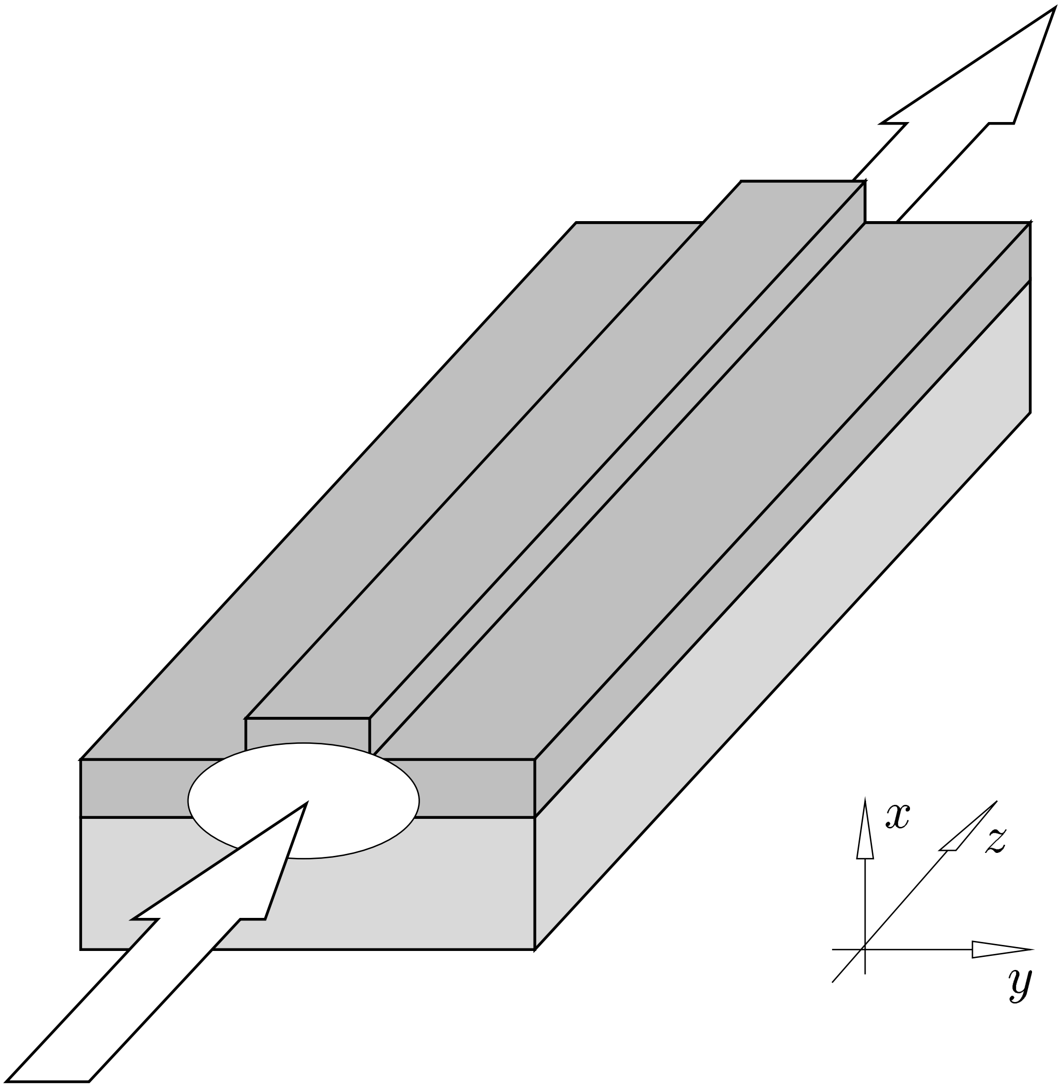
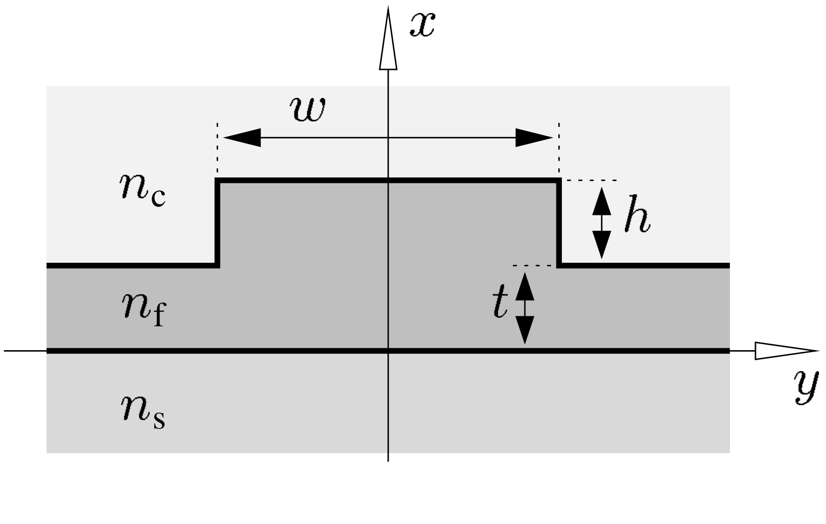




$n_\mathrm{s} = 1.9$
$n_\mathrm{f} = 2.2$
$n_\mathrm{c} = 1.0$
$\lambda = 1.3\,\mu\mathrm{m}$
$t = 0.5\,\mu\mathrm{m}$
$h = 0.5\,\mu\mathrm{m}$
$w = 1.0\,\mu\mathrm{m}$

Guided waves in rectangular integrated magnetooptic devices

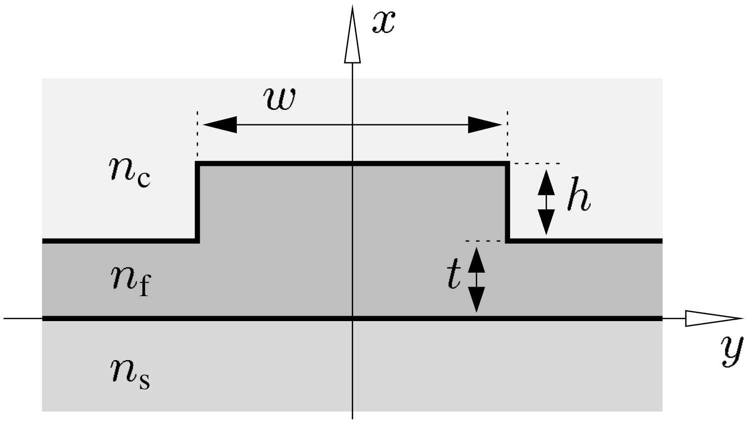
M. Lohmeyer, N. Bahlmann, O. Zhuromskyy, H. Dötsch, P. Hertel*

UNIVERSITÄT OSNABRÜCK

Department of Physics, Osnabrück, Germany

- Rectangular waveguides
- Guided modes
- Wave-matching method
  - Semivectorial calculations
  - Vectorial simulations
- Magnetooptic waveguides
  - Isolator concept
  - Permittivity
  - Coupled mode theory
  - Unidirectional polarization conversion
    - Polarization splitter
    - Circulator device
- Summary

* *Department of Physics, University of Osnabrück* *D–49069 Osnabrück, Germany*
Phone: 0541/969-2641 *Fax: 0541/969-2351* *e-mail: manfred.lohmeyer@physik.uni-osnabrueck.de*



Ansatz: $\epsilon = n^2, \quad k = \omega/c = 2\pi/\lambda$

$$\boldsymbol{\mathcal{E}}(x,y,z,t) = \begin{pmatrix} E_x \\ E_y \\ \mathrm{i}E_z \end{pmatrix}(x,y)\,\mathrm{e}^{\mathrm{i}(\omega t - \beta z)}\,,$$
$$\boldsymbol{\mathcal{H}}(x,y,z,t) = \begin{pmatrix} H_x \\ H_y \\ \mathrm{i}H_z \end{pmatrix}(x,y)\,\mathrm{e}^{\mathrm{i}(\omega t - \beta z)}\,,$$

$E_x, \ldots, H_z$: real mode profile components,
β: propagation constants.

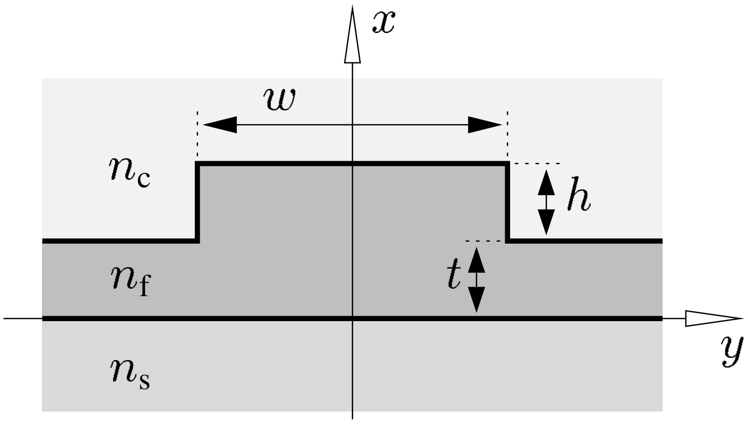
Guided modes:

$$P = \frac{1}{2}\iint (E_x H_y - E_y H_x)\,\mathrm{d}x\,\mathrm{d}y < \infty\,.$$

Mode equations: $X = \frac{1}{\epsilon}(\partial_x \epsilon), \quad Y = \frac{1}{\epsilon}(\partial_y \epsilon),$

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_x + \partial_x X E_x + \partial_x Y E_y = 0\,,$$

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_y + \partial_y X E_x + \partial_y Y E_y = 0\,.$$



Mode equations for locally constant permittivity:

$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_x = 0$$

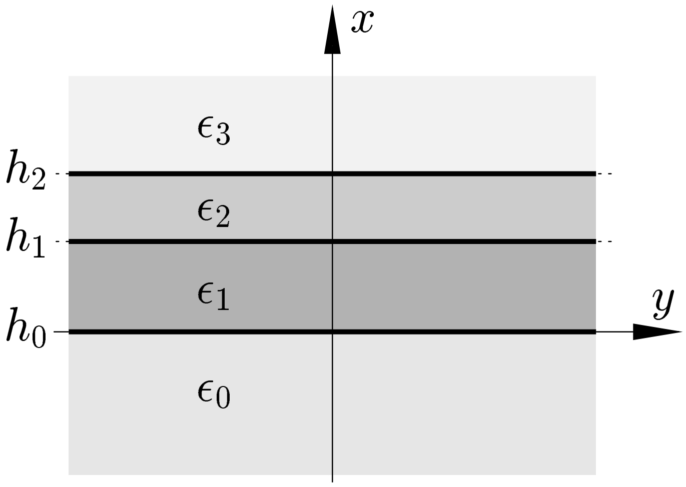
$$(\partial_x^2 + \partial_y^2 + k^2\epsilon - \beta^2)E_y = 0$$

Continuity conditions:

orientation of the boundary	continuous quantities
horizontal	ϵE_x, E_y, E_z, H_x, H_y, H_z
vertical	E_x, ϵE_y, E_z, H_x, H_y, H_z

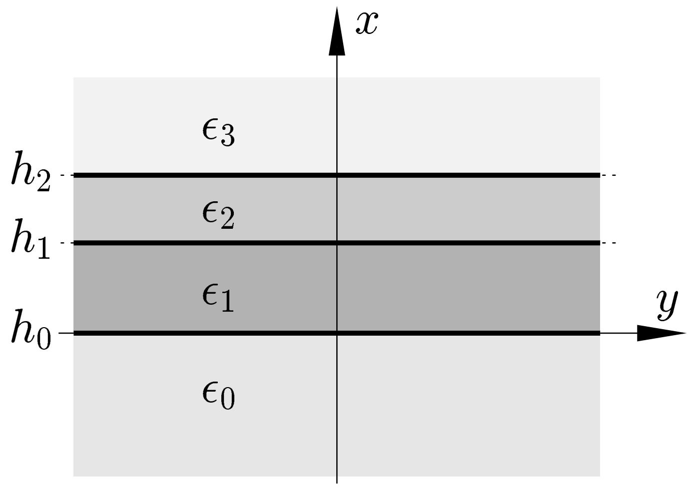
$$\begin{pmatrix} E_x \\ E_y \\ \mathrm{i}E_z \end{pmatrix} = \begin{pmatrix} E_x \\ E_y \\ -\mathrm{i}\beta^{-1}(\partial_x E_x + \partial_y E_y) \end{pmatrix}$$

$$\begin{pmatrix} H_x \\ H_y \\ \mathrm{i}H_z \end{pmatrix} = \frac{1}{\omega\mu_0} \begin{pmatrix} -\beta E_y + \beta^{-1}(\partial_{xy}E_x + \partial_y^2 E_y) \\ \beta E_x - \beta^{-1}(\partial_x^2 E_x + \partial_{xy}E_y) \\ \mathrm{i}(\partial_x E_y - \partial_y E_x) \end{pmatrix}$$



$$\partial_y \epsilon = 0, \quad \partial_y E_x = 0, \ldots, \partial_y H_z = 0$$

TE Modes	**TM Modes**
$\boldsymbol{E} = \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix}$	$\boldsymbol{E} = \dfrac{1}{\omega \epsilon_0 \epsilon} \begin{pmatrix} \beta H_y \\ 0 \\ -\mathrm{i}\partial_x H_y \end{pmatrix}$
$\boldsymbol{H} = \dfrac{1}{\omega \mu_0} \begin{pmatrix} -\beta E_y \\ 0 \\ \mathrm{i}\partial_x E_y \end{pmatrix}$	$\boldsymbol{H} = \begin{pmatrix} 0 \\ H_y \\ 0 \end{pmatrix}$
$(\partial_x^2 + k^2 \epsilon - \beta^2) E_y = 0$	$(\partial_x^2 + k^2 \epsilon - \beta^2) H_y = 0$
E_y, $\partial_x E_y$ continuous at $x = h_j$	H_y, $\dfrac{1}{\epsilon}\partial_x H_y$ continuous at $x = h_j$
$P = \dfrac{\beta}{2\omega\mu_0} \displaystyle\int E_y^2 \,\mathrm{d}x < \infty$	$P = \dfrac{\beta}{2\omega\epsilon_0} \displaystyle\int \dfrac{1}{\epsilon} H_y^2 \,\mathrm{d}x < \infty$



$$\partial_x^2 \phi = (\beta^2 - k^2 \epsilon_l)\phi, \quad l = 0, \dots, N+1$$

- Ansatz: For $h_{l-1} < x < h_l$ choose

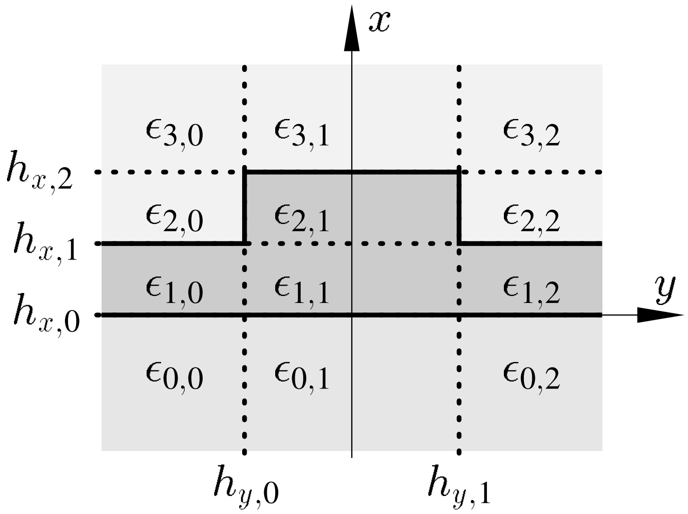
$$\phi(x) = \begin{cases} a_l \exp(q_l x) + b_l \exp(-q_l x), & \text{if } \beta^2 > k^2 \epsilon_l\,, \\ a_l \sin(q_l x) + b_l \cos(q_l x), & \text{otherwise}\,, \end{cases}$$

with $q_l = \sqrt{|\beta^2 - k^2 \epsilon_l|}$ and $\beta^2 > k^2 \epsilon_0$, $\beta^2 > k^2 \epsilon_{N+1}$,
$b_0 = 0$, $a_{N+1} = 0$.

