Simulations of vertically-coupled circular microresonators by 3-D vectorial coupled mode theory

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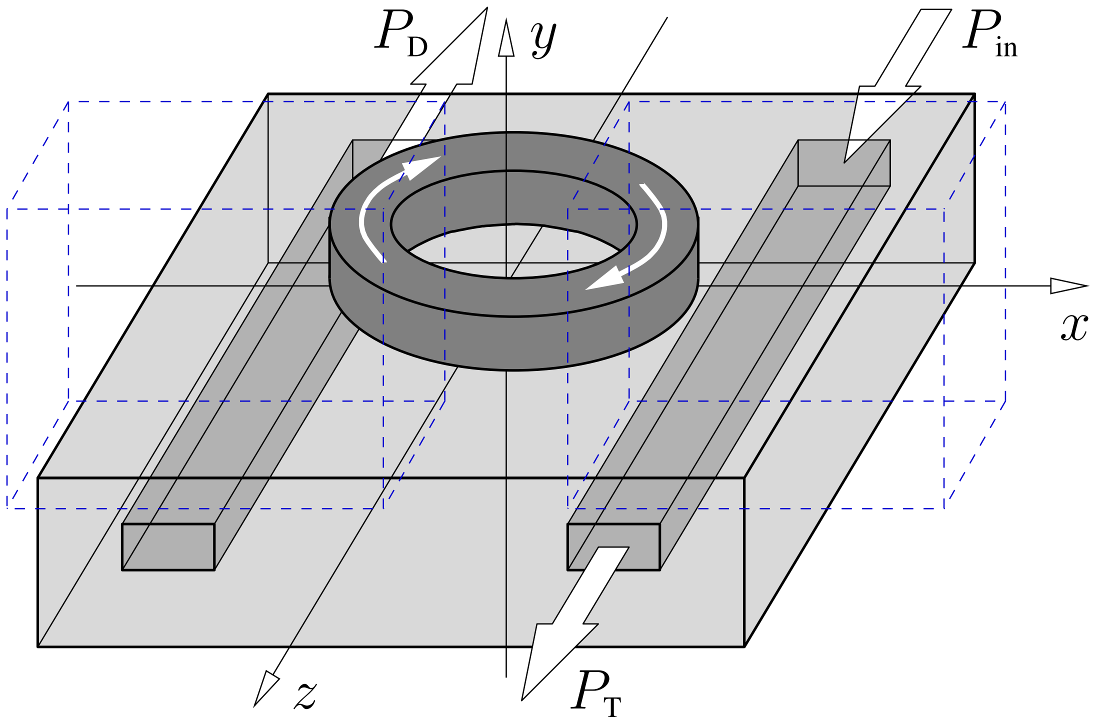
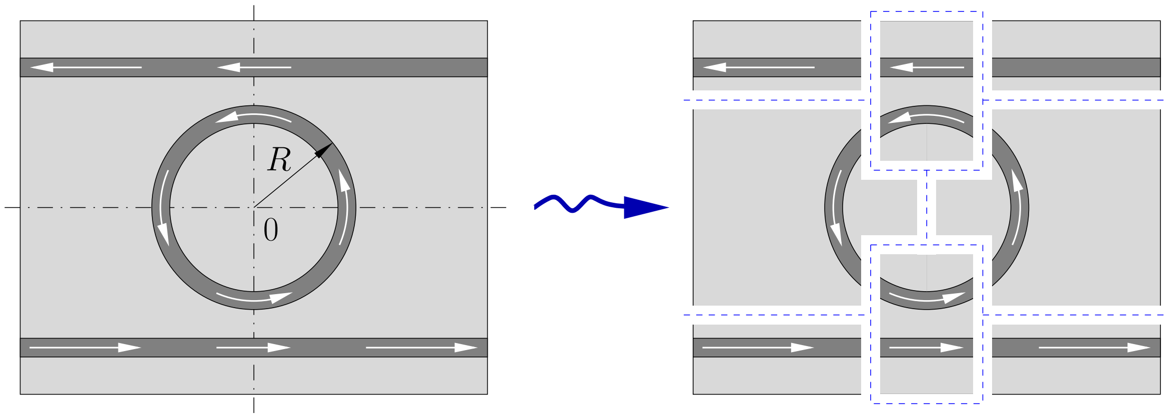
Ringresonator modeling





- Ringresonator
  $\approx$ 2 couplers
  $+$ 2 cavity segments.

