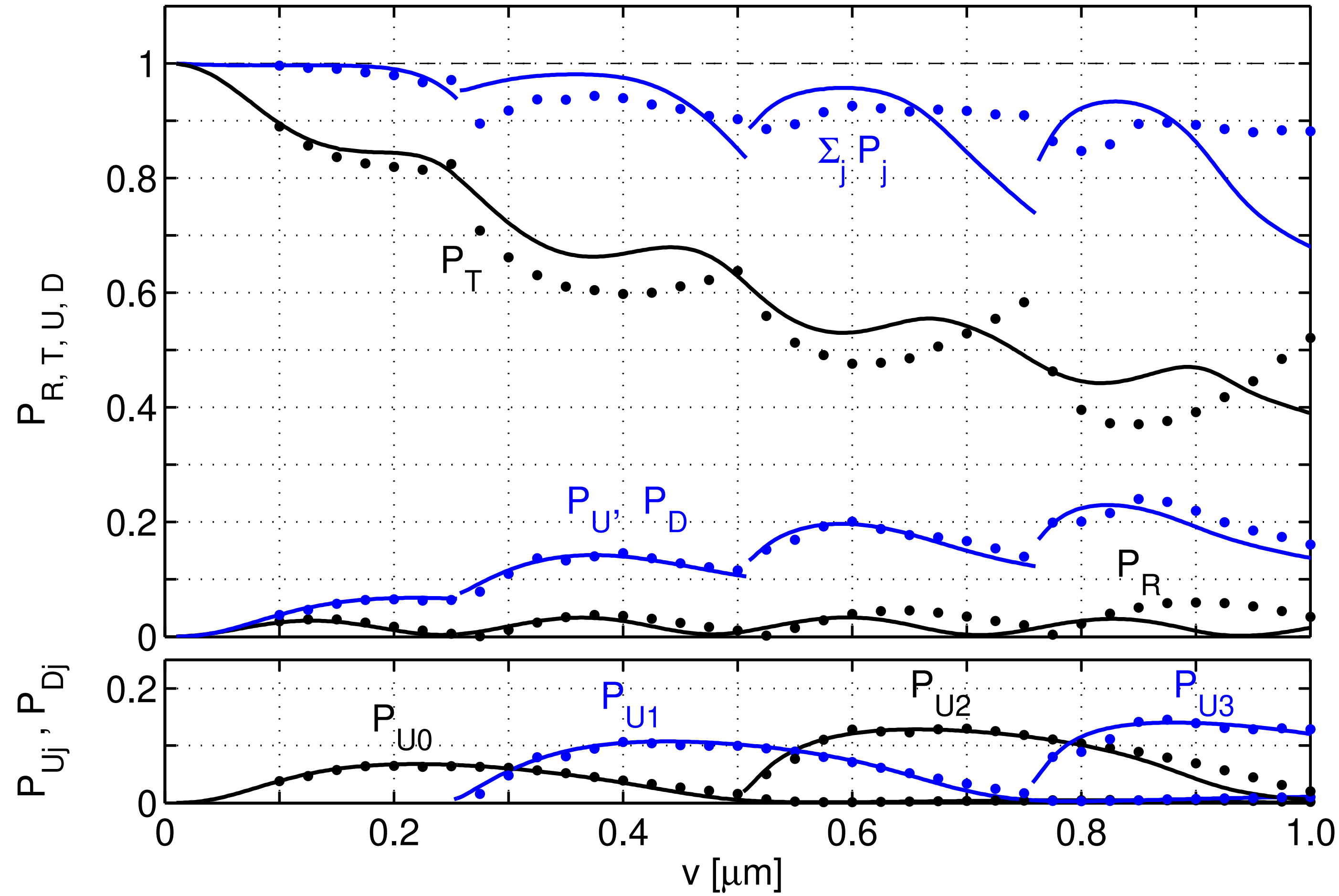
HCMT basis fields:
guided modes
$+$ 4 Gaussian beams,
outgoing along the diagonals.