- Continuity requirements lead to a system of linear equations:

$$\mathsf{M}(\beta)\, \boldsymbol{a} = 0, \quad \boldsymbol{a} = (a_0, a_1, b_1, \dots, a_N, b_N, b_{N+1})^{\mathsf{T}}\,.$$

- Solutions $\boldsymbol{a} \neq 0$ exist only for discrete values $\beta \dots$



$$(\partial_x^2 + \partial_y^2)\phi = (\beta^2 - k^2\epsilon_{lm})\phi, \quad \begin{array}{l} l = 0, \ldots, N_x + 1 \\ m = 0, \ldots, N_y + 1 \end{array}$$

- Ansatz:
  - separately for two basic components
  - separately for each rectangle

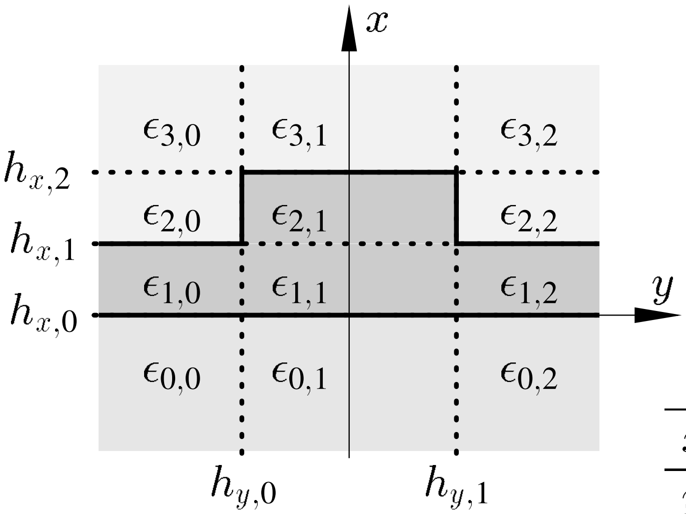
$$\phi(x, y) = \sum_j a_j F_j(p_j x) G_j(q_j y).$$

$F, G \in \{\sin, \cos, \exp\}$
p, q: transverse wavevector-components with

$$\pm p^2 \pm q^2 = \beta^2 - k^2\epsilon\,.$$

- Spectral discretization:
  For each combination of F and G select a number of parameters $\{(p_j, q_j)\}$, such that the trial functions $F(p_j x)\, G(q_j y)$
  - are representative for that set of functions,
  - are numerically linearly independent.

- On unbounded rectangles:
  Consider only outwards exponentially decaying trial functions.



	Continuous quantities
$x = h_{x,l}$	$\epsilon E_x,\ E_y,\ E_z,\ H_x,\ H_y,\ H_z$
$y = h_{y,m}$	$E_x,\ \epsilon E_y,\ E_z,\ H_x,\ H_y,\ H_z$

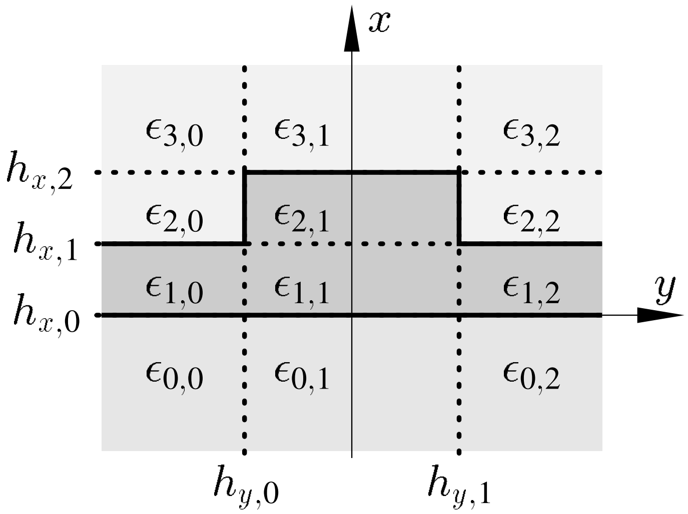
Least squares mismatch on the rectangle boundaries:

$$D_\beta(\boldsymbol{a}) = \sum_{\substack{\text{continuous}\\\text{quantities}}} \sum_{\substack{\text{horizontal}\\\text{/ vertical}\\\text{boundary}\\\text{segments}}} \int_{\substack{\text{boundary}\\\text{line}}} \begin{pmatrix}\text{suitable}\\\text{weighting}\\\text{factors}\end{pmatrix} \bullet \left| \begin{matrix}\text{difference}\\\text{in fields /}\\\text{derivatives}\end{matrix} \right|^2 \mathrm{d}s$$

Constraint: normalized fields

$$N_\beta(\boldsymbol{a}) = \sum_{\text{rectangles}} \iint_{\substack{\text{rectangle}\\\text{domain}}} \begin{pmatrix}\text{suitable}\\\text{weighting}\\\text{factors}\end{pmatrix} \bullet \left| \quad \text{field} \quad \right|^2 \mathrm{d}x\,\mathrm{d}y$$

$$\boxed{\text{Guided modes:}\quad (\beta, \boldsymbol{a}) \text{ with } D_\beta(\boldsymbol{a}) = 0 \text{ and } N_\beta(\boldsymbol{a}) = 1.}$$



- D_β and N_β are quadratic forms:

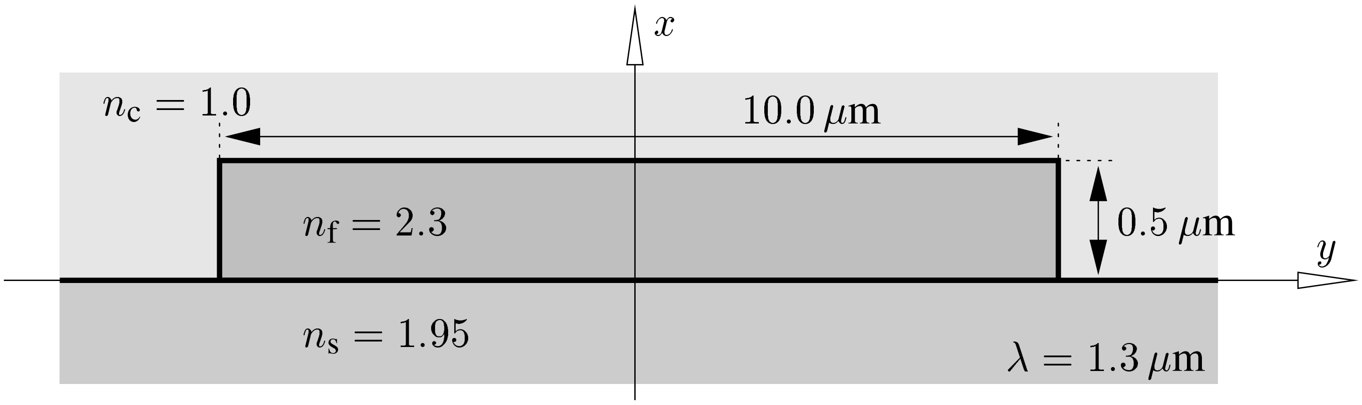
$$D_\beta(\boldsymbol{a}) = \boldsymbol{a}^\mathsf{T} \mathsf{D}_\beta \boldsymbol{a}\,, \qquad N_\beta(\boldsymbol{a}) = \boldsymbol{a}^\mathsf{T} \mathsf{N}_\beta \boldsymbol{a}\,.$$

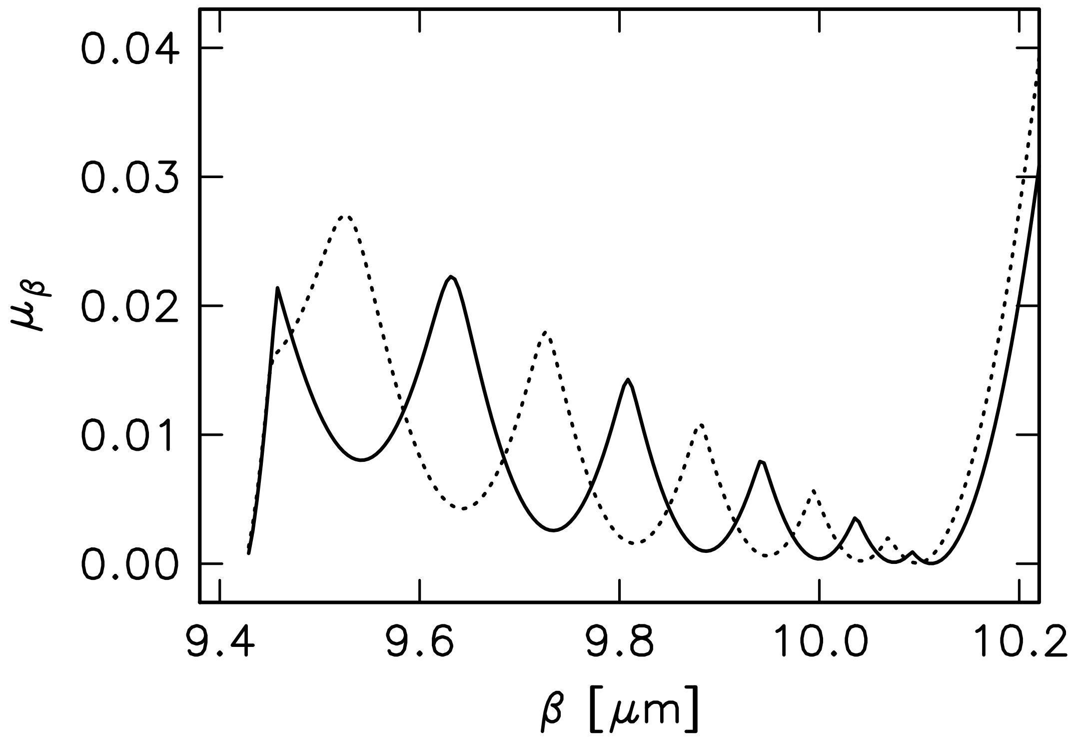
- For fixed β find the set of coefficients $\boldsymbol{a}$ with minimum error μ_β
  $\Leftrightarrow$ solve the eigenvalue equation

$$\mathsf{D}_\beta \boldsymbol{a} = \mu_\beta \mathsf{N}_\beta \boldsymbol{a}\,.$$

- Minima in μ_β with respect to β
  $\leadsto$ approximations of valid propagation constants.

$\longrightarrow$ Implementation

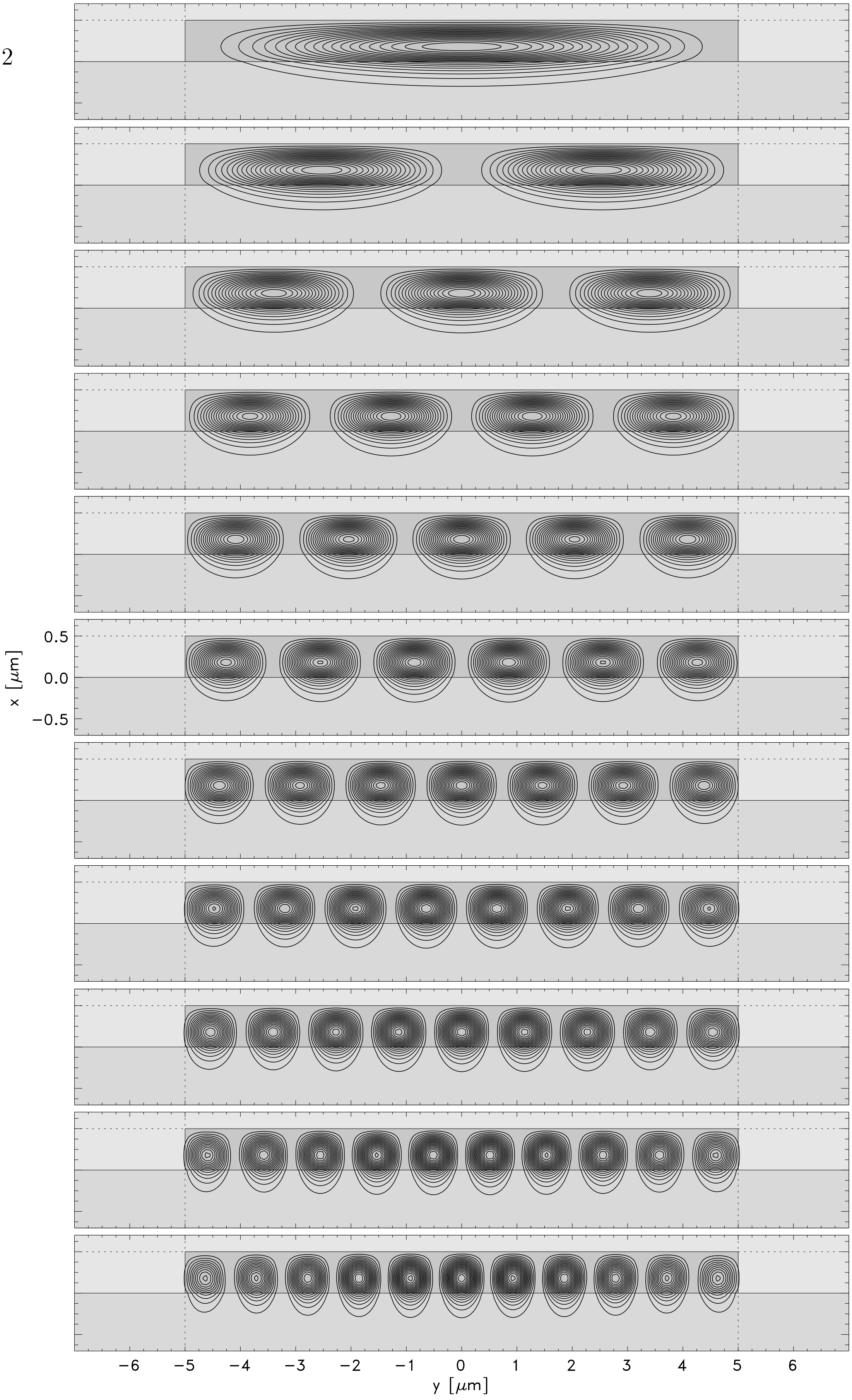


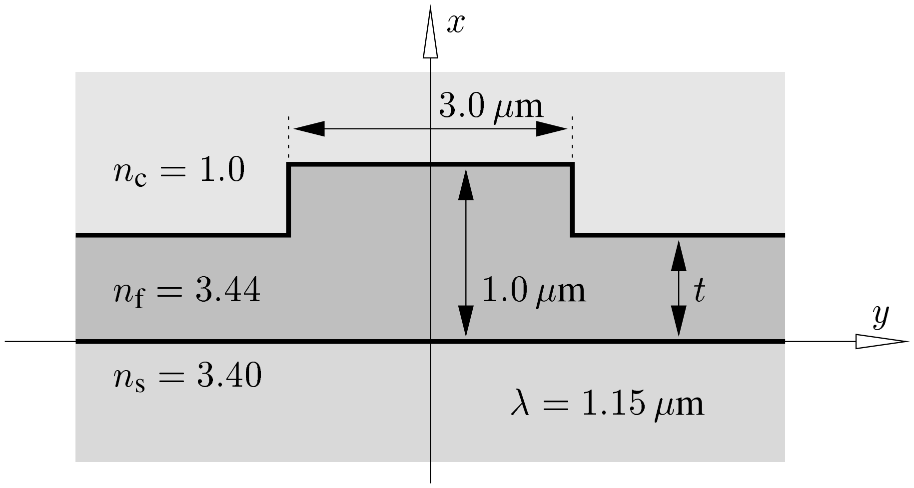


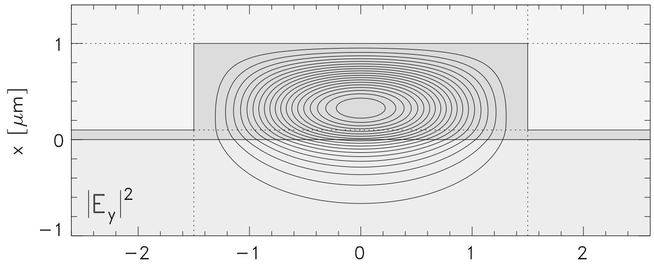
——— TM, symmetrical field $H_y(x, y) = H_y(x, -y)$