- CW description :
  $\boldsymbol{E}, \boldsymbol{H} \sim \mathrm{e}^{\mathrm{i}\omega t}$,
  $\omega = k\,\mathrm{c}, \;\; k = 2\pi/\lambda.$

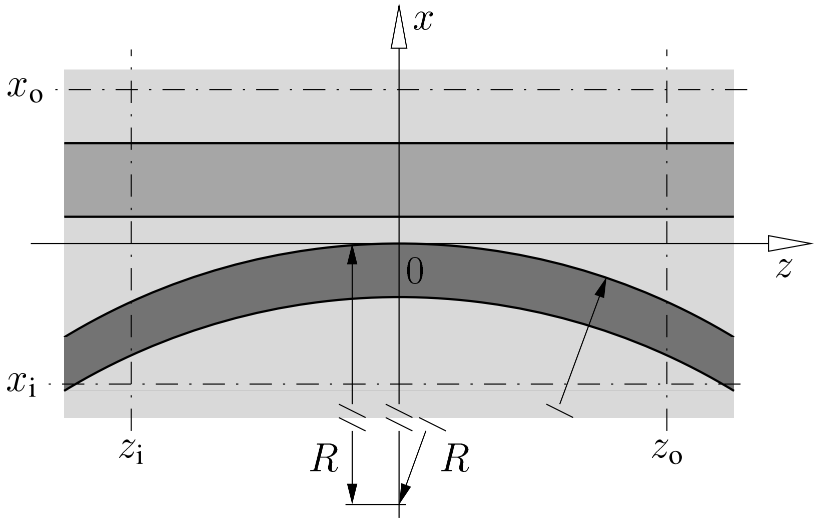
Ab initio 3-D model ...

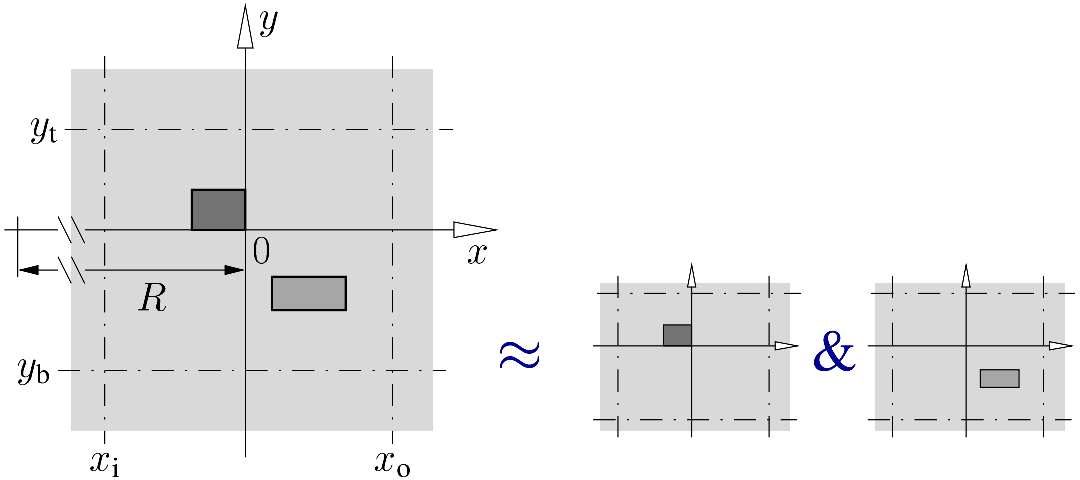
3-D CMT simulations of circular microresonators

- CMT ringresonator model
  - Coupler region: Basis fields, CMT ansatz
  - Coupler region: CMT equations, solution
  - Resonator: Power transmission, spectra