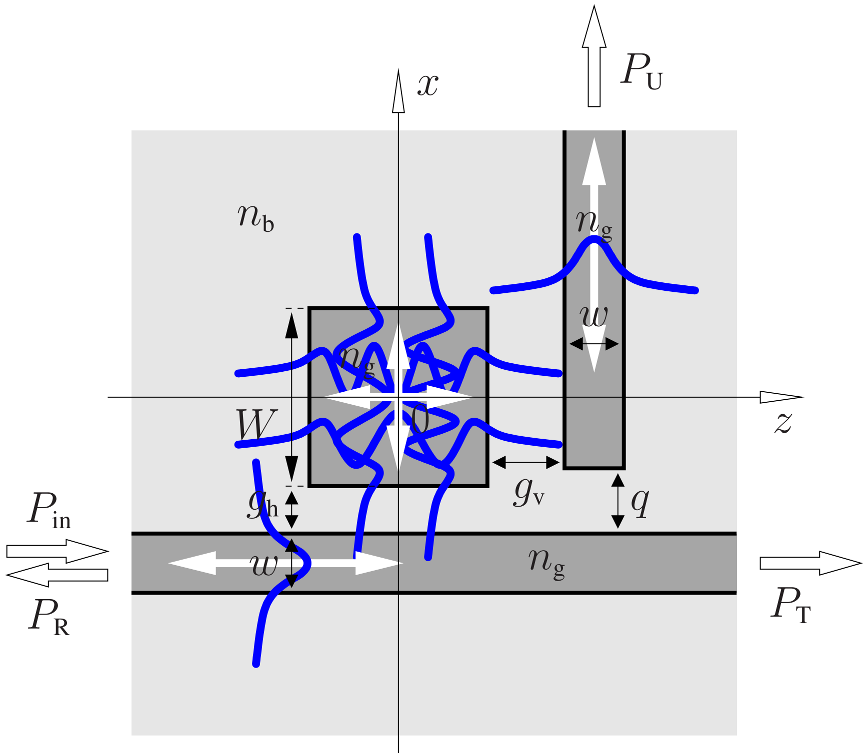


Waveguide crossing, power transfer (II)