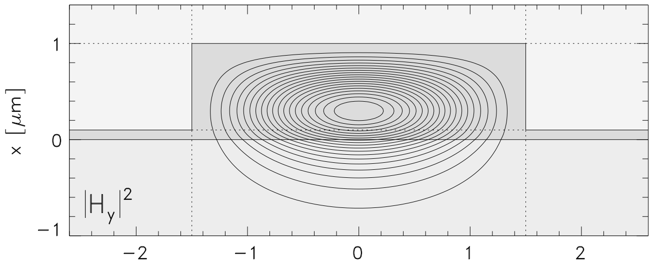
· · · · · · · · · · · · TM, antisymmetrical field $H_y(x, y) = -H_y(x, -y)$

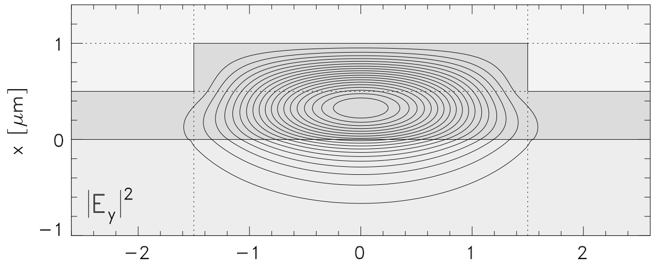
TM, $|H_y|^2$

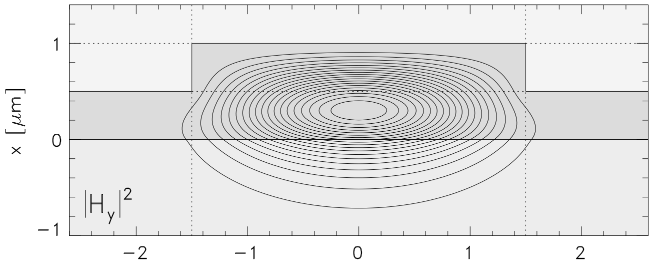


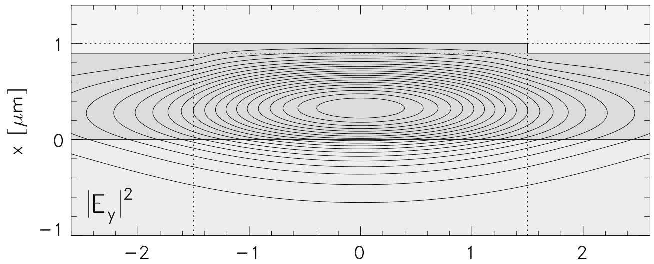


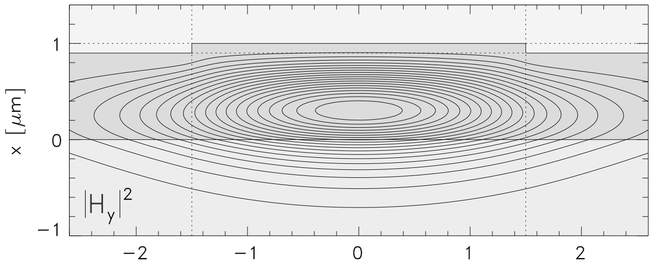

t = 0.1 μm, TE


t = 0.1 μm, TM


t = 0.5 μm, TE


t = 0.5 μm, TM

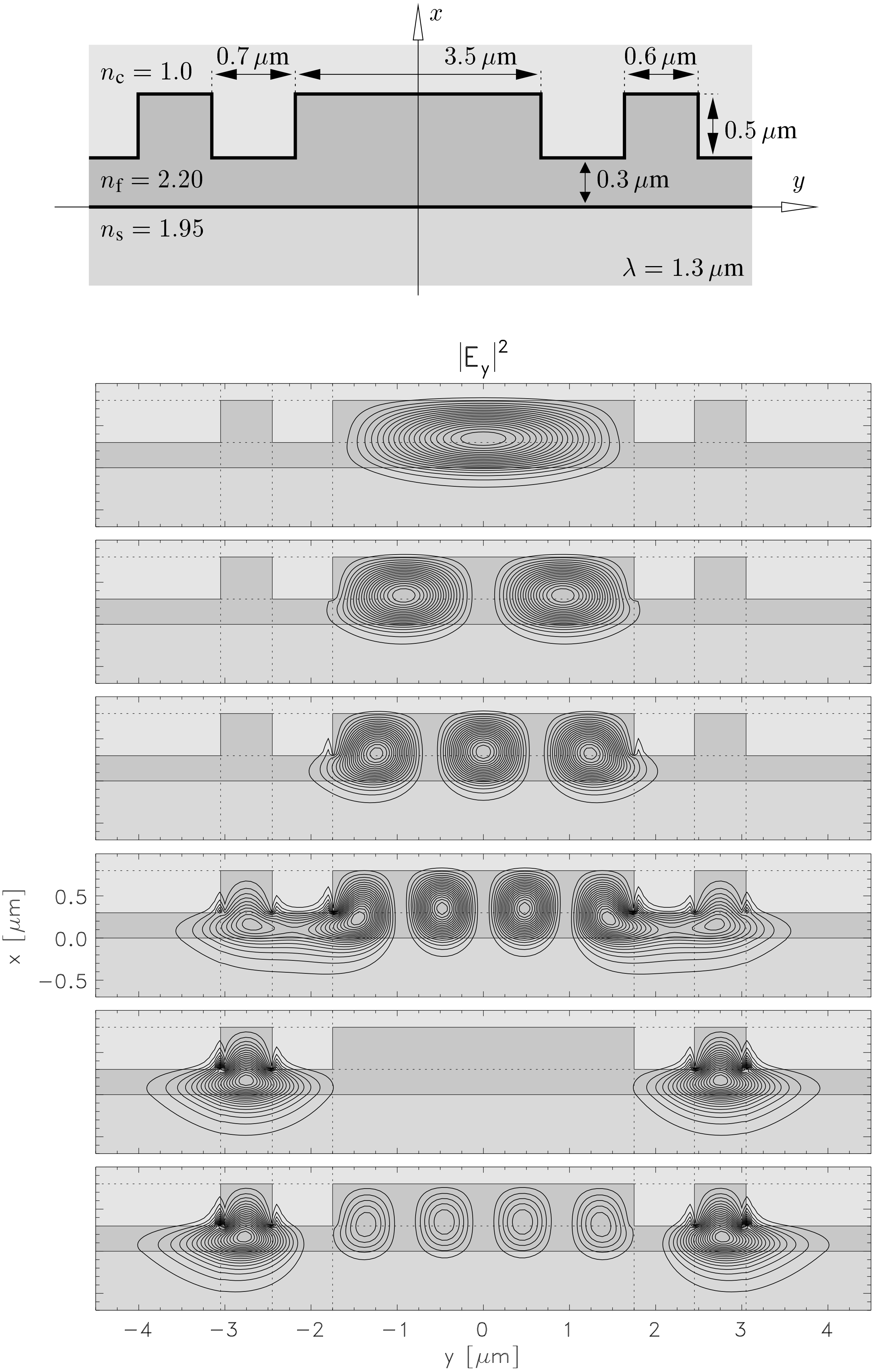

t = 0.9 μm, TE

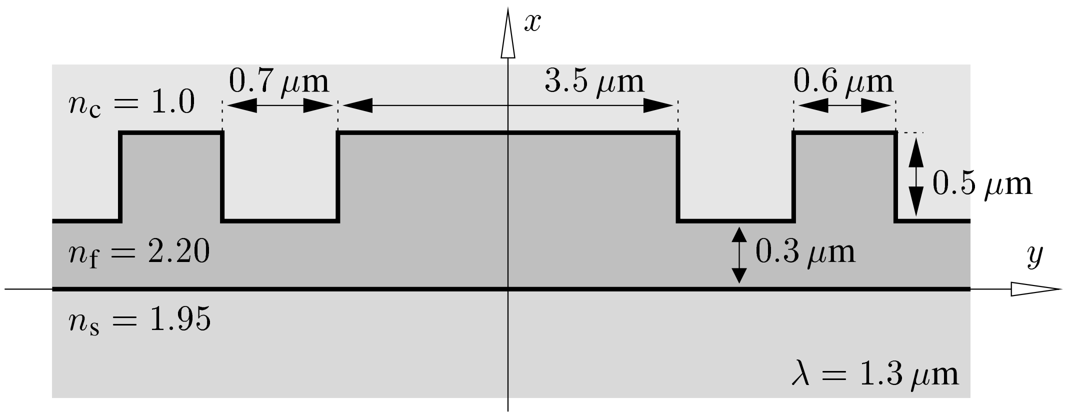

t = 0.9 μm, TM

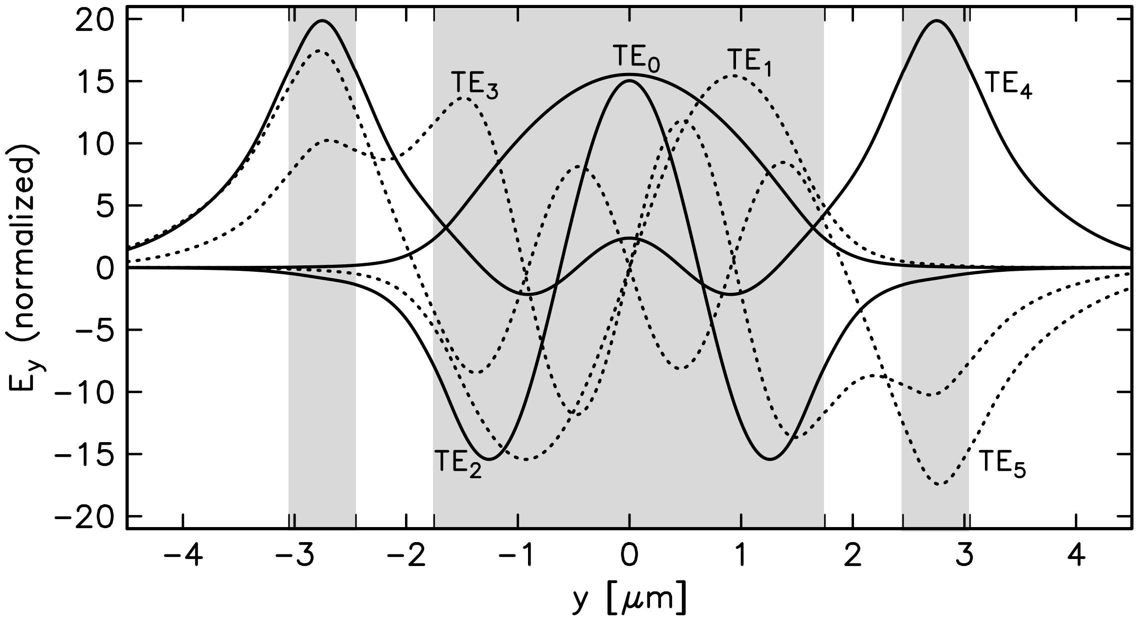
$$\frac{(\beta/k)^2 - n_s^2}{n_f^2 - n_s^2}$$

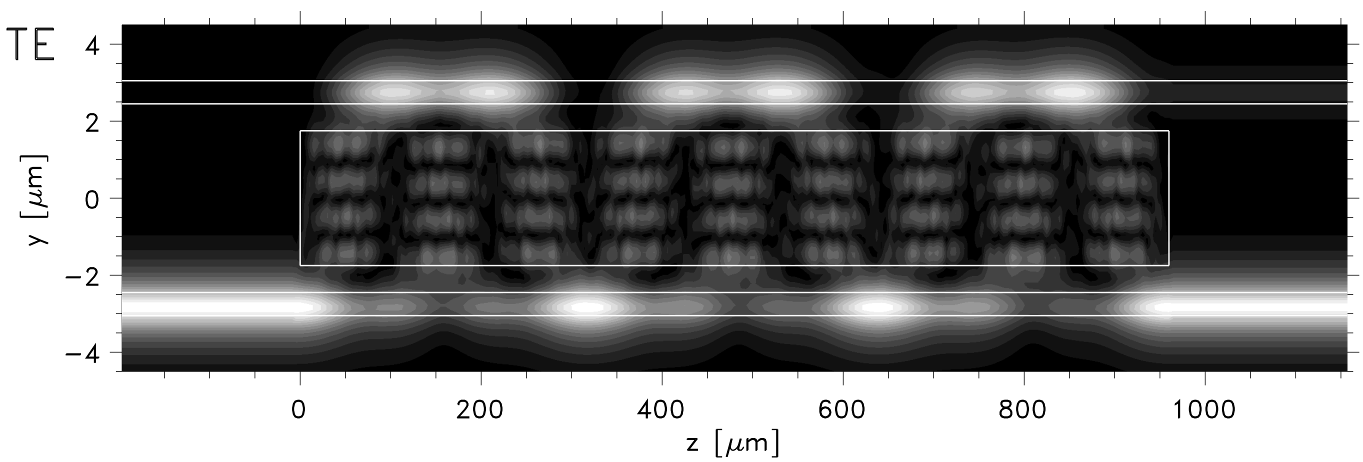
t/μm		0.1	0.3	0.5	0.7	0.9
TE	MTRM	0.3019	0.3110	0.3270	0.3512	0.3883
	WMM	0.3024	0.3112	0.3268	0.3508	0.3881
TM	MTRM	0.2674	0.2751	0.2890	0.3107	0.3455
	WMM	0.2676	0.2759	0.2892	0.3102	0.3451

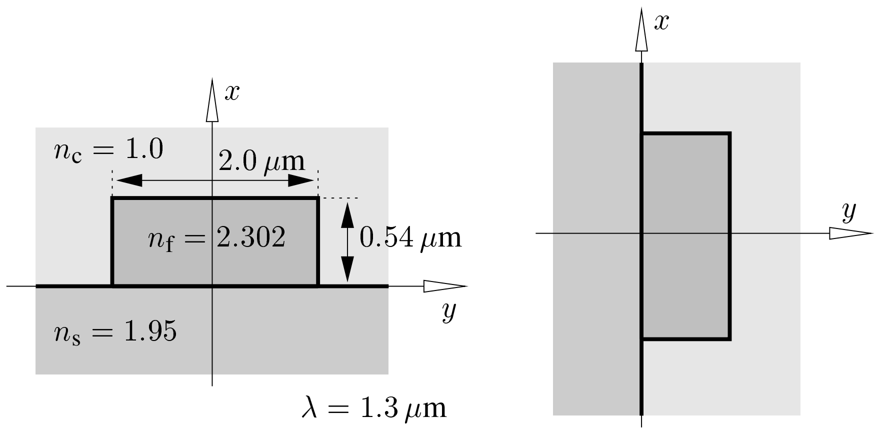
MTRM: Modal Transverse Resonance Method by A. S. Sudbø;
Data from C. Vassallo, *Optical and Quantum Electronics* **29**, 1997