- Numerical results
  - Vertically coupled microdisk-resonator
  - Resonator with “hybrid” ring-cavity

Coupler region: Basis fields & CMT ansatz





Numerical basis fields: (FD, FMM)

- Cavity: 3-D (2-D) bend modes (ϵ_m)

$$\begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, y, z) = \begin{pmatrix} \boldsymbol{E}_{m,0} \\ \boldsymbol{H}_{m,0} \end{pmatrix}(r(x, z), y)\, \mathrm{e}^{-\mathrm{i}\gamma_m R\theta(x, z)},$$

- Straight bus core: 3-D (2-D) guided modes (ϵ_n)

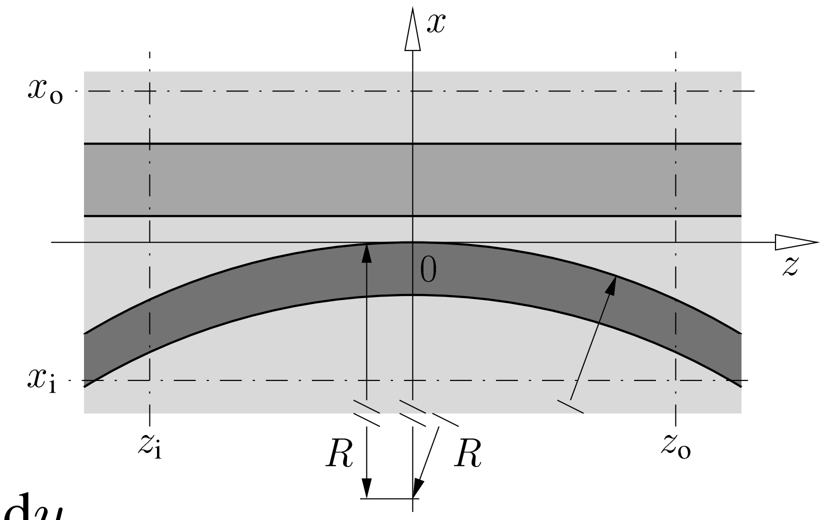
$$\begin{pmatrix} \boldsymbol{E}_n \\ \boldsymbol{H}_n \end{pmatrix}(x, y, z) = \begin{pmatrix} \boldsymbol{E}_{n,0} \\ \boldsymbol{H}_{n,0} \end{pmatrix}(x, y)\, \mathrm{e}^{-\mathrm{i}\beta_n z}.$$

Coupled mode ansatz:

$$\begin{pmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{H}} \end{pmatrix}(x, y, z, t) = \frac{1}{2}\mathrm{Re} \sum_m A_m(z) \begin{pmatrix} \boldsymbol{E}_m \\ \boldsymbol{H}_m \end{pmatrix}(x, y, z)\, \mathrm{e}^{\mathrm{i}\omega t}.$$

Coupler region: CMT equations, solution

Suitable integral form of Maxwell equations



$$\mathsf{O}(z)\,\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{A}(z) = \mathsf{K}(z)\,\boldsymbol{A}(z)\,,$$

$$\boldsymbol{A} = (A_m)\,,\quad \mathsf{O} = (\sigma_{lm})\,,\quad \mathsf{K} = (\kappa_{lm})\,,$$

$$\sigma_{lm} = \frac{1}{4}\iint \boldsymbol{e}_z \cdot (\boldsymbol{E}_m \times \boldsymbol{H}_l^* + \boldsymbol{E}_l^* \times \boldsymbol{H}_m)\,\mathrm{d}x\,\mathrm{d}y\,,$$

$$\kappa_{lm} = -\mathrm{i}\,\frac{\omega\epsilon_0}{4}\iint \boldsymbol{E}_l^* \cdot (\epsilon - \epsilon_m)\,\boldsymbol{E}_m\,\mathrm{d}x\,\mathrm{d}y\,.$$