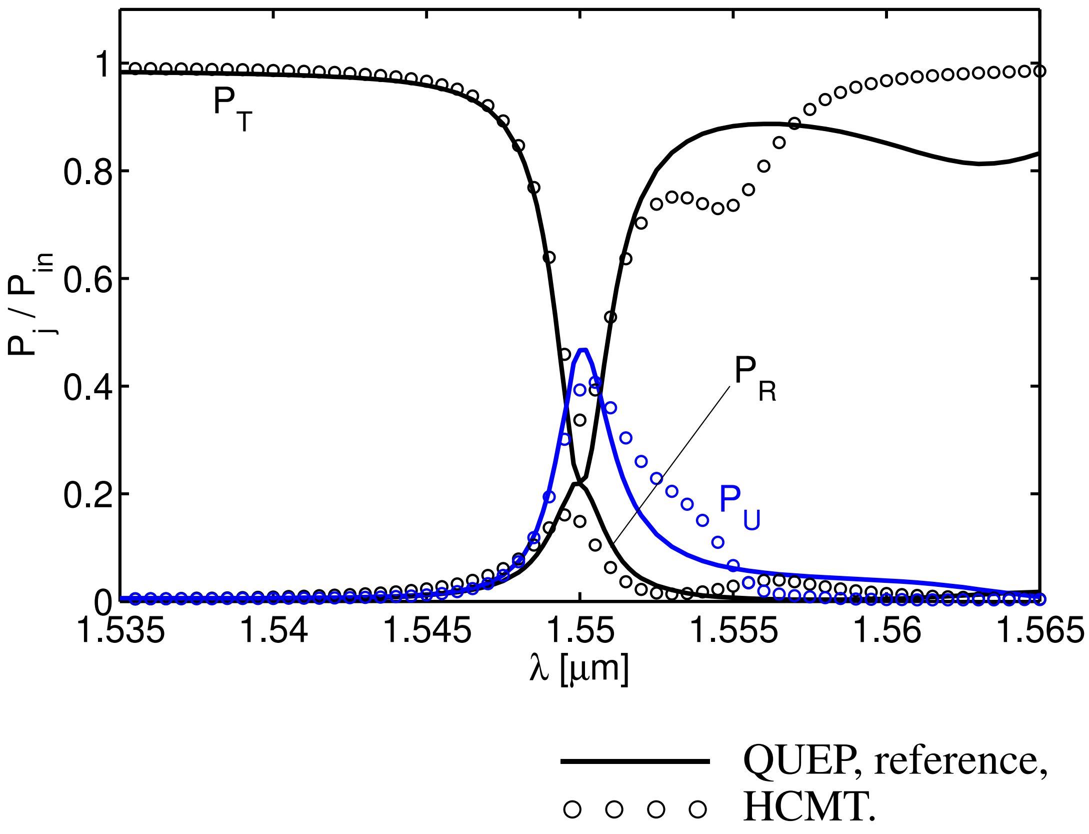




Square resonator

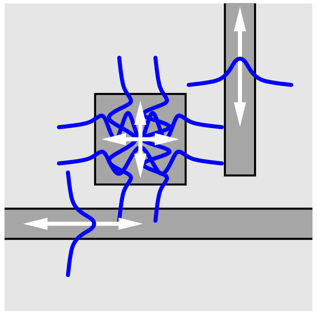


TE, $n_{\mathrm{g}} = 3.4$, $n_{\mathrm{b}} = 1.0$,
$W = 1.786\,\mu\mathrm{m}$, $w = 0.1\,\mu\mathrm{m}$,
$g_{\mathrm{v}} = 0.385\,\mu\mathrm{m}$, $g_{\mathrm{h}} = q = 0.355\,\mu\mathrm{m}$.

