On effective index approximations of photonic crystal slabs

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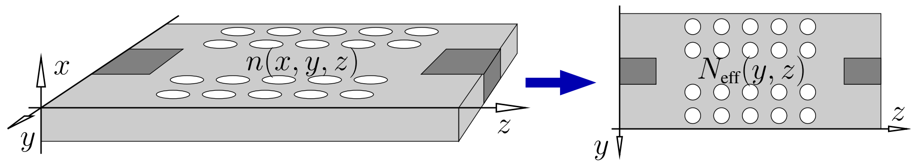
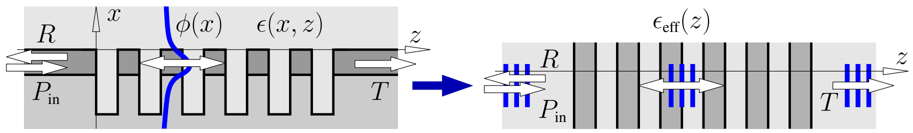
Effective index approximations

$3D \longrightarrow 2D$



$2D \longrightarrow 1D$



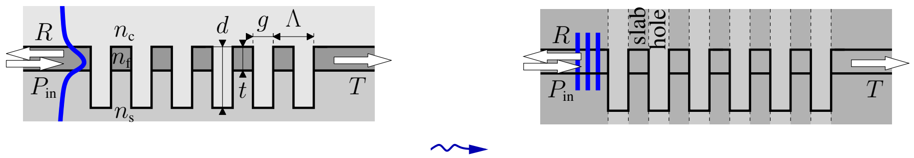
Outline

Effective index approximations

- $2D \longrightarrow 1D$, examples
- A variational view on the effective index method
- vEIM $2D \longrightarrow 1D$, examples
- $3D \longrightarrow 2D$, scalar approximation

- Outlook: vectorial formalism

2D -> 1D



$(n_s, n_f, n_c) = (1.45, 2.0, 1.0)$,

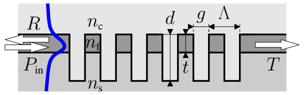
$\Lambda = 0.21\,\mu\text{m}$, $g = 0.11\,\mu\text{m}$,
$d = 0.6\,\mu\text{m}$, $t = 0.2\,\mu\text{m}$,

TE, $\lambda \in [0.4, 0.9]\,\mu\text{m}$.

$2D \longrightarrow 1D$
effective index approximation:

$N_{\text{eff}}^{\text{slab}} \in [1.67, 1.87]$,
$N_{\text{eff}}^{\text{holes}} = \,?$

Deeply etched waveguide grating



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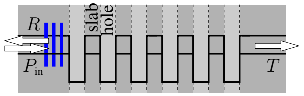
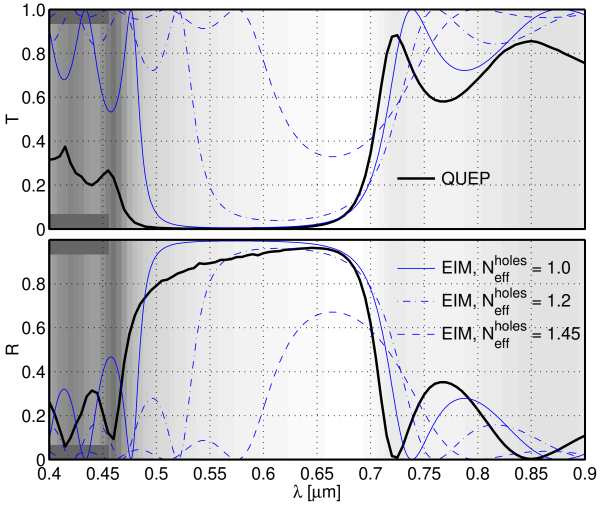
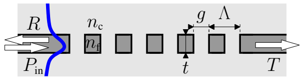


$N_{\rm eff}^{\rm slab} \in [1.67, 1.87]$.