Numerical solution $[z_\mathrm{i}, z_\mathrm{o}]$, projection $|_{z_\mathrm{i}}$, $|_{z_\mathrm{o}}$

Coupler scattering matrix S;

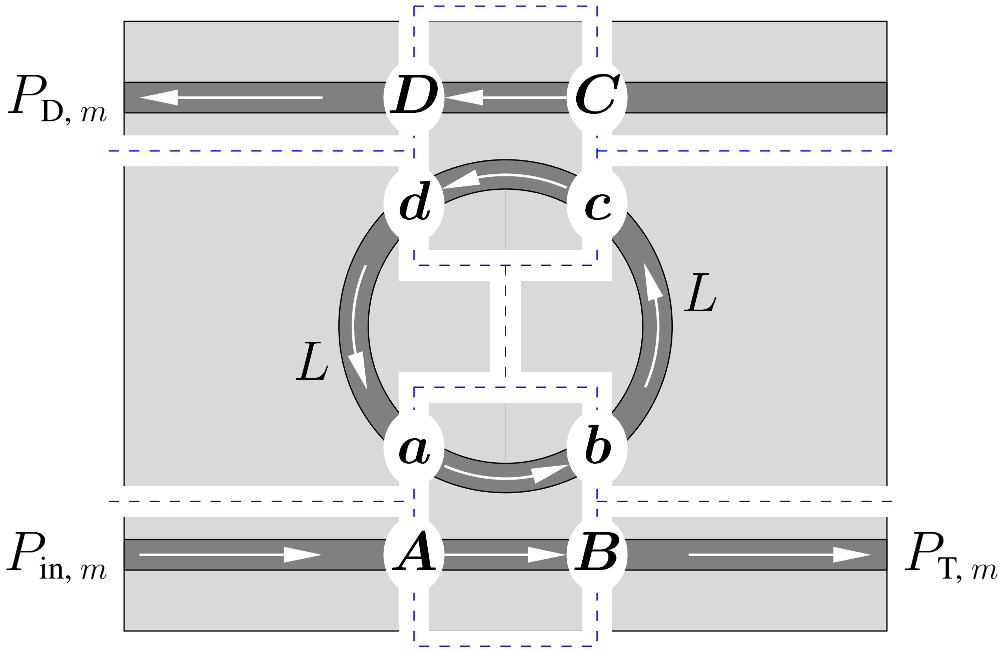
$$\sum_o |S_{o\,i}|^2 \leq 1\,,\quad S_{o\,i} = S_{i\,o}\quad \left(\begin{array}{l}\text{normalized modes,}\\ \text{symmetric setting}\end{array}\right).$$

Resonator: Power transmission, spectra

$$\begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{b} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\mathrm{ss}} & \mathsf{S}_{\mathrm{sb}} \\ \mathsf{S}_{\mathrm{bs}} & \mathsf{S}_{\mathrm{bb}} \end{pmatrix} \begin{pmatrix} \boldsymbol{A} \\ \boldsymbol{a} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathsf{S}_{\mathrm{ss}} & \mathsf{S}_{\mathrm{sb}} \\ \mathsf{S}_{\mathrm{bs}} & \mathsf{S}_{\mathrm{bb}} \end{pmatrix} \begin{pmatrix} \boldsymbol{C} \\ \boldsymbol{c} \end{pmatrix},$$

$$\boldsymbol{c} = \mathsf{G}\boldsymbol{b}, \quad \boldsymbol{a} = \mathsf{G}\boldsymbol{d}, \qquad \mathsf{G} = \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}\gamma_m L}\right).$$



$P_{\mathrm{in},m}$, $\boldsymbol{A}$ given, $\boldsymbol{C} = 0$:

$$\boldsymbol{b} = \Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}, \qquad \Omega = 1 - \mathsf{S}_{\mathrm{bb}}\mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}, \qquad P_{\mathrm{D},m} = |D_m|^2,$$