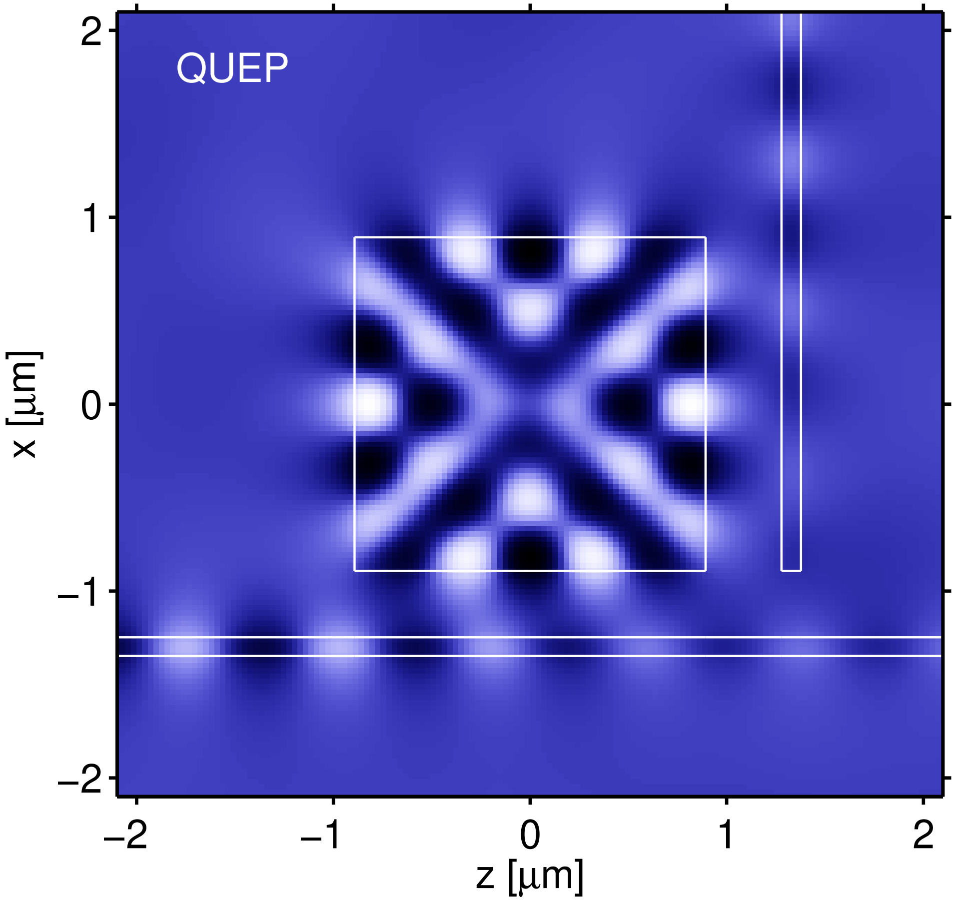


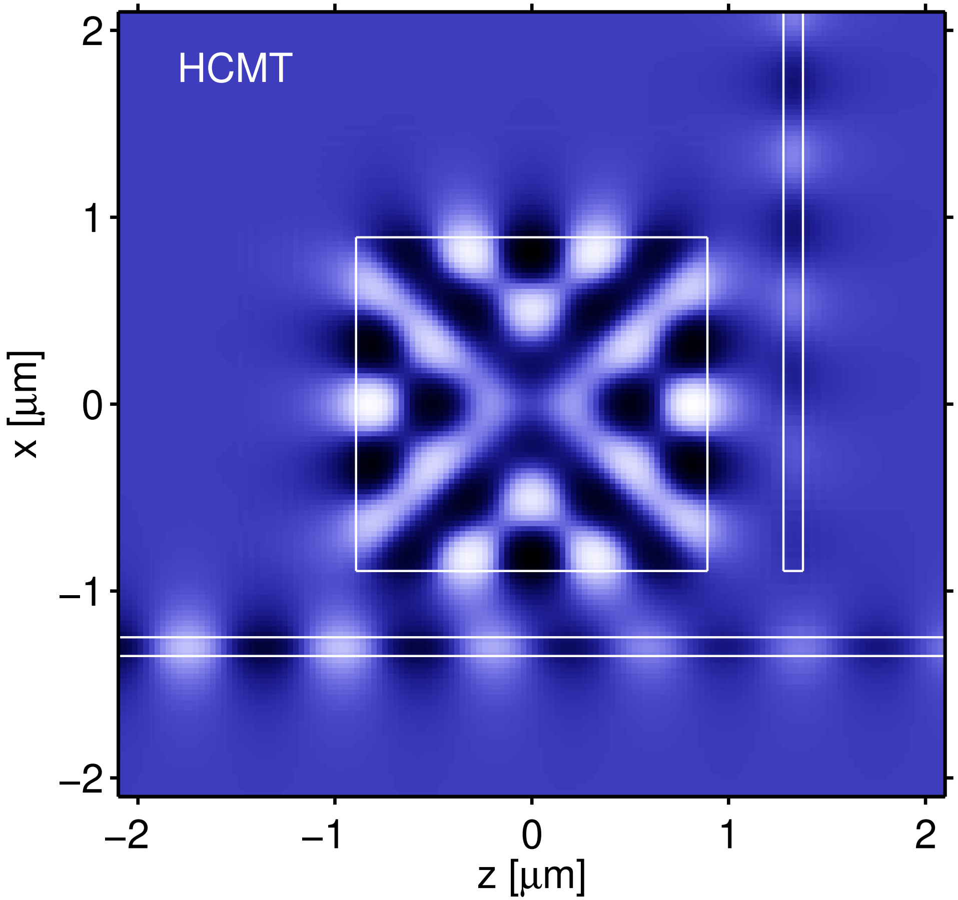
Square cavity, resonance

$\lambda = 1.55\,\mu$m:

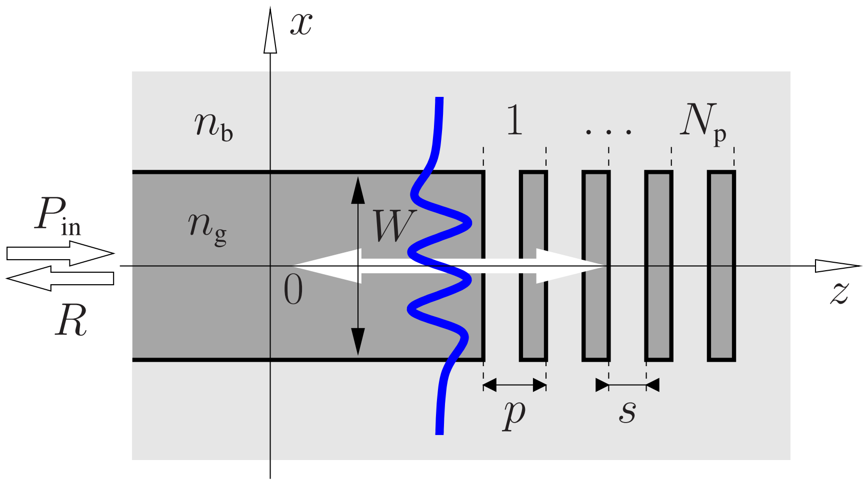


reference

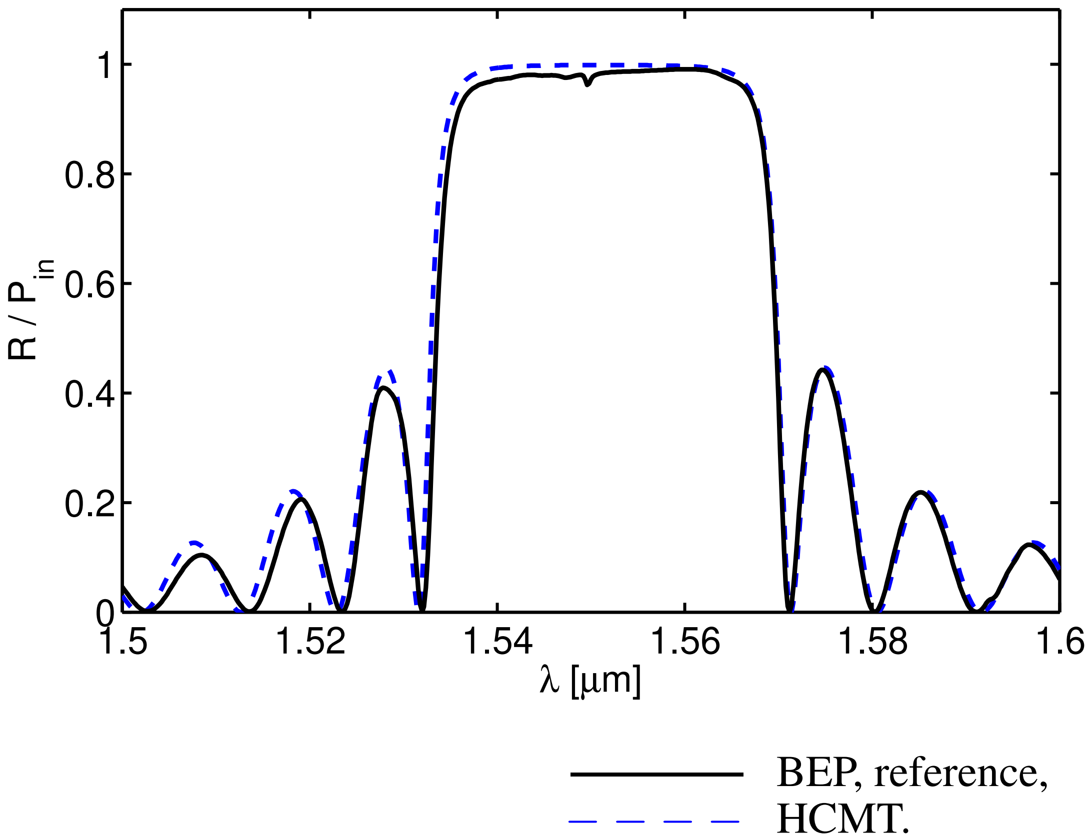




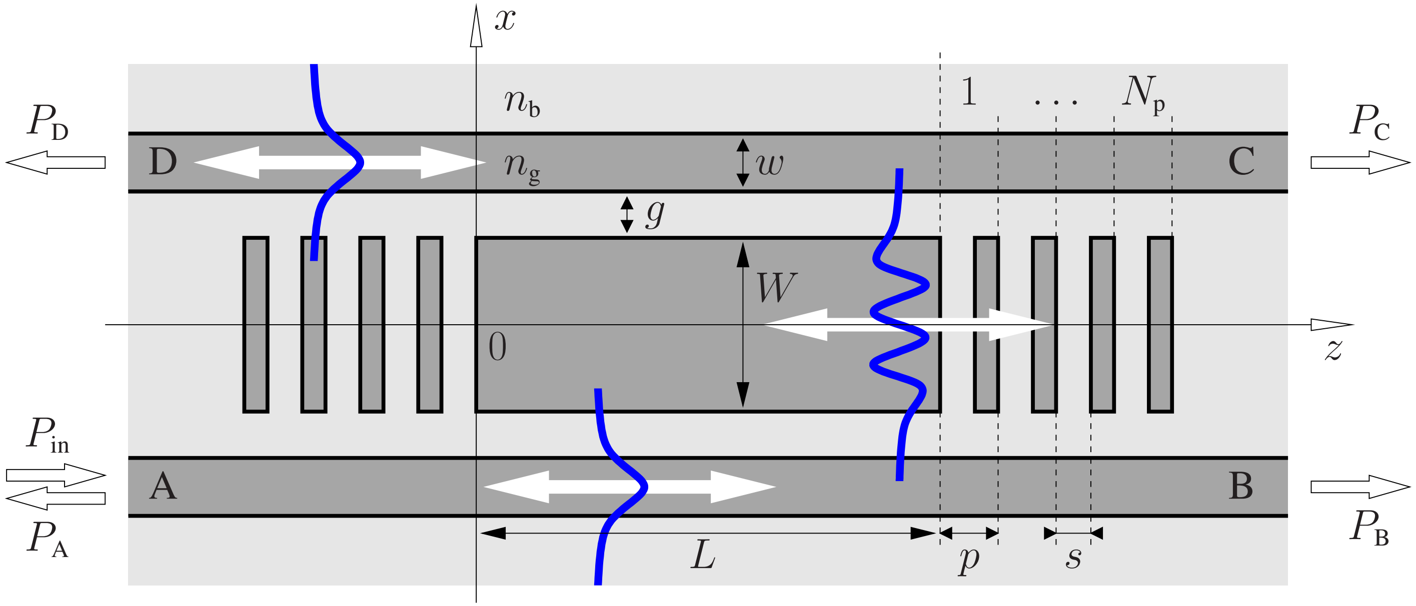
Waveguide Bragg reflector



TE, $n_g = 1.6$, $n_b = 1.45$,
$p = 1.538\,\mu$m, $s = 0.281\,\mu$m,
$N_p = 40$, $W = 9.955\,\mu$m.