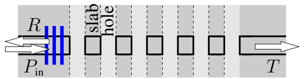


High contrast PC membrane

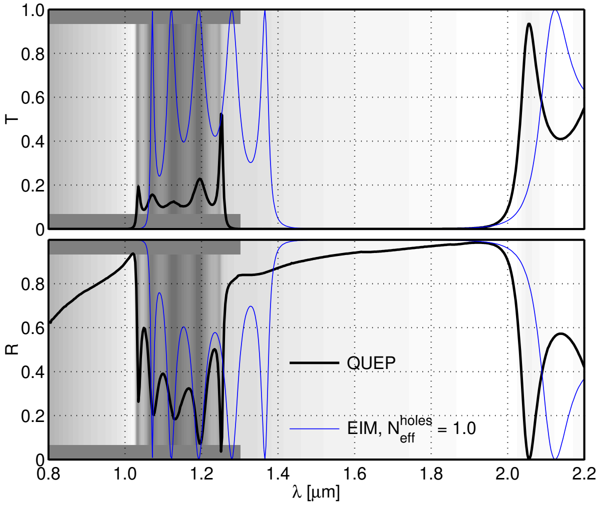


$(n_{\rm c}, n_{\rm f}, n_{\rm c}) = (1.0, 3.4, 1.0)$,

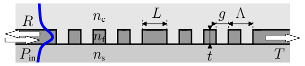
$\Lambda = 0.45\,\mu$m, $g = 0.225\,\mu$m,
$t = 0.2\,\mu$m.



$N_{\rm eff}^{\rm slab} \in [2.33, 3.09]$,
$N_{\rm eff}^{\rm holes} = n_{\rm c} = 1.0$.

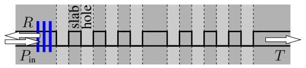


Defect cavity

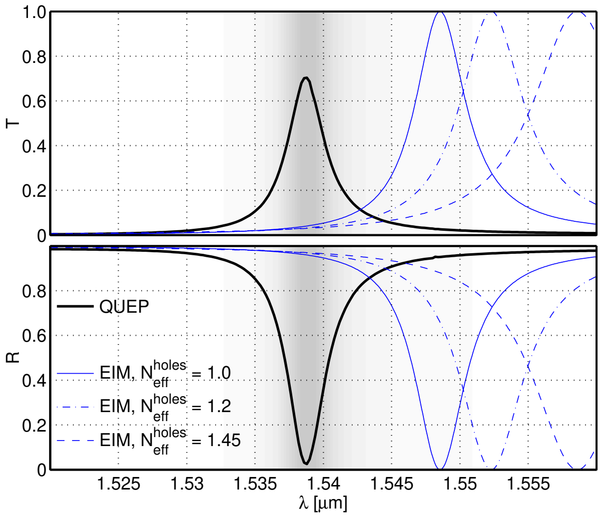


$(n_{\rm s}, n_{\rm f}, n_{\rm c}) = (1.45, 3.4, 1.0)$,

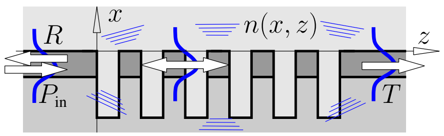
$t = 0.22\,\mu$m, $\Lambda = 0.31\,\mu$m,
$g = 0.135\,\mu$m,
$L = 1.515\,\mu$m.



$N_{\rm eff}^{\rm slab} \in [2.75, 2.77]$.



2D propagation problems



2D, TE, frequency domain problems,
vacuum wavelength $\lambda = 2\pi/k$,
permittivity $\epsilon(x, z) = n^2(x, z)$,
principal electric component $E_y(x, z)$.