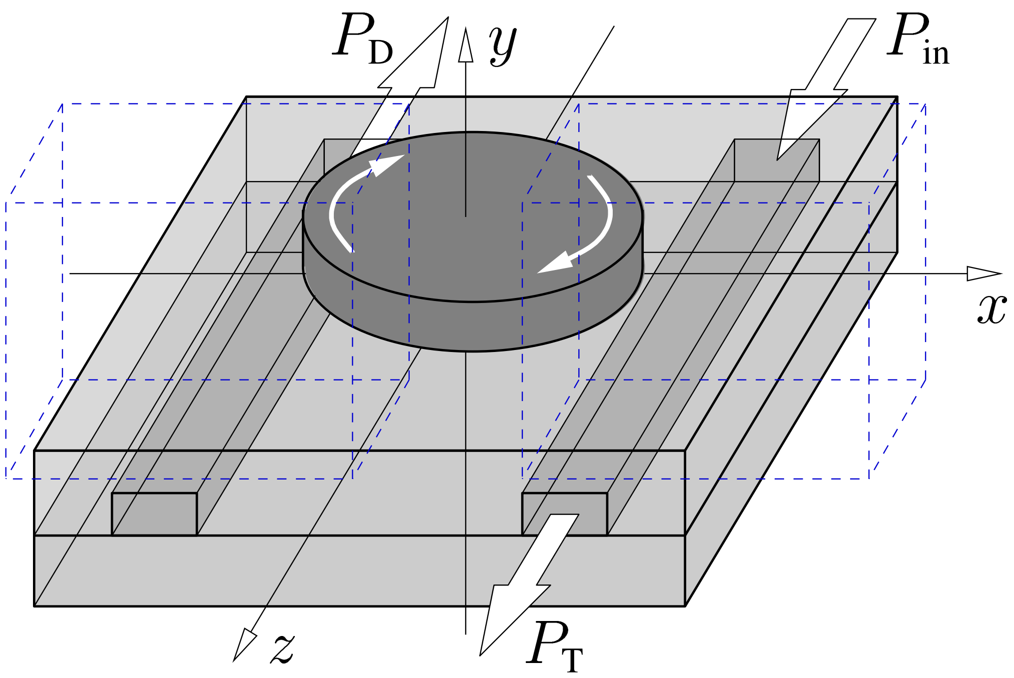
$$\boldsymbol{c} = \mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}, \qquad P_{\mathrm{T},m} = |B_m|^2,$$

$$\boldsymbol{d} = \mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}, \qquad \boldsymbol{D} = \mathsf{S}_{\mathrm{sb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A},$$

$$\boldsymbol{a} = \mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}}\boldsymbol{A}, \qquad \boldsymbol{B} = (\mathsf{S}_{\mathrm{sb}}\mathsf{G}\mathsf{S}_{\mathrm{bb}}\mathsf{G}\Omega^{-1}\mathsf{S}_{\mathrm{bs}} + \mathsf{S}_{\mathrm{ss}})\boldsymbol{A}.$$

Spectrum: $\mathsf{S}(\cancel{\lambda})_{L\to 2\pi R/2}$, $\gamma_m(\lambda)$ interpolated $\rightsquigarrow$ $P_{\mathrm{T},m}(\lambda)$, $P_{\mathrm{D},m}(\lambda)$.