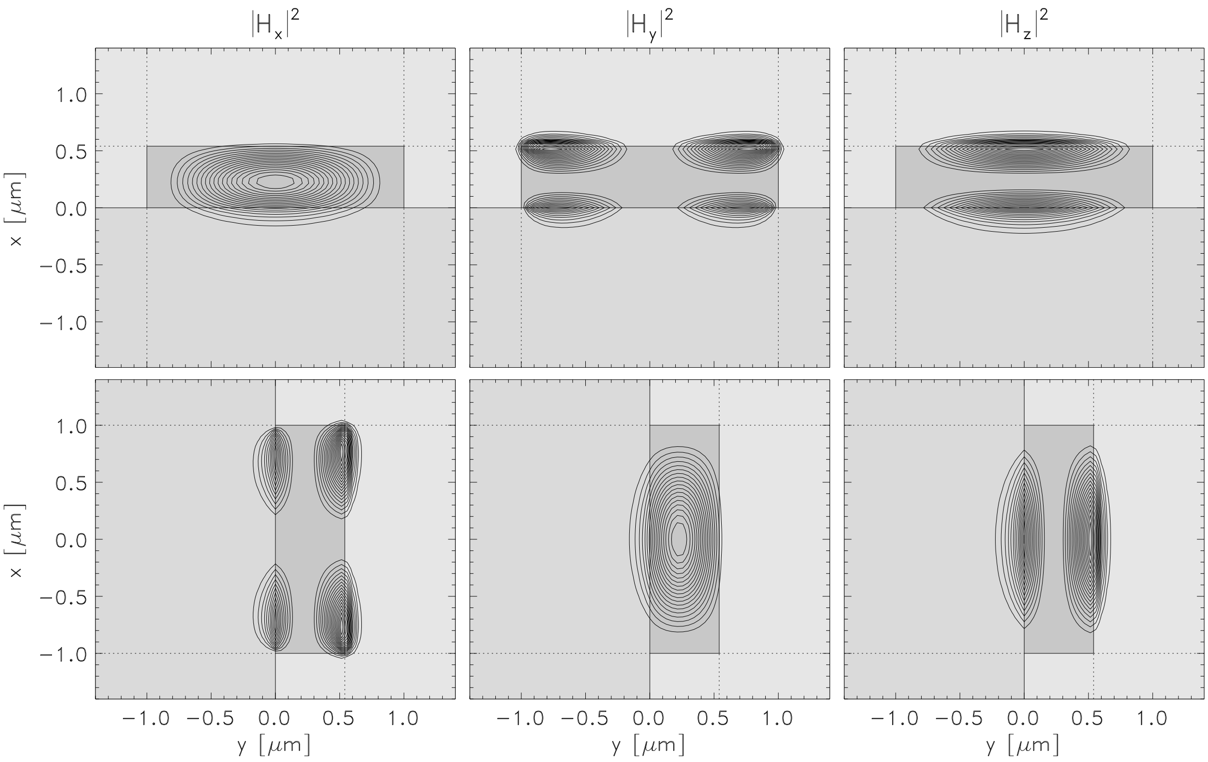

x
$n_c = 1.0$
$0.7\,\mu$m
$3.5\,\mu$m
$0.6\,\mu$m
$0.5\,\mu$m
$n_f = 2.20$
$0.3\,\mu$m
y
$n_s = 1.95$
$\lambda = 1.3\,\mu$m
$|E_y|^2$
0.5
0.0
−0.5
x [μm]
−4 −3 −2 −1 0 1 2 3 4
y [μm]

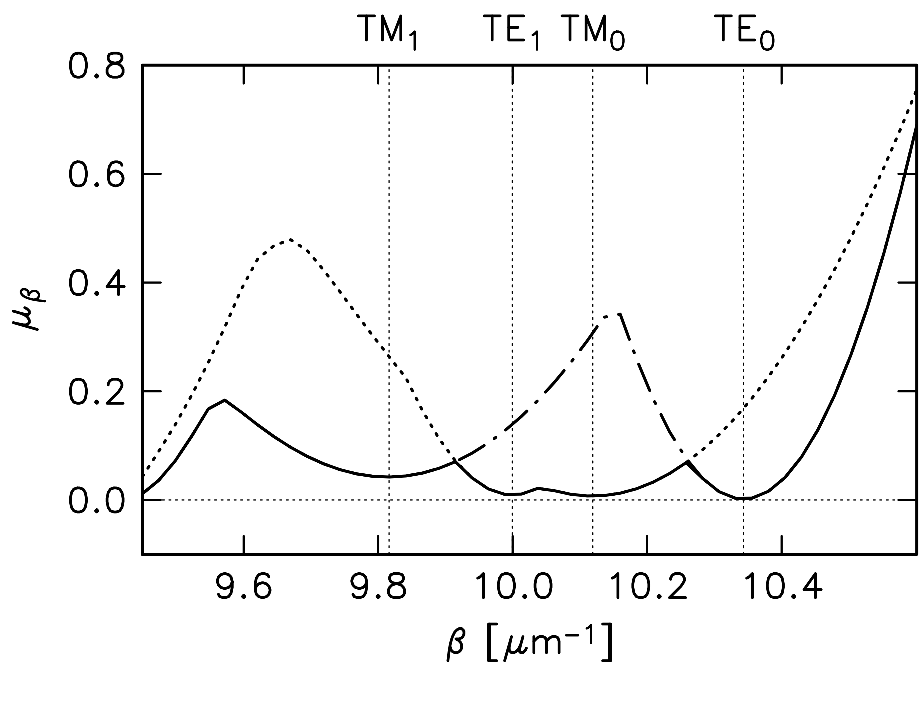

x
$n_c = 1.0$
$0.7\,\mu$m
$3.5\,\mu$m
$0.6\,\mu$m
$0.5\,\mu$m
$n_f = 2.20$
$0.3\,\mu$m
y
$n_s = 1.95$
$\lambda = 1.3\,\mu$m

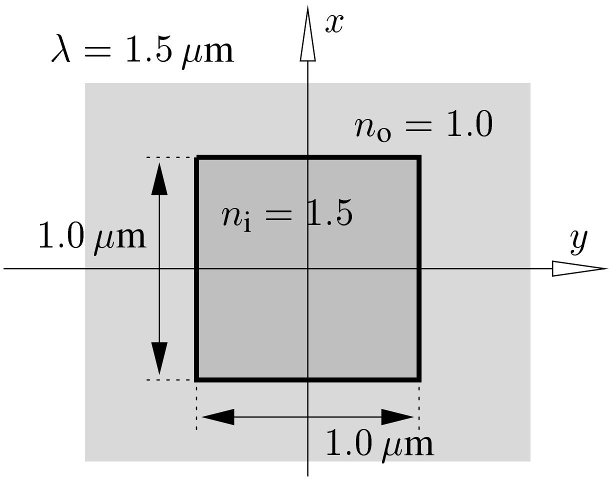

E_y (normalized)
y [μm]
TE_0
TE_1
TE_2
TE_3
TE_4
TE_5

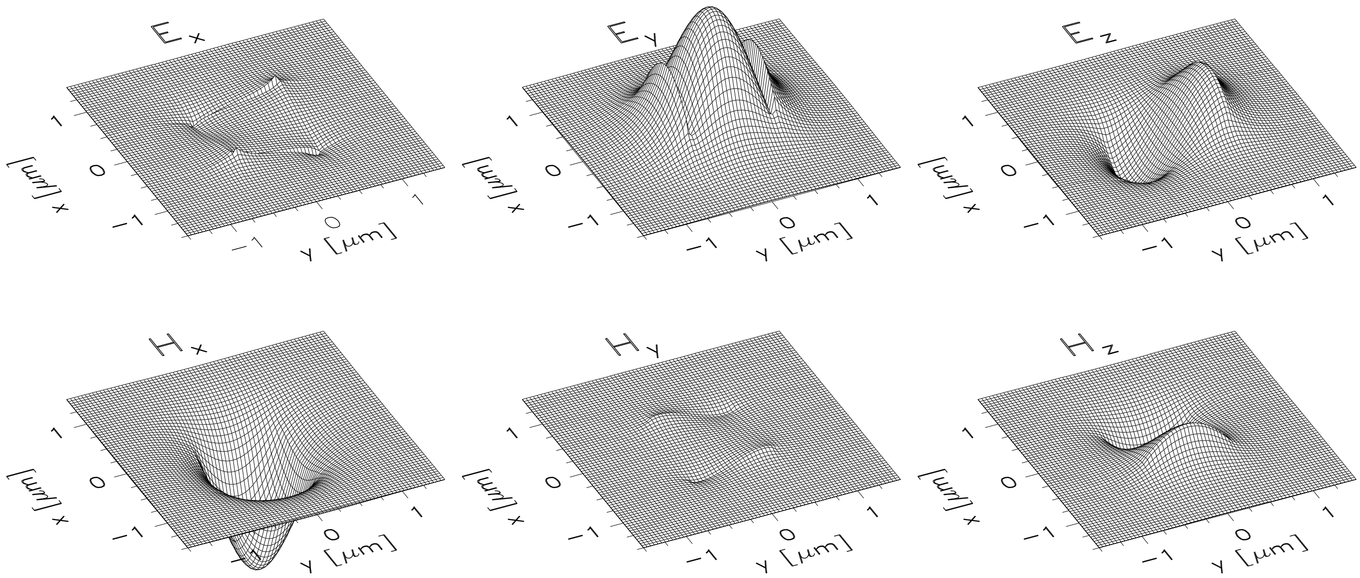

TE
y [μm]
z [μm]

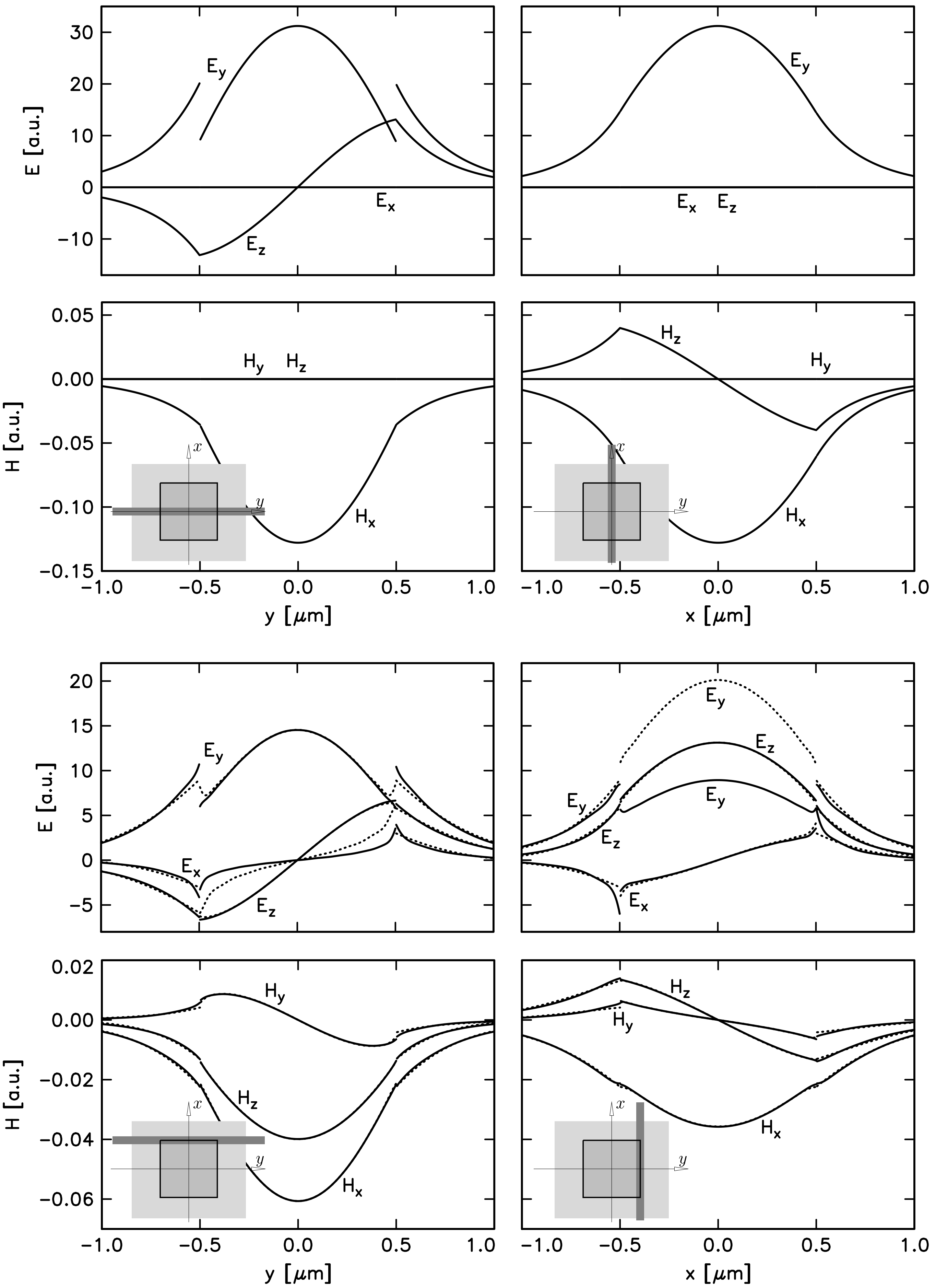

x
$n_c = 1.0$
$2.0\,\mu$m
$n_f = 2.302$
$0.54\,\mu$m
y
$n_s = 1.95$
$\lambda = 1.3\,\mu$m
x
y

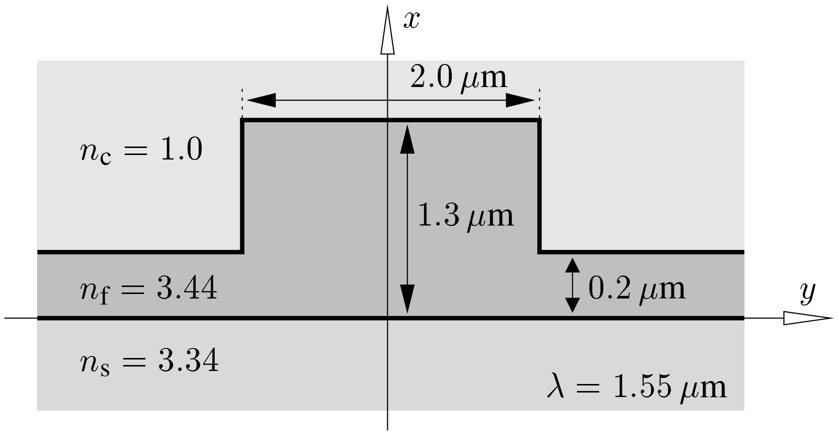

$|H_x|^2$
$|H_y|^2$
$|H_z|^2$
x [μm]
y [μm]


TM_1
TE_1
TM_0
TE_0
μ_β
β [μm^{-1}]