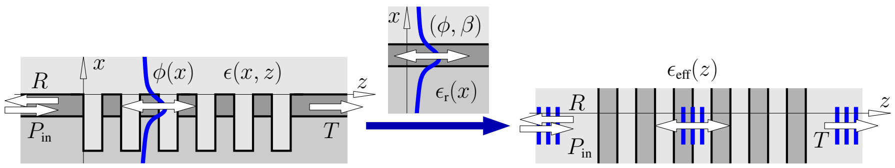
If the functional

$$\mathcal{H}(E_y) := \frac{1}{2}\iint_\Omega \left((\partial_x E_y)^2 + (\partial_z E_y)^2 - k^2\epsilon E_y^2\right) \mathrm{d}x\,\mathrm{d}z$$

is stationary for E_y, then E_y satisfies the Helmholtz equation

$$\partial_x^2 E_y + \partial_z^2 E_y + k^2\epsilon\, E_y = 0, \qquad (x, z) \in \Omega.$$

Variational effective index approximation



Reference permittivity $\epsilon_{\text{r}}(x)$ with guided mode $\phi(x)$, mode index β/k:

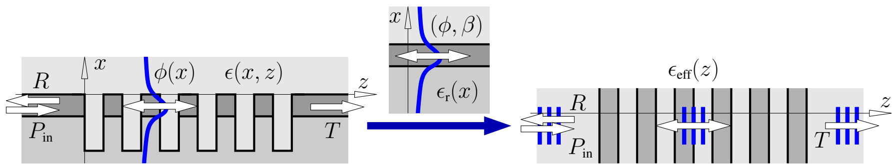
$$\partial_x^2\phi + (k^2\epsilon_{\text{r}} - \beta^2)\,\phi = 0\,.$$

Assumption:

ϕ constitutes a reasonable approximation for the vertical field shape on the entire horizontal axis,

$$E_y(x,z) = \psi(z)\,\phi(x)\,, \qquad \psi = ?$$

Variational effective index approximation



Restriction of $\mathcal{H}$ to $E_y(x,z) = \psi(z)\,\phi(x)$ $\rightsquigarrow$ stationarity condition

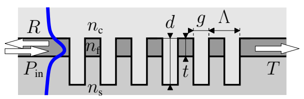
$$\partial_z^2\psi + k^2\epsilon_{\text{eff}}(z)\,\psi = 0$$

with effective permittivity

$$\epsilon_{\text{eff}}(z) = (\beta/k)^2 + \frac{\int (\epsilon(x,z) - \epsilon_{\text{r}}(x))\,\phi^2(x)\,\mathrm{d}x}{\int \phi^2(x)\,\mathrm{d}x}\,, \qquad \epsilon_{\text{eff}}(z) = N_{\text{eff}}^2(z).$$

Variational Effective Index Method vEIM

Deeply etched waveguide grating



$(n_{\text{s}}, n_{\text{f}}, n_{\text{c}}) = (1.45, 2.0, 1.0)$,

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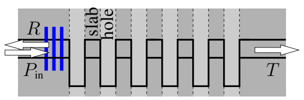
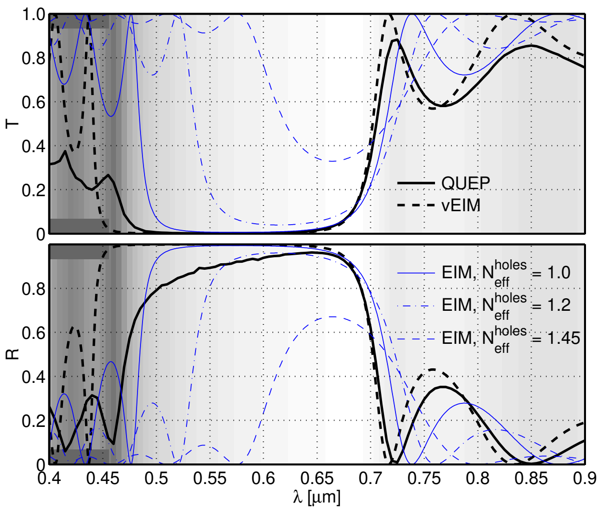
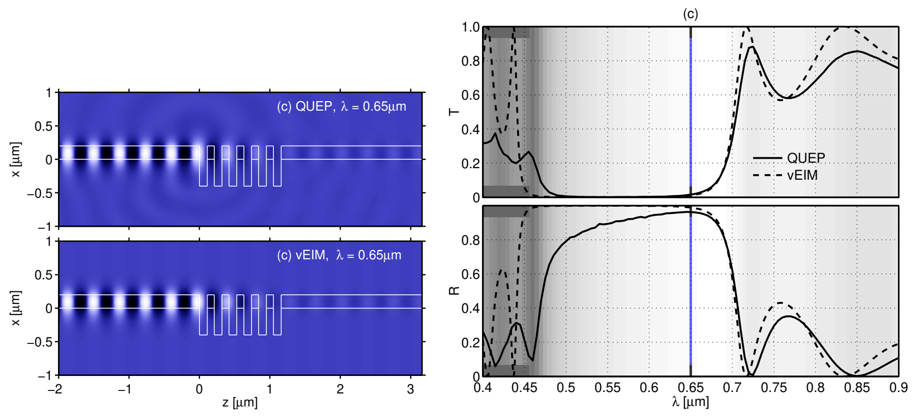