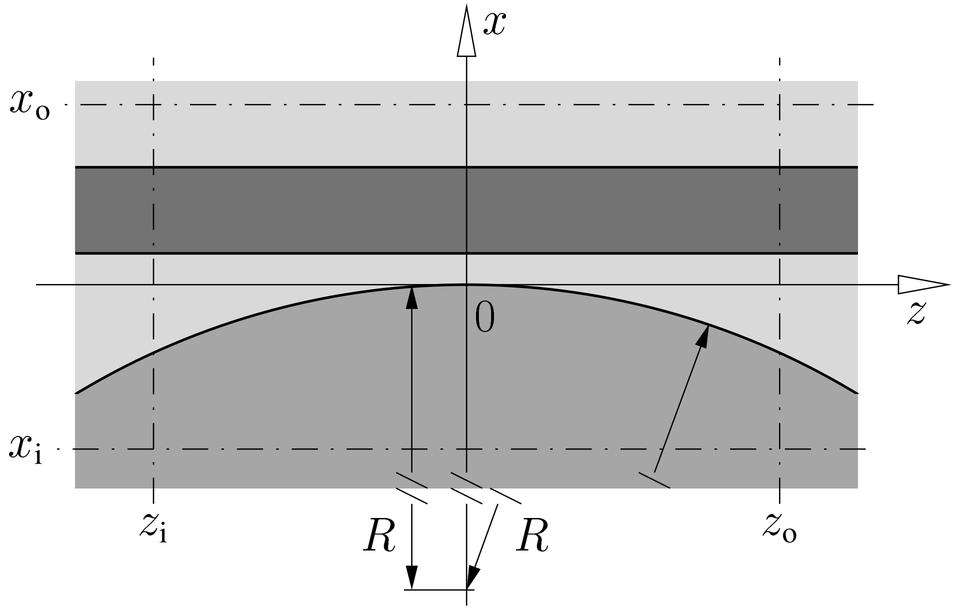
Vertically coupled multimode microdisk-resonator

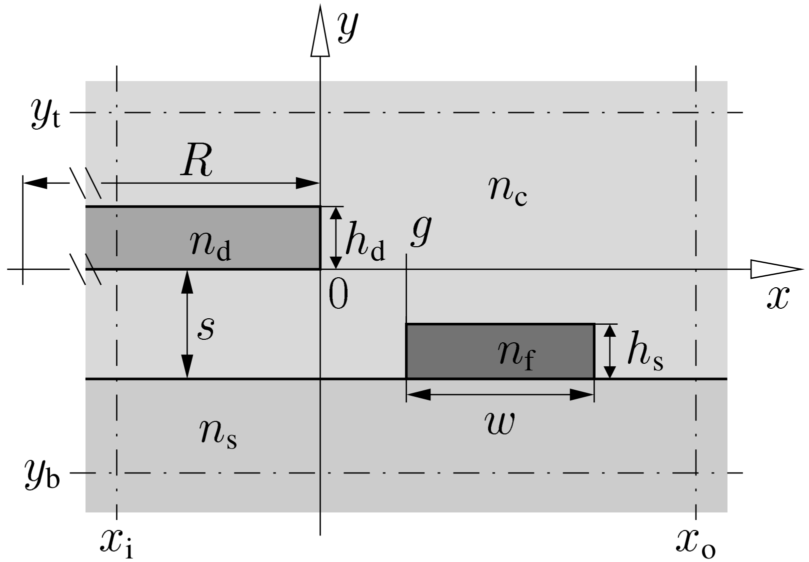


$w = 2.0\,\mu$m, $h_s = 140\,$nm,
$n_f = 1.98$, $n_s = 1.45$,
$n_c = 1.4017$, $n_d = 1.6062$,
$h_d = 1.0\,\mu$m, $R = 100\,\mu$m;