$\lambda = 1.5\,\mu$m
x
$n_o = 1.0$
$n_i = 1.5$
$1.0\,\mu$m
y
$1.0\,\mu$m

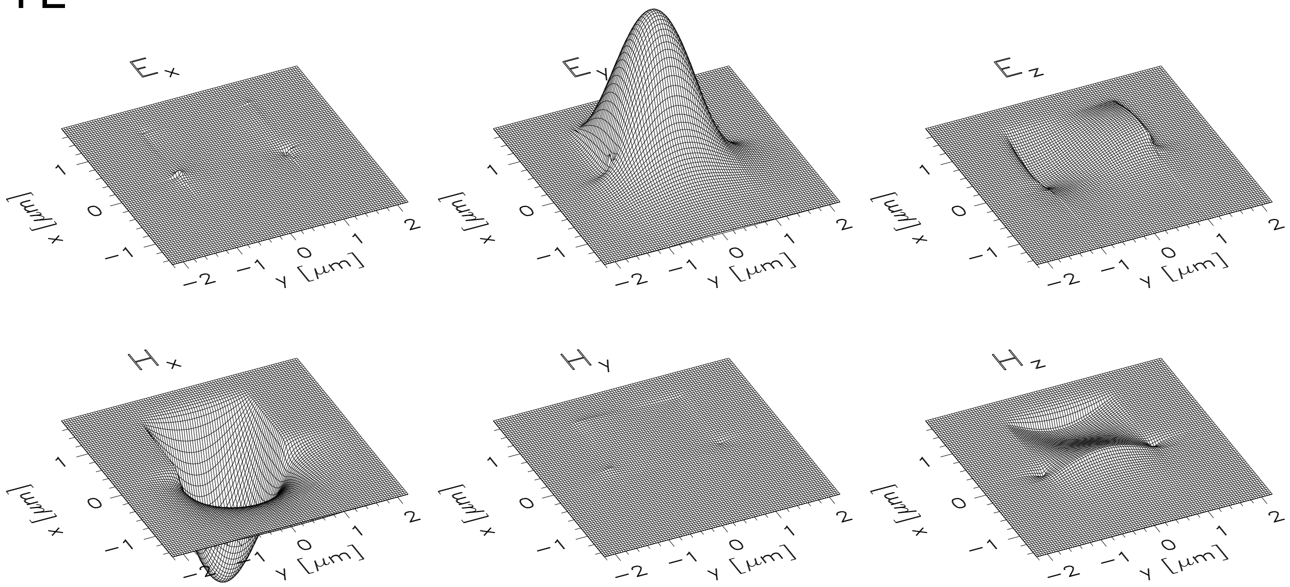

E_x
E_y
E_z
H_x
H_y
H_z
x [μm]
y [μm]


E [a.u.]
H [a.u.]
y [μm]
x [μm]
E_y
E_x
E_z
E_x E_z
H_y H_z
H_x
H_y
H_z
x
y
−1.0
−0.5
0.0
0.5
1.0
30
20
10
0
−10
0.05
0.00
−0.05
−0.10
−0.15
15
5
−5
0.02
−0.02
−0.04
−0.06

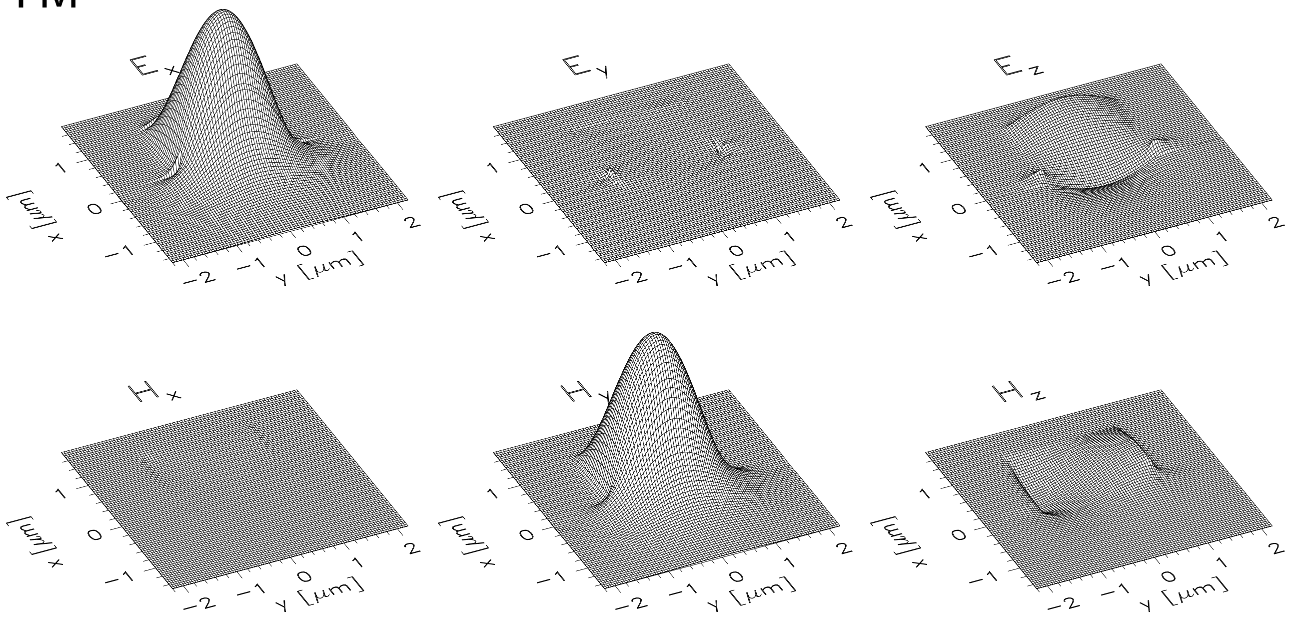


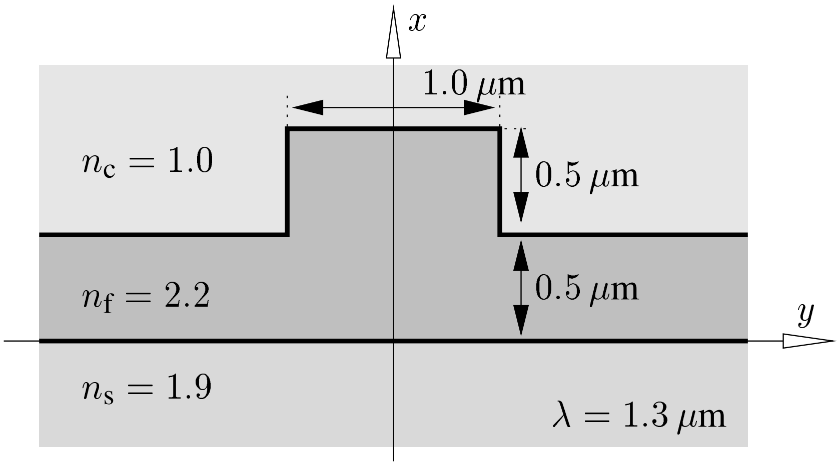
β/k	TE	TM
SV	3.38866	3.38783
VEC	3.38867	3.38782

TE



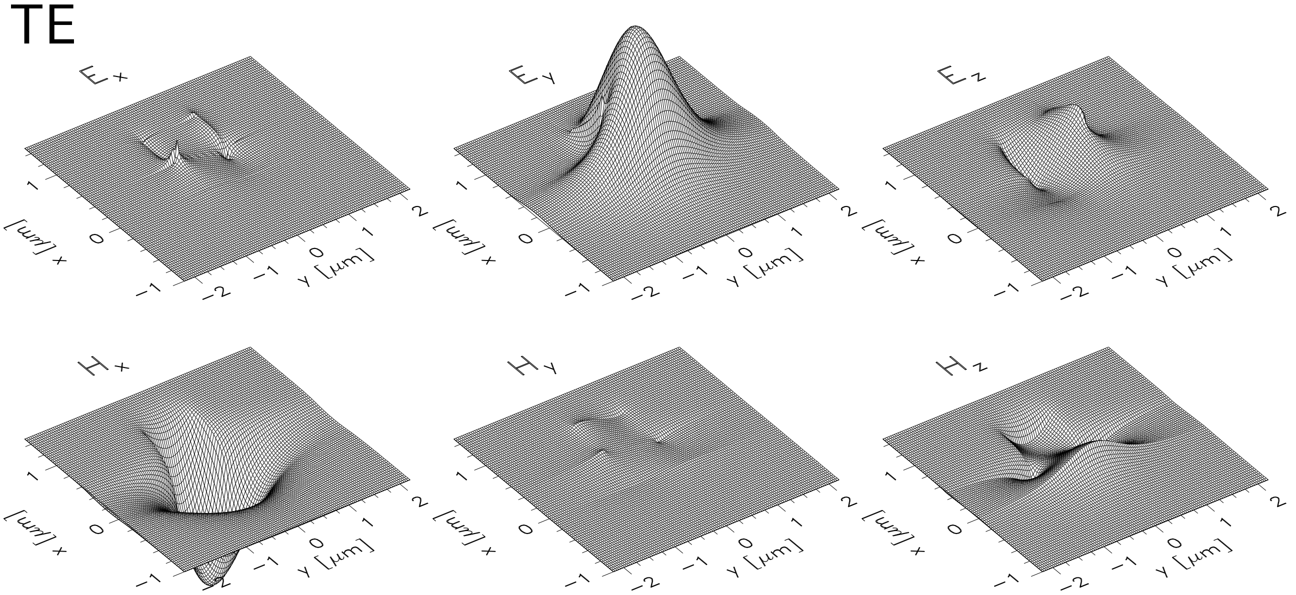
TM



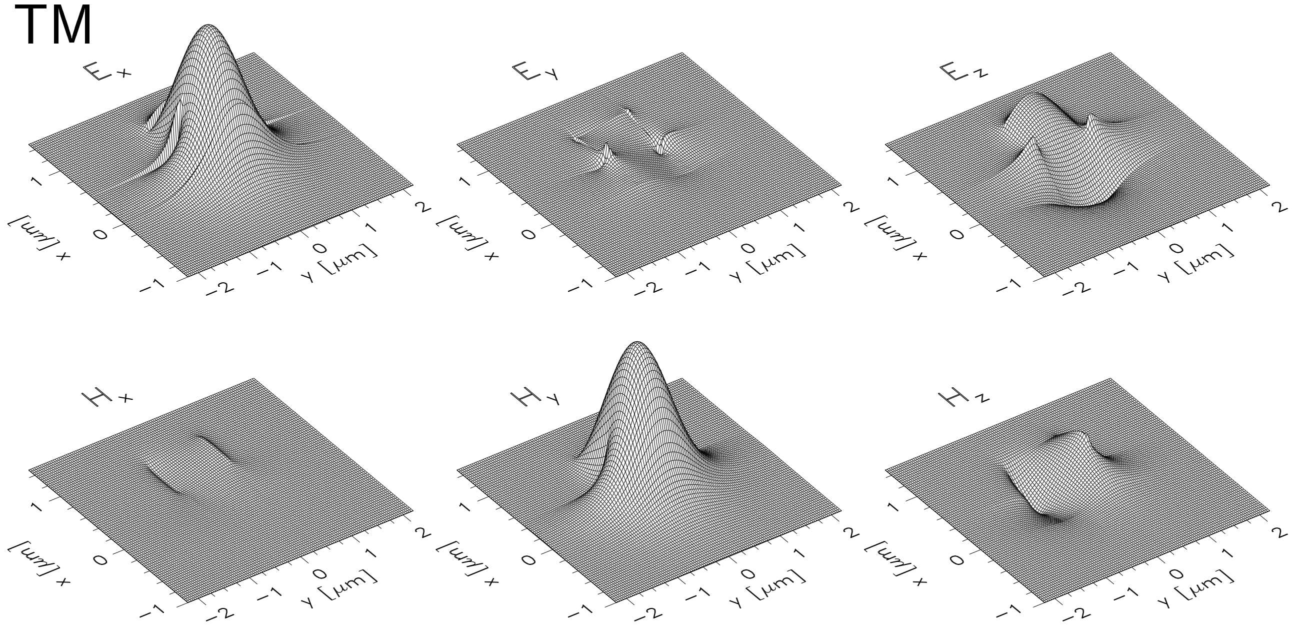


β/k	TE	TM
SV	2.08941	2.07997
VEC	2.08963	2.08009

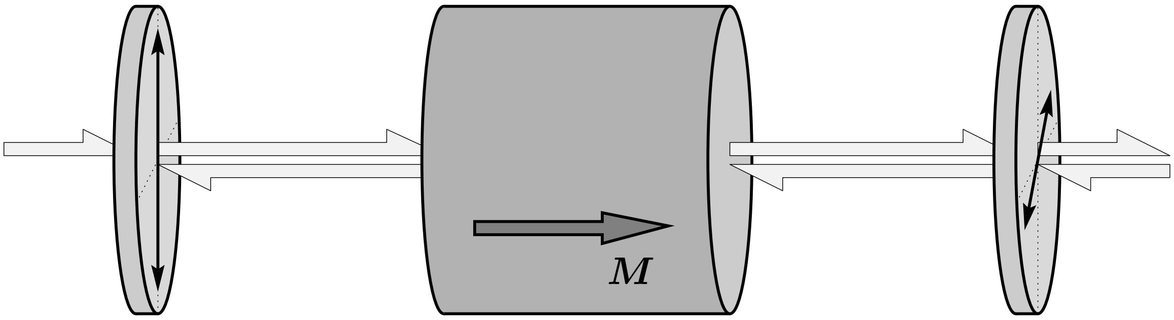
TE



TM

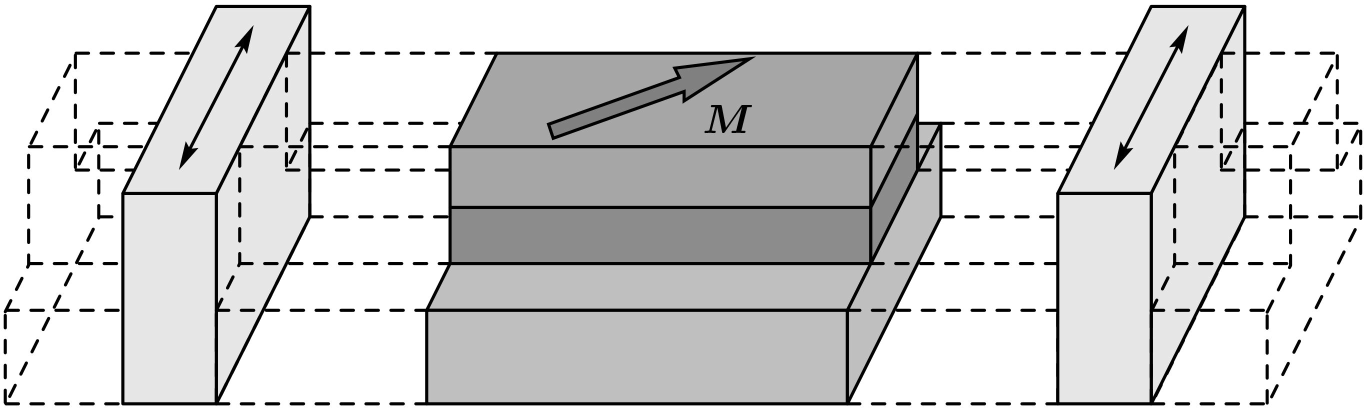


Conventional microoptic setup:



Polarizer, $0°$ | $45°$ nonreciprocal Faraday-rotator | Polarizer, $45°$

Integrated optical concept*:



Polarizer, transparent for TE light

Unidirectional magnetooptic polarization converter
TE $\longrightarrow$ TE
TM $\longleftarrow$ TE

Polarizer, transparent for TE light

* An investigation of planar devices:
T. Mizumoto, Y. Naito, *Waveguide-type optical isolator with in plane magnetization structure*, Vol. 1274 of SPIE Proceedings, 220–228 (1990).