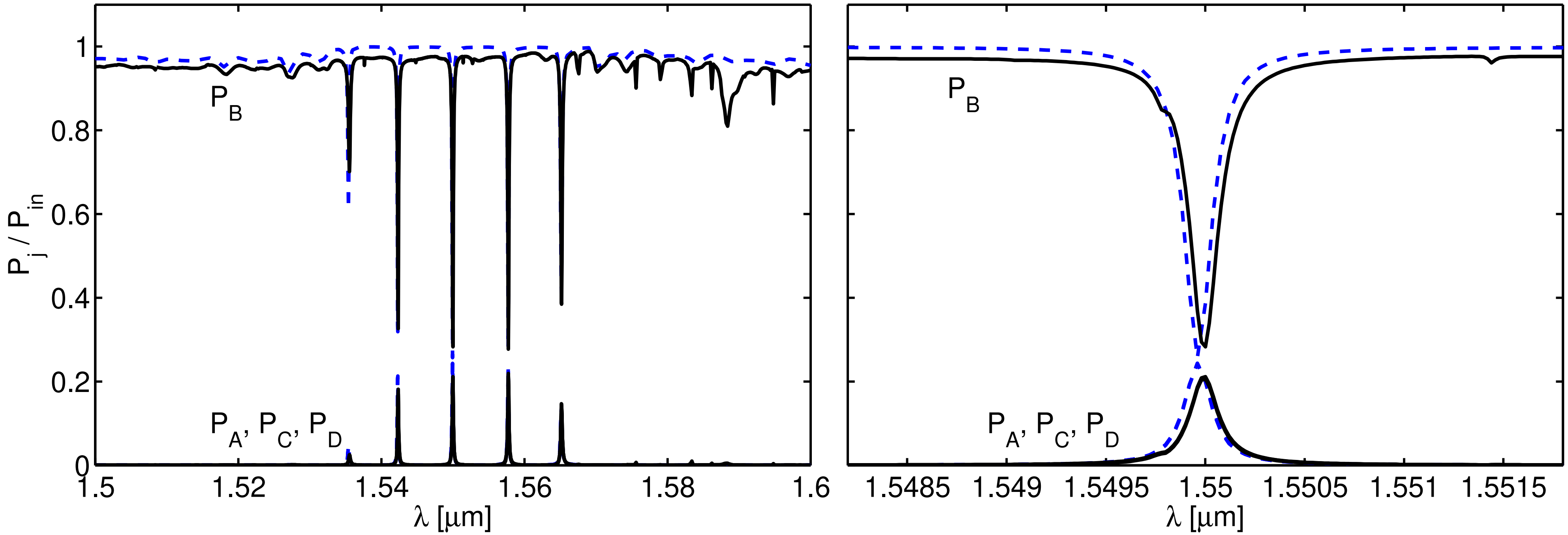


Grating-assisted rectangular resonator



$n_g = 1.60$, $n_b = 1.45$,
$w = 1.0\,\mu$m, $g = 1.6\,\mu$m,
$W = 9.955\,\mu$m,
$L = 79.985\,\mu$m,
$p = 1.538\,\mu$m,
$s = 0.281\,\mu$m,
$N_p = 40$.

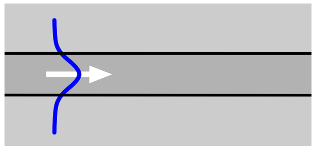
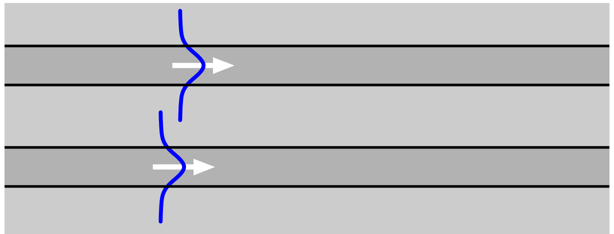
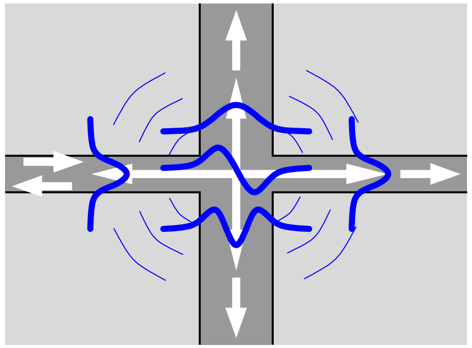
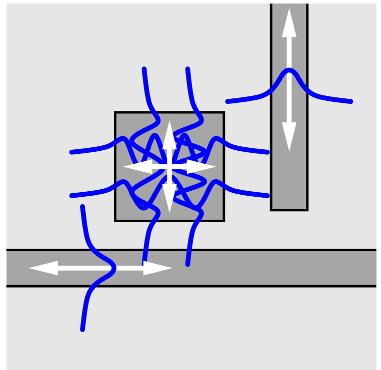
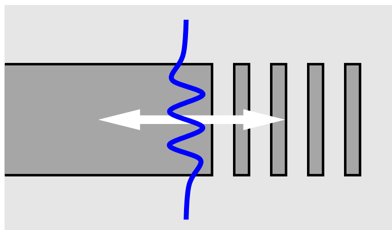
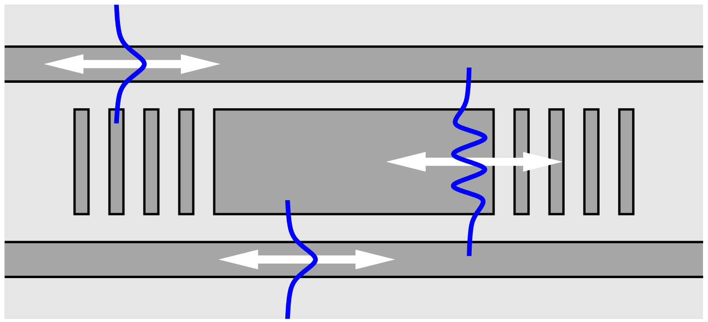
BEP, reference,
HCMT.



Hybrid analytical / numerical coupled mode modeling

HCMT:

- a quantitative, quite general CMT variant, alternatively
- a numerical (FEM) approach with highly specialized base functions,
- extension to 3-D (?): numerical basis fields, still moderate effort,
- reasonably versatile:

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