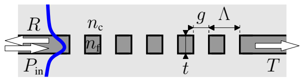
$N_{\text{eff}}^{\text{slab}} \in [1.67, 1.87]$,
$N_{\text{eff}}^{\text{holes}} \in [0.71, 0.82]$ (vEIM).



Deeply etched waveguide grating, fields

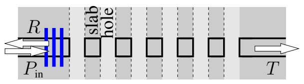


High contrast PC membrane

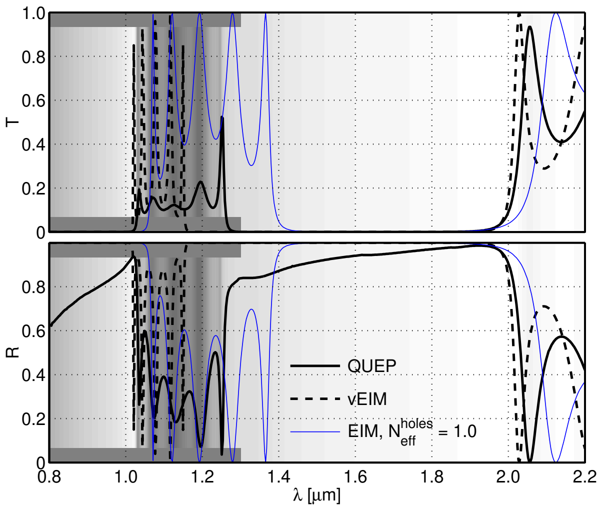


$(n_c, n_f, n_c) = (1.0, 3.4, 1.0)$,

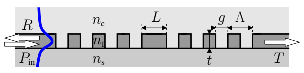
$\Lambda = 0.45\,\mu$m, $g = 0.225\,\mu$m,
$t = 0.2\,\mu$m.



$N_{\rm eff}^{\rm slab} \in [2.33, 3.09]$,
$\epsilon_{\rm eff}^{\rm holes} \in [-1.30, -0.41]$ (vEIM).



Defect cavity



$(n_s, n_f, n_c) = (1.45, 3.4, 1.0)$,

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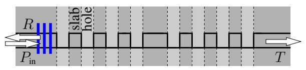
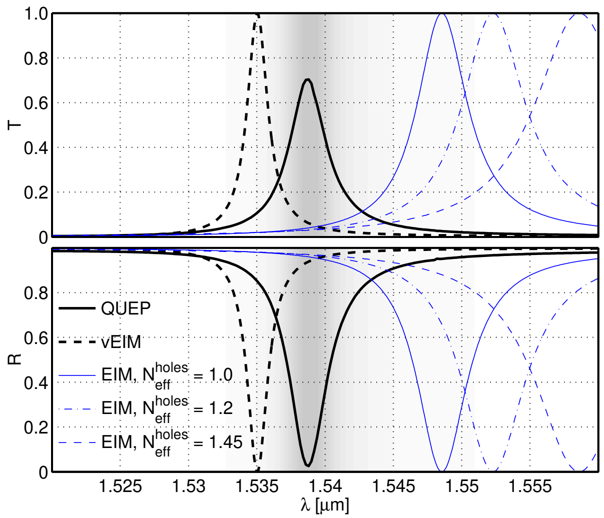
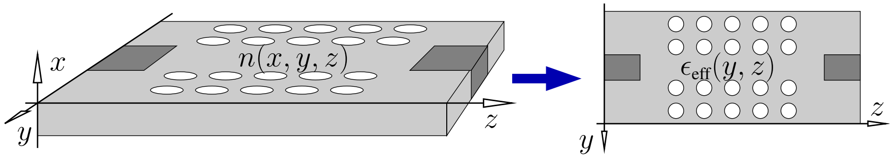