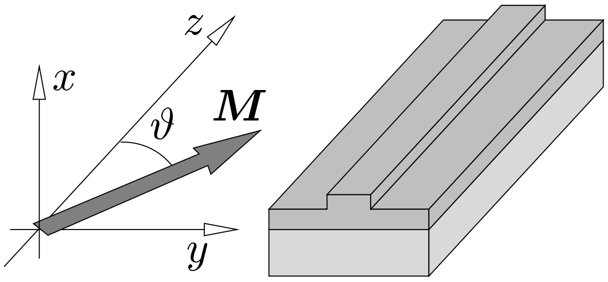
Permittivity of a waveguide made of magnetooptic garnet films:

n^2 — Guiding isotropic refractive index profile,

$+\,\delta\hat{\epsilon}_{\mathrm{mo}}$ — Contribution due to the first order magnetooptic effect.

The magnetization

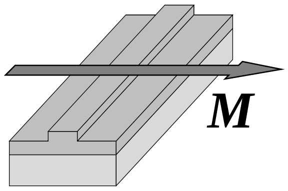
$$\boldsymbol{M} = M(0, \sin\vartheta, \cos\vartheta)$$

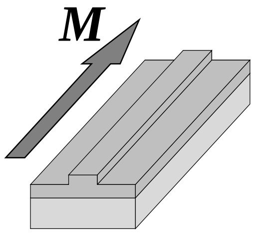


leads to the permittivity perturbation

$$\delta\hat{\epsilon}_{\mathrm{mo}} = \sin\vartheta\ \delta\hat{\epsilon}^{y}_{\mathrm{mo}} + \cos\vartheta\ \delta\hat{\epsilon}^{z}_{\mathrm{mo}}\,,$$

where

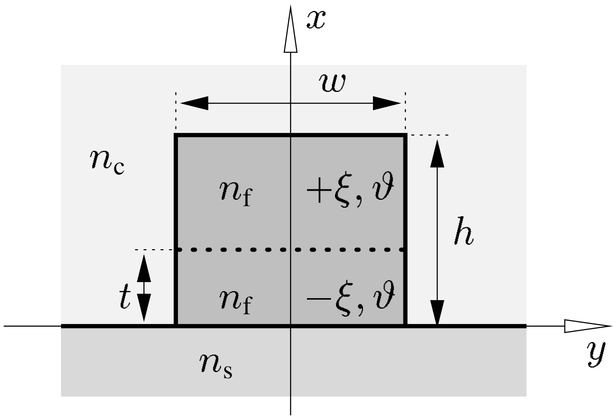


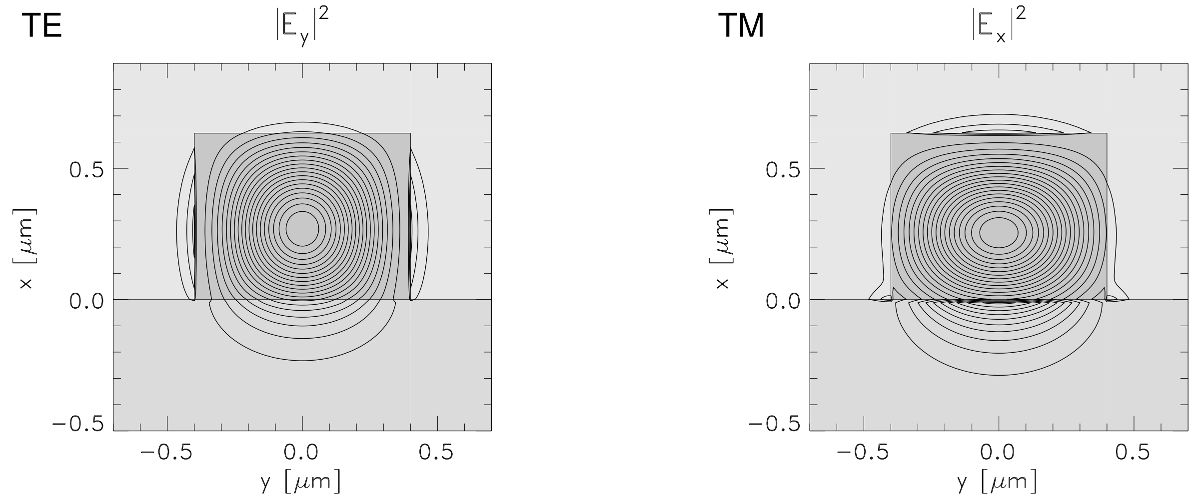


$$\delta\hat{\epsilon}^{y}_{\mathrm{mo}} = \begin{pmatrix} 0 & 0 & -\mathrm{i}\xi \\ 0 & 0 & 0 \\ \mathrm{i}\xi & 0 & 0 \end{pmatrix}, \qquad \delta\hat{\epsilon}^{z}_{\mathrm{mo}} = \begin{pmatrix} 0 & \mathrm{i}\xi & 0 \\ -\mathrm{i}\xi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$KM/\mathrm{i} = \xi = n\lambda\Theta_{\mathrm{F}}/\pi$ with vacuum wavelength λ and specific Faraday-rotation Θ_{F}.

$$|\xi| \ll n^2 \quad \longrightarrow \quad \text{perturbation theory} \ldots$$





$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, y, z, t) = \sum_m C_m(z) \frac{1}{\sqrt{P_m}} \begin{pmatrix} \boldsymbol{E_m} \\ \boldsymbol{H_m} \end{pmatrix}(x, y)\, \mathrm{e}^{\mathrm{i}\omega t}, \quad m = \mathrm{TE}, \mathrm{TM}\,,$$

$$\boldsymbol{C} = \begin{pmatrix} C_{\rm TE} \\ C_{\rm TM} \end{pmatrix}.$$

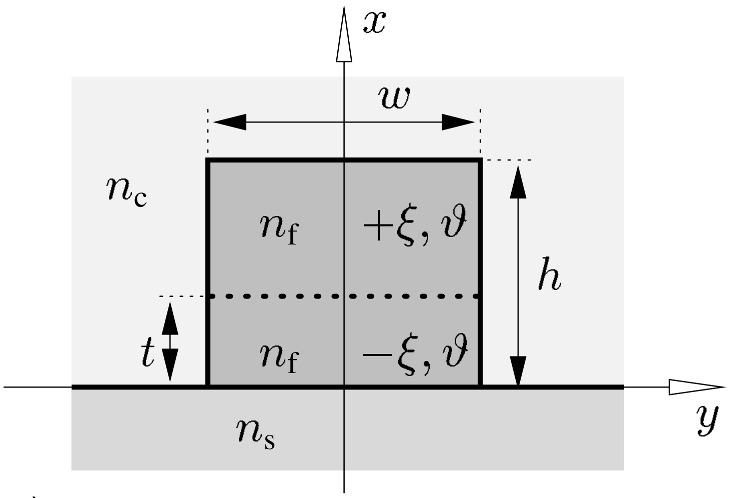
- Light propagation in the isotropic waveguide: $(\delta\hat{\epsilon}_{\rm mo} = 0)$:

$$\mathrm{i}\partial_z \boldsymbol{C} = \begin{pmatrix} \beta_{\rm TE} & 0 \\ 0 & \beta_{\rm TM} \end{pmatrix} \boldsymbol{C} \quad \rightarrow \quad \boldsymbol{C}(z) = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\beta_{\rm TE} z} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\beta_{\rm TM} z} \end{pmatrix} \boldsymbol{C}(0)\,.$$

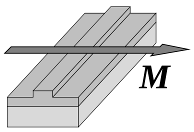
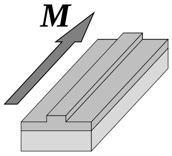
- Magnetooptic effects included:

$$\mathrm{i}\partial_z \boldsymbol{C} = \begin{pmatrix} \beta_{\rm TE} & 0 \\ 0 & \beta_{\rm TM} \end{pmatrix} \boldsymbol{C} + \begin{pmatrix} \kappa_{\rm TE,TE} & \kappa_{\rm TE,TM} \\ \kappa^*_{\rm TE,TM} & \kappa_{\rm TM,TM} \end{pmatrix} \boldsymbol{C}\,,$$

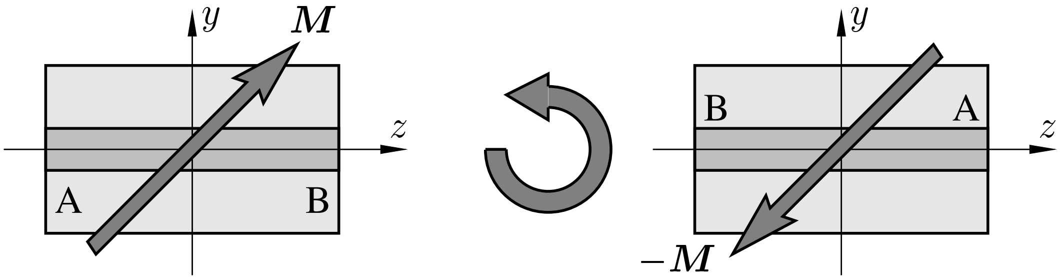
$$\kappa_{l,m} = \frac{\omega\epsilon_0}{4\sqrt{P_l P_m}} \iint \boldsymbol{E}_l^* \delta\hat{\epsilon}_{\rm mo} \boldsymbol{E}_m \,\mathrm{d}x\,\mathrm{d}y\,.$$

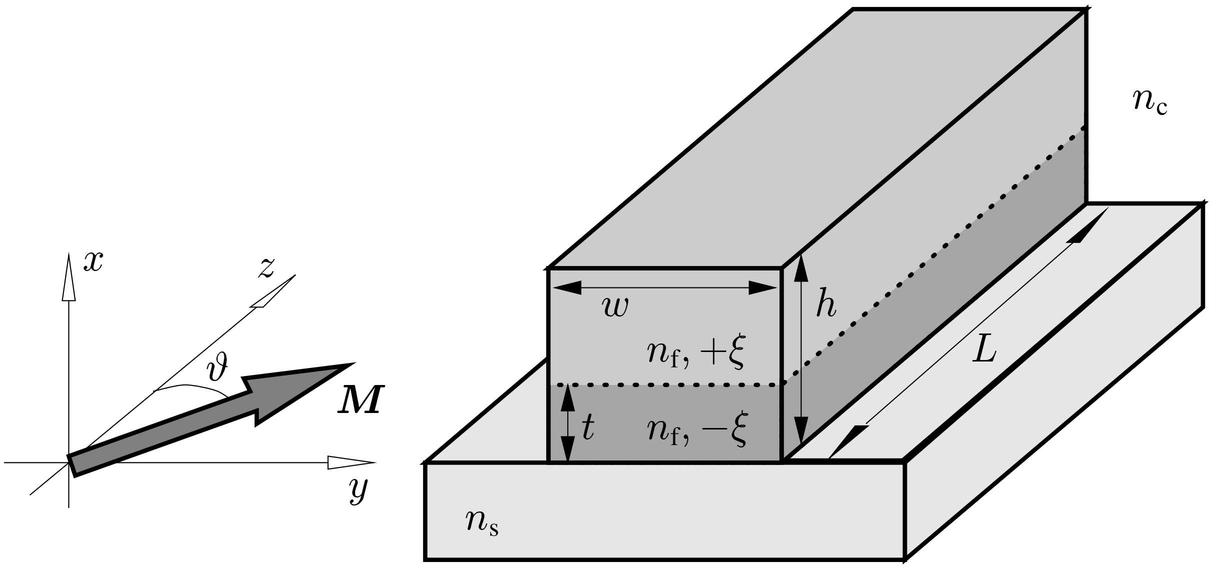


$$\begin{pmatrix} n \\ \delta\hat{\epsilon}^j_{\rm mo} \end{pmatrix}(y) = \begin{pmatrix} n \\ \delta\hat{\epsilon}^j_{\rm mo} \end{pmatrix}(-y) \qquad\qquad \gamma = \frac{\omega\epsilon_0}{4\sqrt{P_l P_m}}\,.$$

	TE-PS $\Delta\beta_{\rm TE}/2 = \kappa_{\rm TE,TE}$	TM-PS $\Delta\beta_{\rm TM}/2 = \kappa_{\rm TM,TM}$	TE/TM-CV $\kappa = \kappa_{\rm TE,TM}$
 $\delta\hat{\epsilon}^y_{\rm mo}$	≈ 0	$2\gamma \iint \xi E_x^{\rm TM} E_z^{\rm TM}\,{\rm d}x\,{\rm d}y$	0
 $\delta\hat{\epsilon}^z_{\rm mo}$	0	0	$\approx -{\rm i}\gamma \iint \xi E_y^{\rm TE} E_x^{\rm TM}\,{\rm d}x\,{\rm d}y$

Propagation in forward/backward direction $\leftrightarrow$ signs of ξ:





Degree of polarization conversion:

$$\eta(z) = \left|\frac{C_\mathrm{TM}(z)}{C_\mathrm{TE}(0)}\right|^2 = \frac{|\kappa|^2}{(\Delta\beta/2)^2 + |\kappa|^2}\,\sin^2\left(\frac{\pi}{2}\,\frac{z}{L_\mathrm{c}}\right)\ ,$$

$C_\mathrm{TM}(0) = 0, \qquad \Delta\beta = \beta_\mathrm{TE} - (\beta_\mathrm{TM} \pm \delta\beta_\mathrm{TM})\ .$

Conversion length:

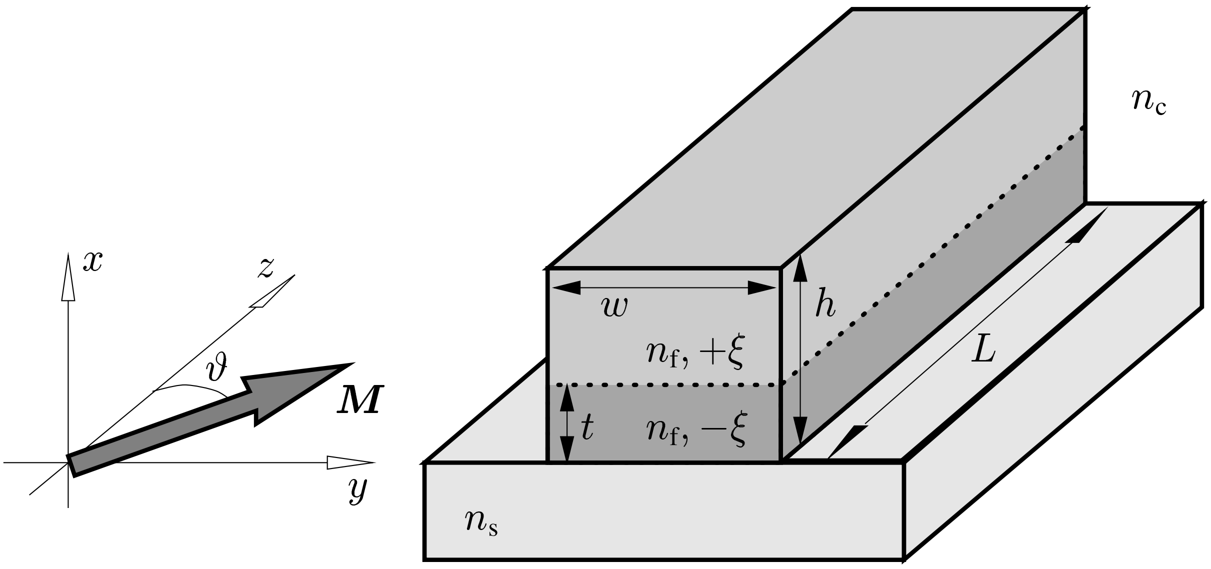
$$L_\mathrm{c} = \frac{\pi}{2\sqrt{(\Delta\beta/2)^2 + |\kappa|^2}}\,.$$