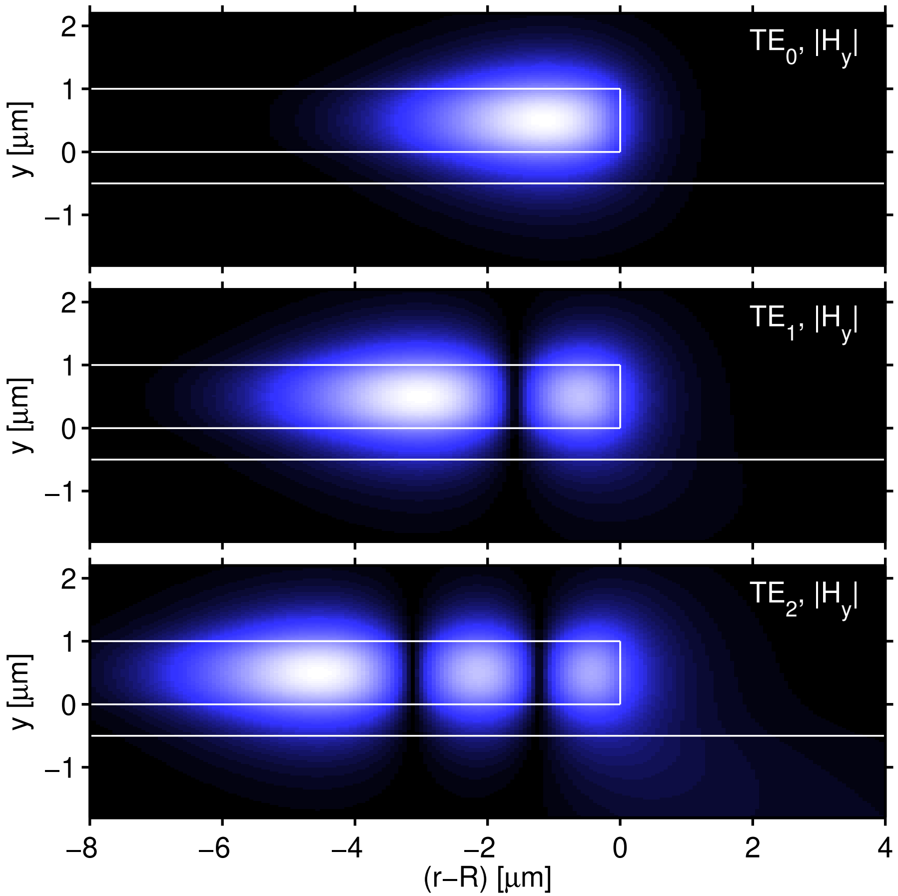
varying g, s;
target wavelength: $\lambda = 1.55\,\mu$m.

CMT computational window:
$[x_i, x_o] = [-12, 4]\,\mu$m,
$[y_b, y_t] = [-4, 4]\,\mu\text{m} - s$,
$[z_i, z_o] = [-30, 30]\,\mu$m.