$N_{\rm eff}^{\rm slab} \in [2.75, 2.77]$,
$\epsilon_{\rm eff}^{\rm holes} \in [-0.96, -0.94]$ (vEIM).



3D -> 2D, scalar



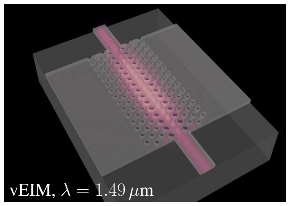
Choose $\epsilon_{\rm r}(x)$, $\phi(x)$, β, with $\partial_x^2\phi + (k^2\epsilon_{\rm r} - \beta^2)\,\phi = 0\,.$

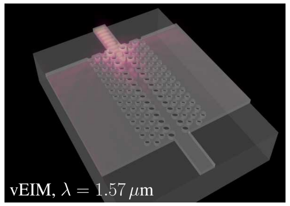
Assume $E(x, y, z) = \psi(y, z)\,\phi(x)$:

$$\partial_y^2\psi + \partial_z^2\psi + k^2\epsilon_{\rm eff}(y, z)\,\psi = 0\,,$$

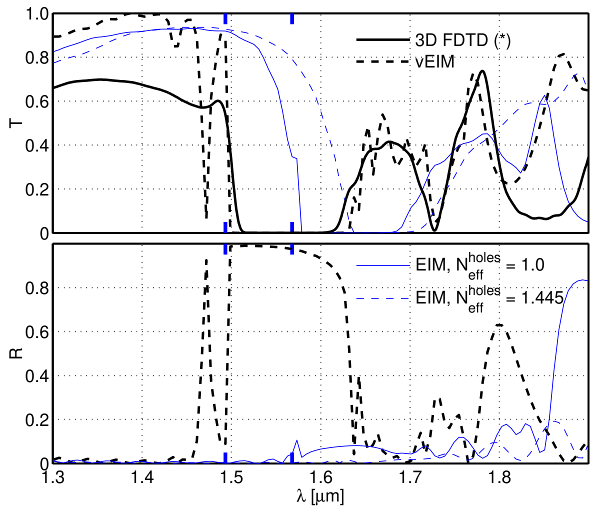
$$\epsilon_{\rm eff}(y, z) = (\beta/k)^2 + \frac{\int (\epsilon(x, y, z) - \epsilon_{\rm r}(x))\,\phi^2(x)\,{\rm d}x}{\int \phi^2(x)\,{\rm d}x}\,, \quad \epsilon_{\rm eff}(y, z) = N_{\rm eff}^2(y, z).$$

Outlook: 3D -> 2D, vectorial





Contrast (1.445 : 3.48 : 1.0).



$N_{\rm eff}^{\rm slab} \in [2.68, 2.96]$, $\epsilon_{\rm eff}^{\rm holes} \in [-0.792, -1.146]$ (vEIM).

Concluding remarks

Effective index treatment of photonic crystal slabs:

- a mere qualitative, sometimes perhaps a crude quantitative approximation,
- where unavoidable:
  use effective permittivity $\epsilon_{\rm eff} = N_{\rm eff}^2$ with perturbational correction term;
  $\epsilon_{\rm eff}$ can well be smaller than one, or be negative,
  less heuristic approach with well-defined field approximation,
- in general: no guarantees.