Constraint for complete conversion:

Phase matching $\Delta\beta = 0$.

$$\Rightarrow \qquad\qquad\qquad\qquad \Leftarrow$$

$$\left.\begin{array}{c} \Delta\beta^\mathrm{f} = \beta_\mathrm{TE} - \beta_\mathrm{TM} - \delta\beta_\mathrm{TM} \\ L_\mathrm{c}^\mathrm{f} \\ \eta_\mathrm{max}^\mathrm{f} \end{array}\right\} \neq \left\{\begin{array}{c} \Delta\beta^\mathrm{b} = \beta_\mathrm{TE} - \beta_\mathrm{TM} + \delta\beta_\mathrm{TM} \\ L_\mathrm{c}^\mathrm{b} \\ \eta_\mathrm{max}^\mathrm{b} \end{array}\right.$$



Choose parameters such that

$$\Delta\beta^{b} = 0, \quad \text{i.e.} \quad \eta^{b}_{\max} = 1,$$

and

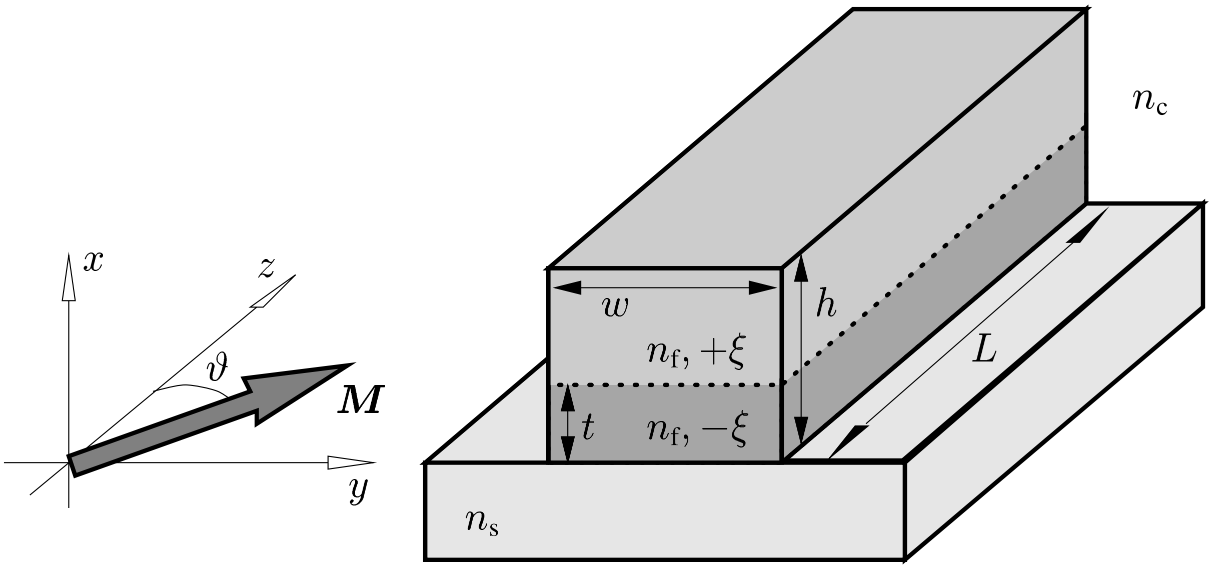
$$L = L^{b}_{c} = 2L^{f}_{c}.$$

$\longrightarrow$ No polarization conversion in forward direction,
but complete conversion for backwards propagating light !

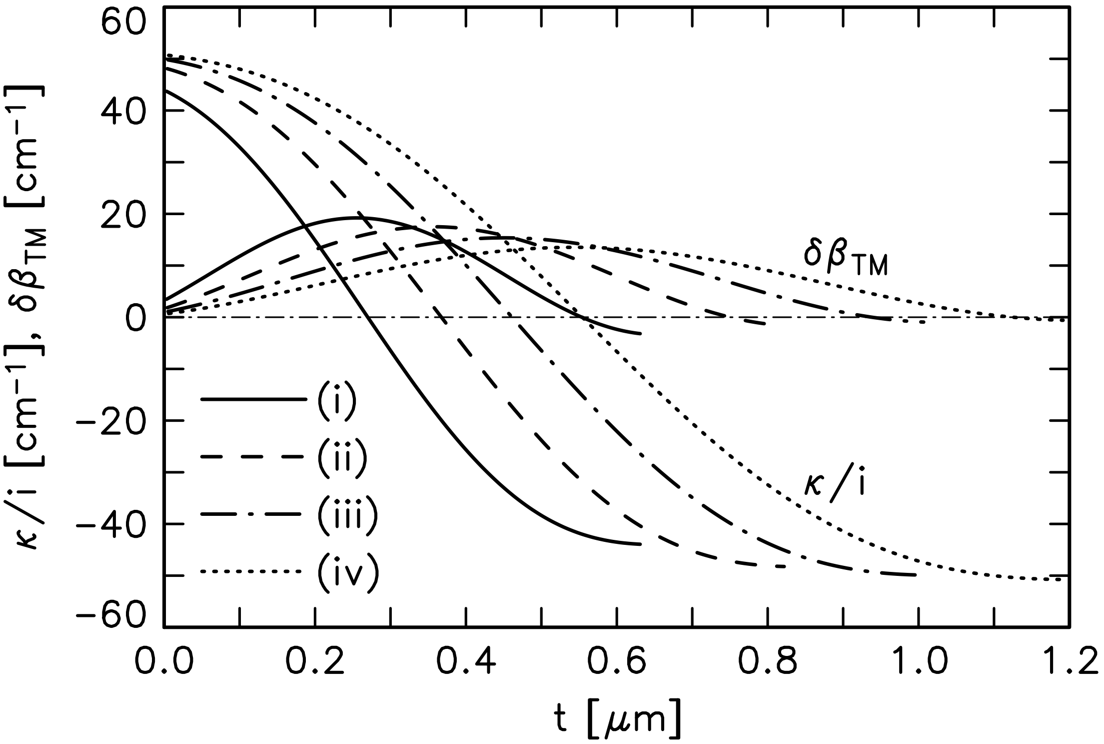
Conditions for ideal unidirectional polarization conversion:

$$|\delta\beta_{\mathrm{TM}}| = \sqrt{3}|\kappa| \qquad \text{and} \qquad \Delta\beta^{b} = 0\,.$$

Device length: $\quad L = \dfrac{\pi}{2|\kappa|}\,.$

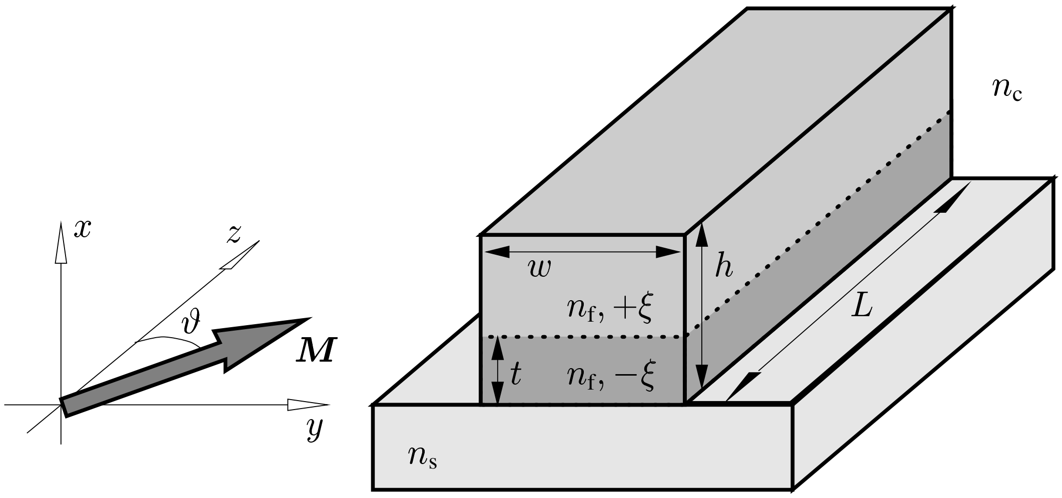


Maximum nonreciprocal TM phase shift $\delta\beta_{\mathrm{TM}} = \kappa_{\mathrm{TM,TM}}$ $(\vartheta = \pi/2)$
and maximum polarization conversion $\kappa = \kappa_{\mathrm{TE,TM}}$ $(\vartheta = 0)$:



	(i)	(ii)	(iii)	(iv)
$h/\,\mu$m	0.634	0.826	1.012	1.198
$w/\,\mu$m	0.798	0.995	1.191	1.387

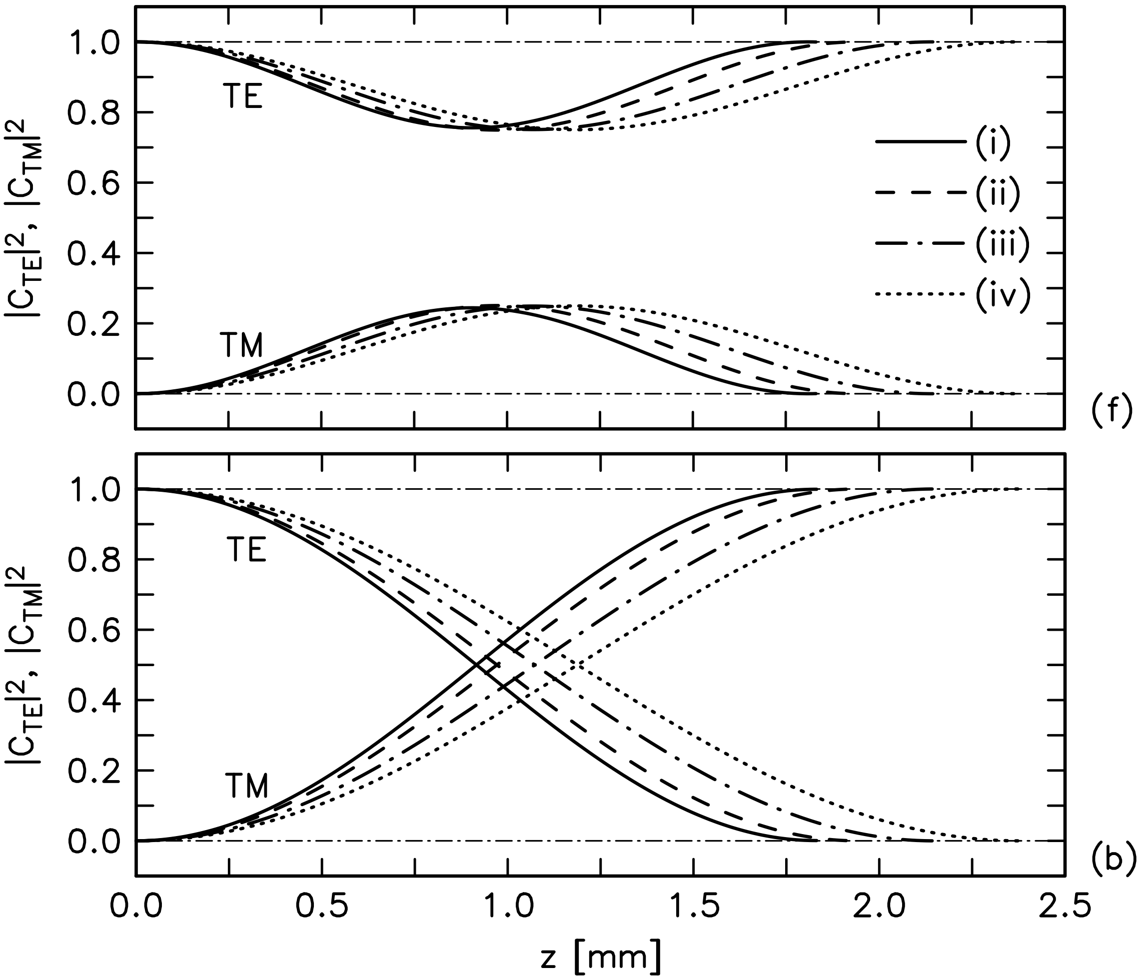
$n_s = 1.95$, $n_f = 2.302$, $n_c = 1.0$,
$\xi = 0.005$, $\lambda = 1.3\,\mu$m.

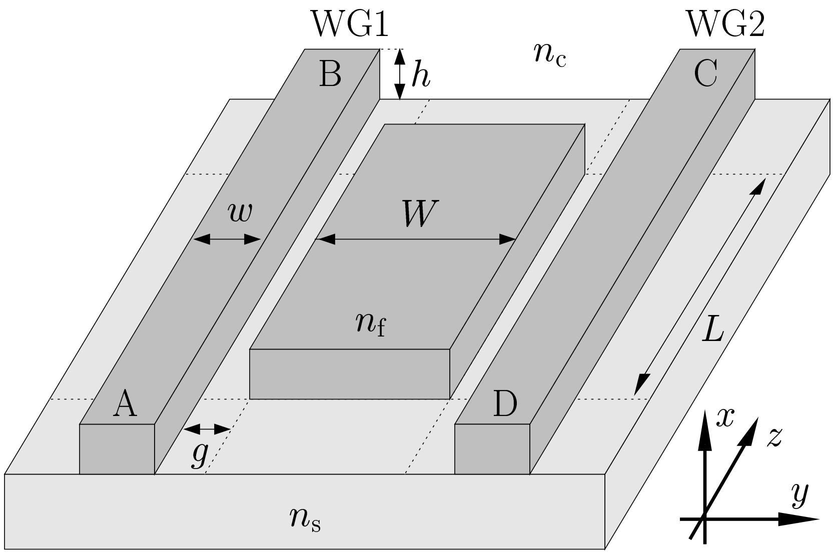


Geometries for unidirectional polarization converters:

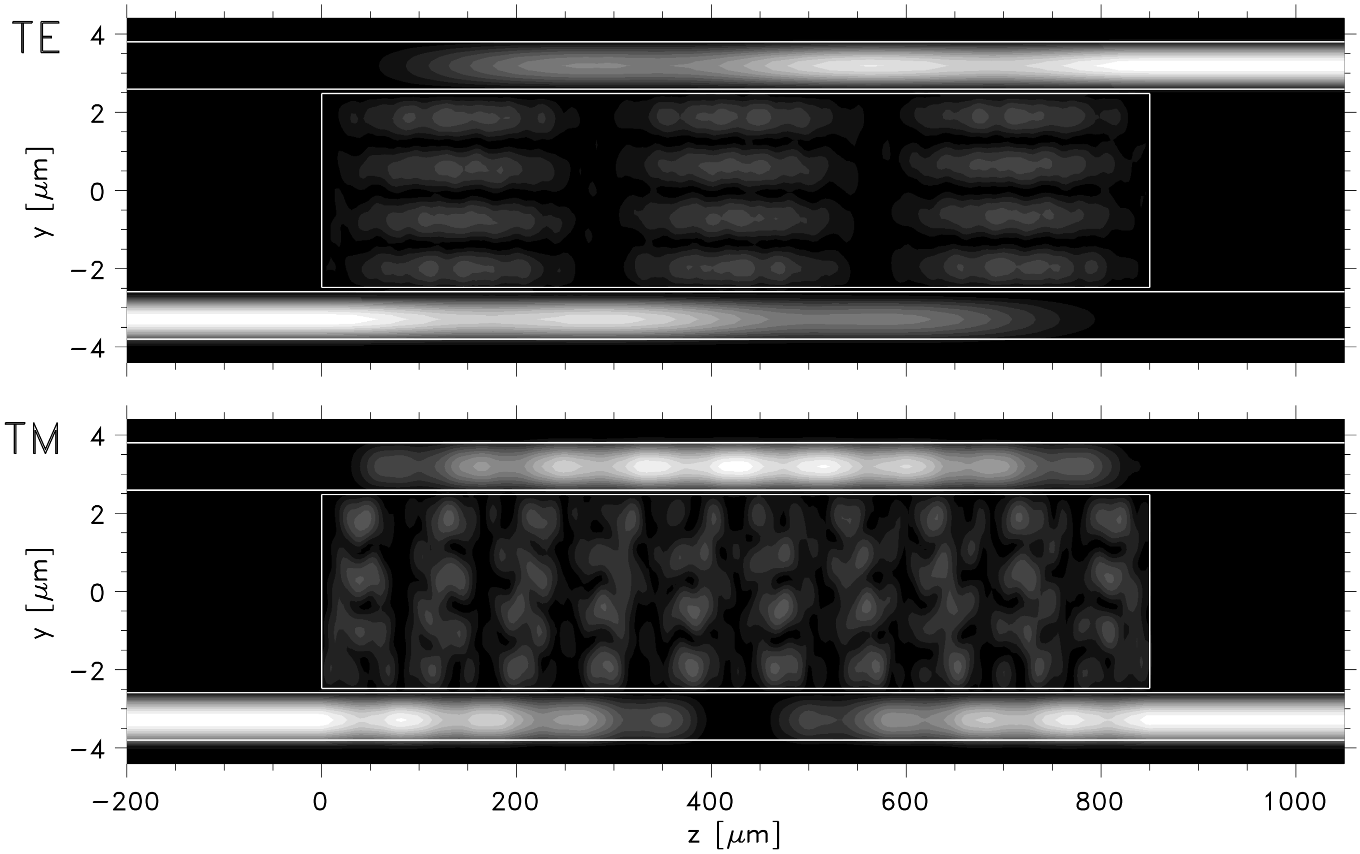
	$h/\,\mu$m	$w/\,\mu$m	$t/\,\mu$m	$\vartheta/^\circ$	$L/\,$mm
(i)	0.634	0.789	0.181	63	1.83
(ii)	0.826	0.995	0.264	65	1.94
(iii)	1.012	1.191	0.349	66	2.14
(iv)	1.198	1.387	0.431	68	2.38

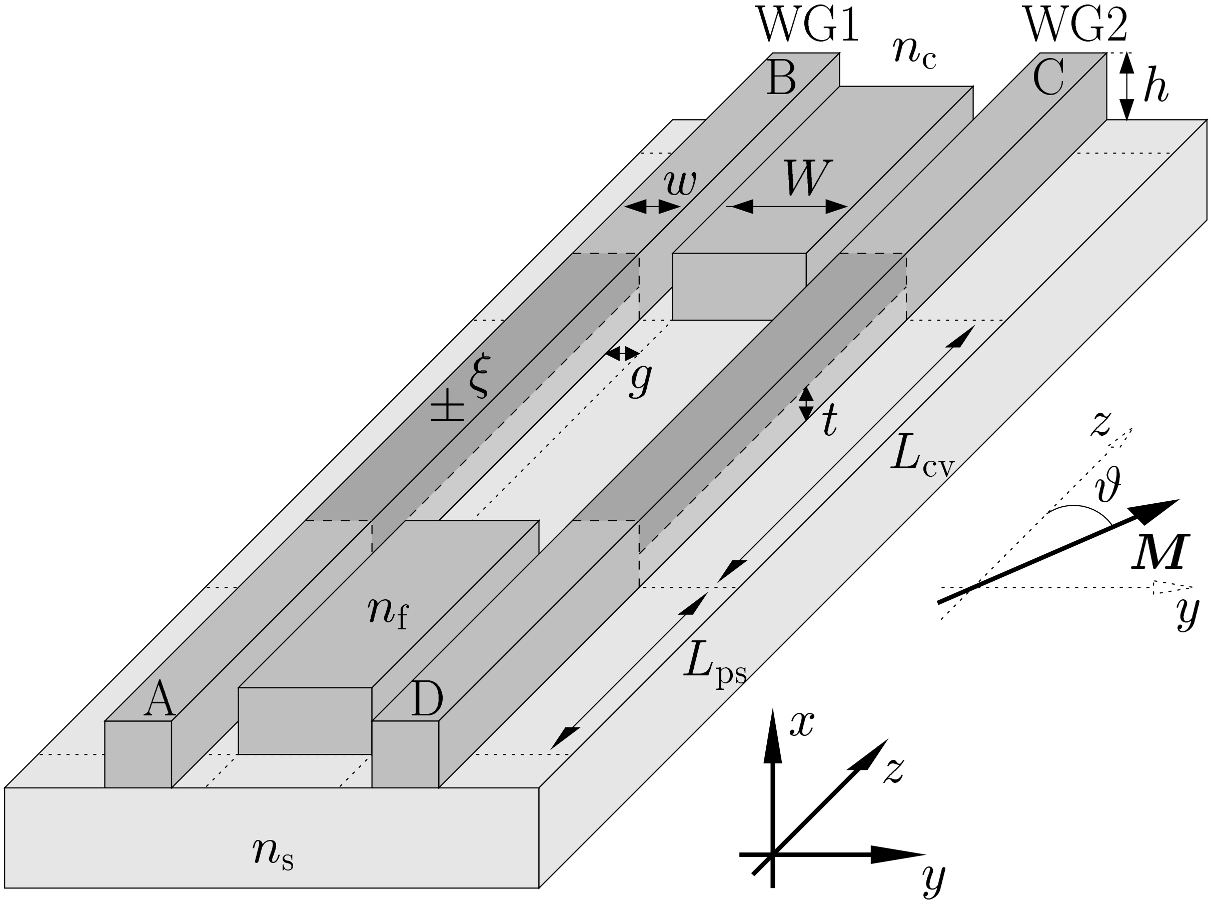
$n_\mathrm{s} = 1.95$,
$n_\mathrm{f} = 2.302$,
$n_\mathrm{c} = 1.0$,
$\xi = 0.005$,
$\lambda = 1.3\,\mu$m.



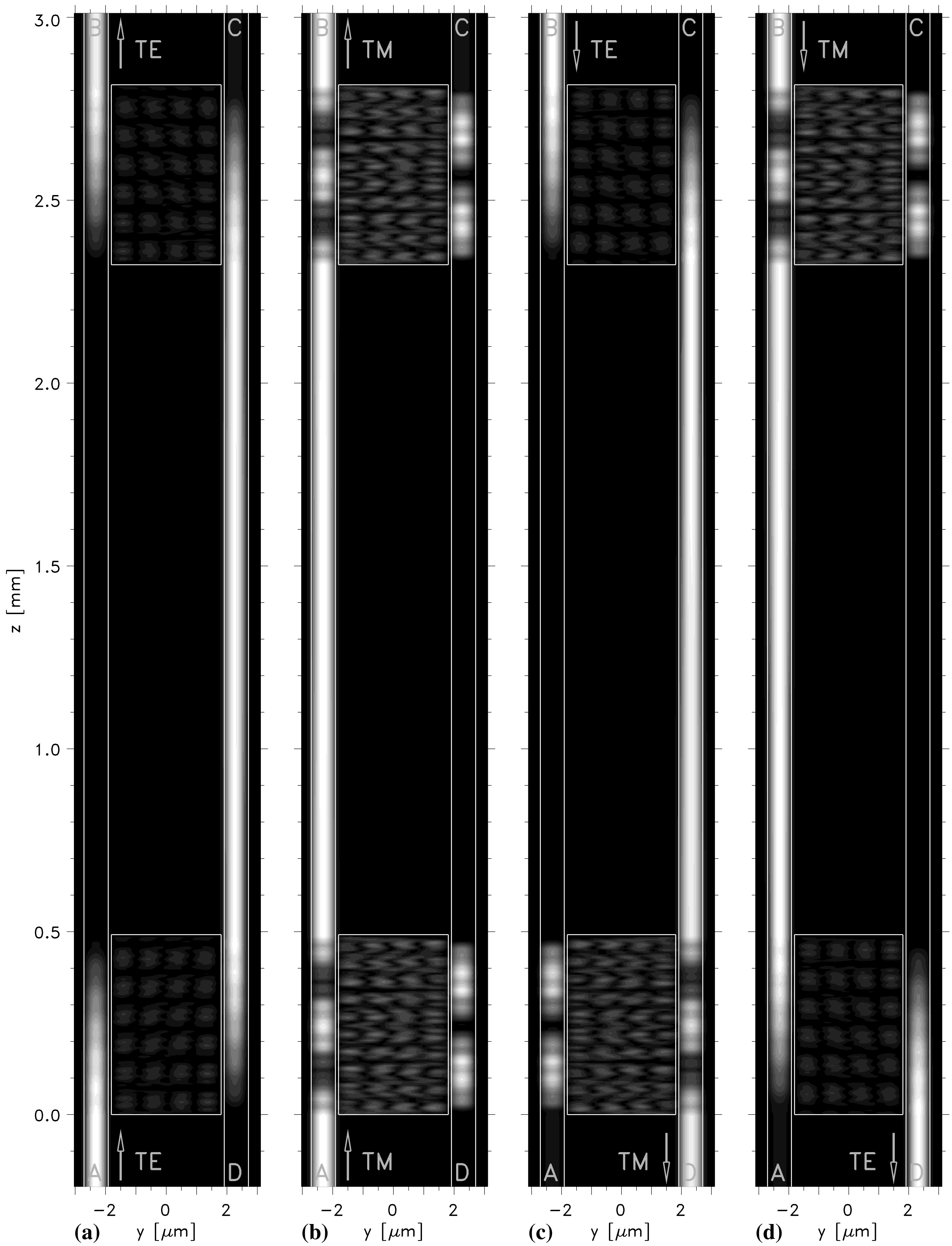


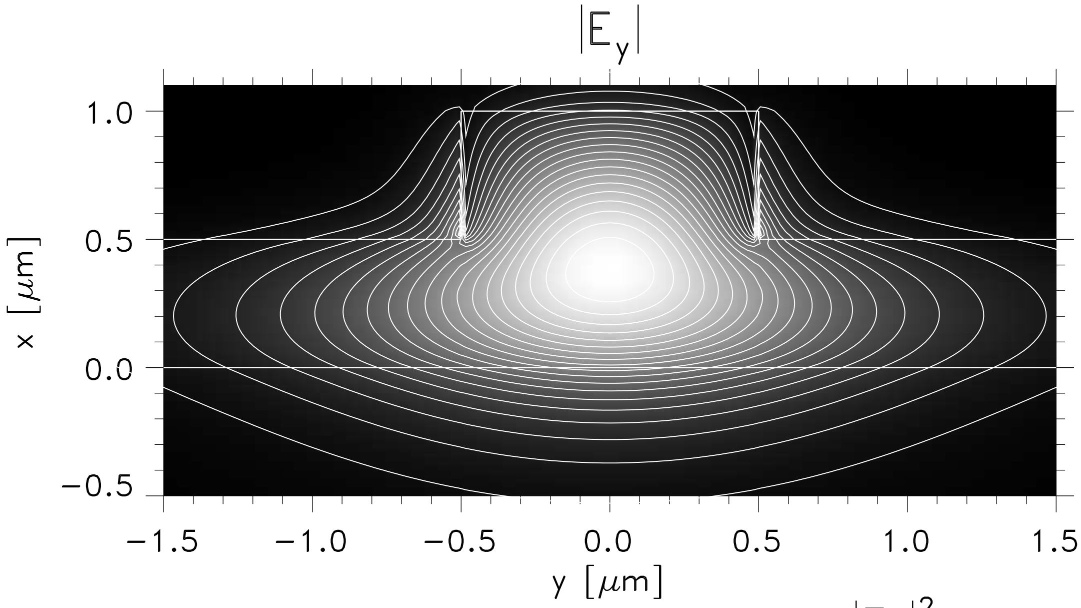
W	w	g	h	λ	L	n_s	n_f	n_c
4.96 μm	1.21 μm	0.11 μm	0.70 μm	1.30 μm	850 μm	1.95	2.2	1.0

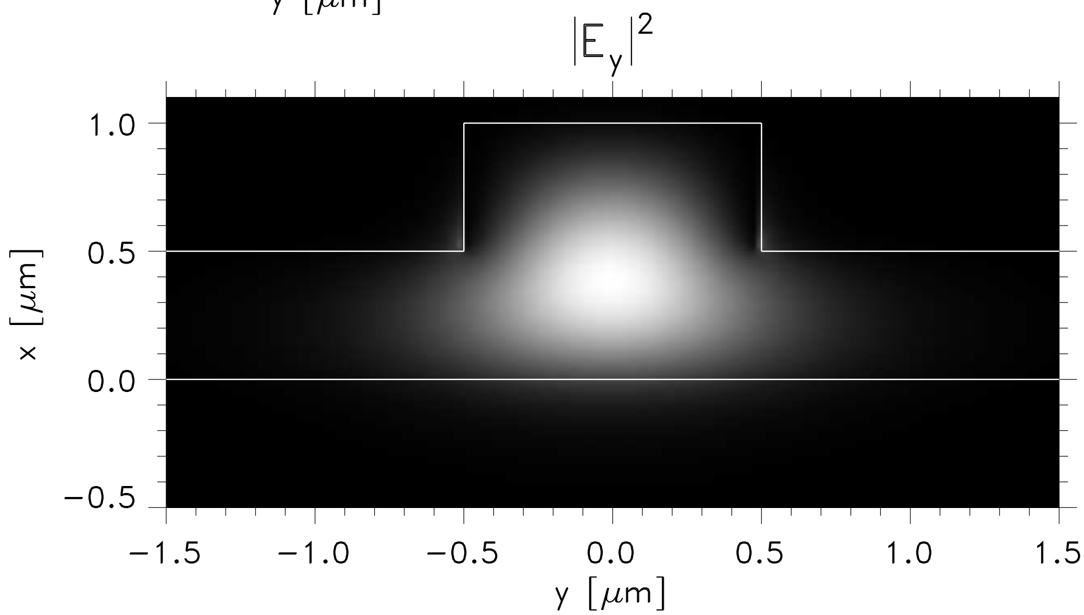




h/μm	w/μm	t/μm	$\vartheta/^\circ$	g/μm	W/μm	L_{ps}/μm	L_{cv}/μm	n_s	n_f	n_c	$\pm\xi$	λ/μm
0.634	0.798	0.181	63	0.103	3.616	492	1832	1.95	2.302	1.0	0.005	1.3


3.0
2.5
2.0
1.5
1.0
0.5
0.0
z [mm]
B
C
A
D
TE
TM
−2
0
2
y [μm]
(a)
(b)
(c)
(d)





- Guided modes for piecewise constant refractive index profiles
- Wave-matching method for numerical mode analysis
- Examples for semivectorial and fully vectorial simulations
- Perturbation theory for magnetooptic waveguides
- Polarisation conversion, phase matching
- Unidirectional polarization conversion
- Concept for a polarization independent circulator