Vertical straight-disk-coupler, CMT basis fields



$s = 0.5\,\mu$m

Cavity disk:
three TE-like bend modes,

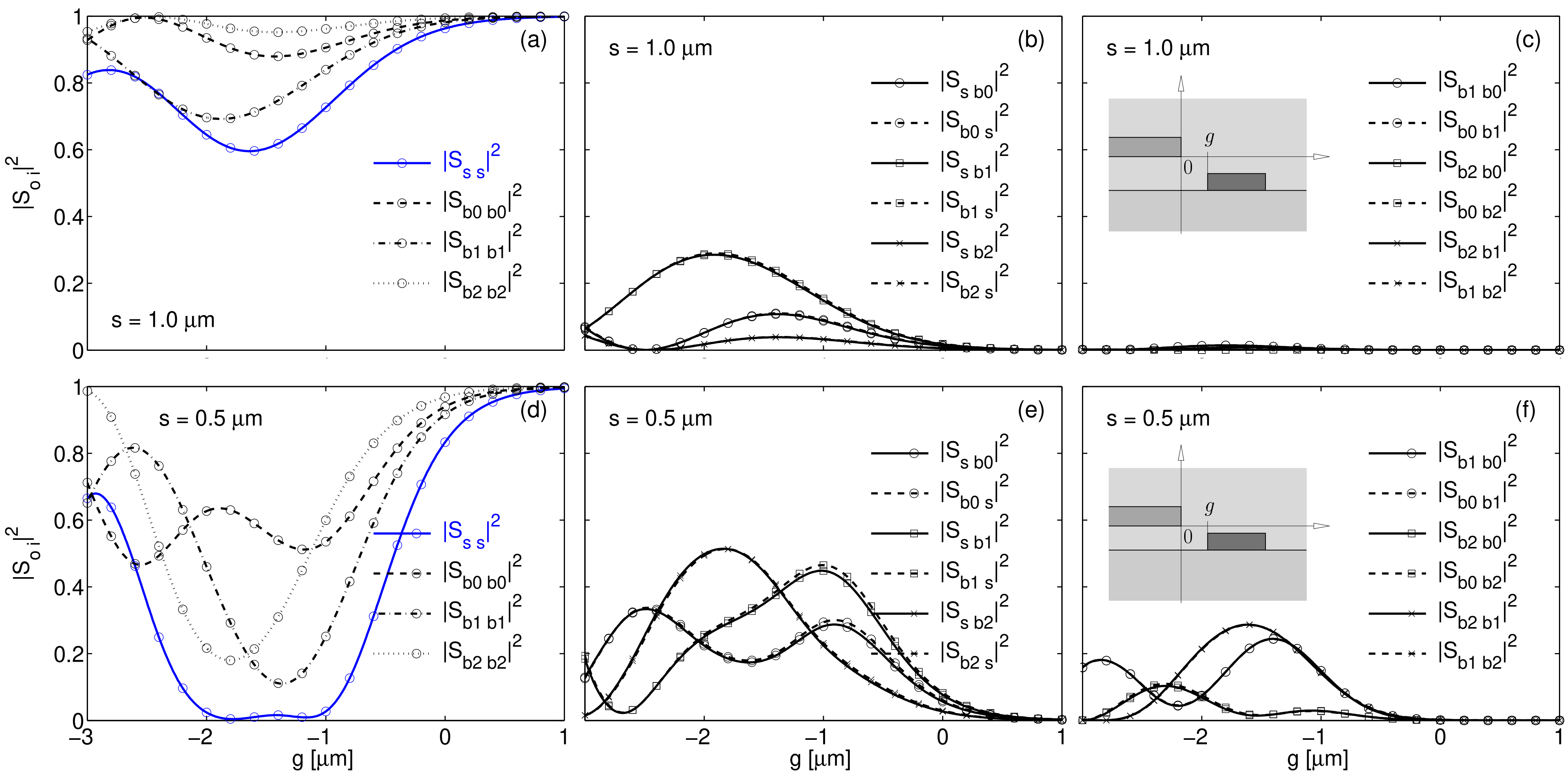
	n_{eff}
b0, TE$_0$	$1.503778 - \mathrm{i}\,1.35 \cdot 10^{-9}$
b1, TE$_1$	$1.474931 - \mathrm{i}\,1.77 \cdot 10^{-6}$
b2, TE$_2$	$1.451487 - \mathrm{i}\,5.05 \cdot 10^{-5}$

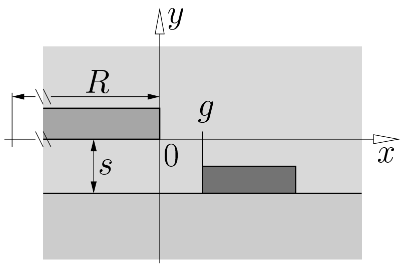
Bus waveguides:
one TE-like mode, s, $n_{\text{eff}} = 1.48229$.

CMT

$$\begin{pmatrix} B_{\mathrm{s}} \\ b_{\mathrm{b0}} \\ b_{\mathrm{b1}} \\ b_{\mathrm{b2}} \end{pmatrix}_{z_{\mathrm{o}}} = \begin{pmatrix} S_{\mathrm{s\,s}} & S_{\mathrm{s\,b0}} & S_{\mathrm{s\,b1}} & S_{\mathrm{s\,b2}} \\ S_{\mathrm{b0\,s}} & S_{\mathrm{b0\,b0}} & S_{\mathrm{b0\,b1}} & S_{\mathrm{b0\,b2}} \\ S_{\mathrm{b1\,s}} & S_{\mathrm{b1\,b0}} & S_{\mathrm{b1\,b1}} & S_{\mathrm{b1\,b2}} \\ S_{\mathrm{b2\,s}} & S_{\mathrm{b2\,b0}} & S_{\mathrm{b2\,b1}} & S_{\mathrm{b2\,b2}} \end{pmatrix} \begin{pmatrix} A_{\mathrm{s}} \\ a_{\mathrm{b0}} \\ a_{\mathrm{b1}} \\ a_{\mathrm{b2}} \end{pmatrix}_{z_{\mathrm{i}}} .$$

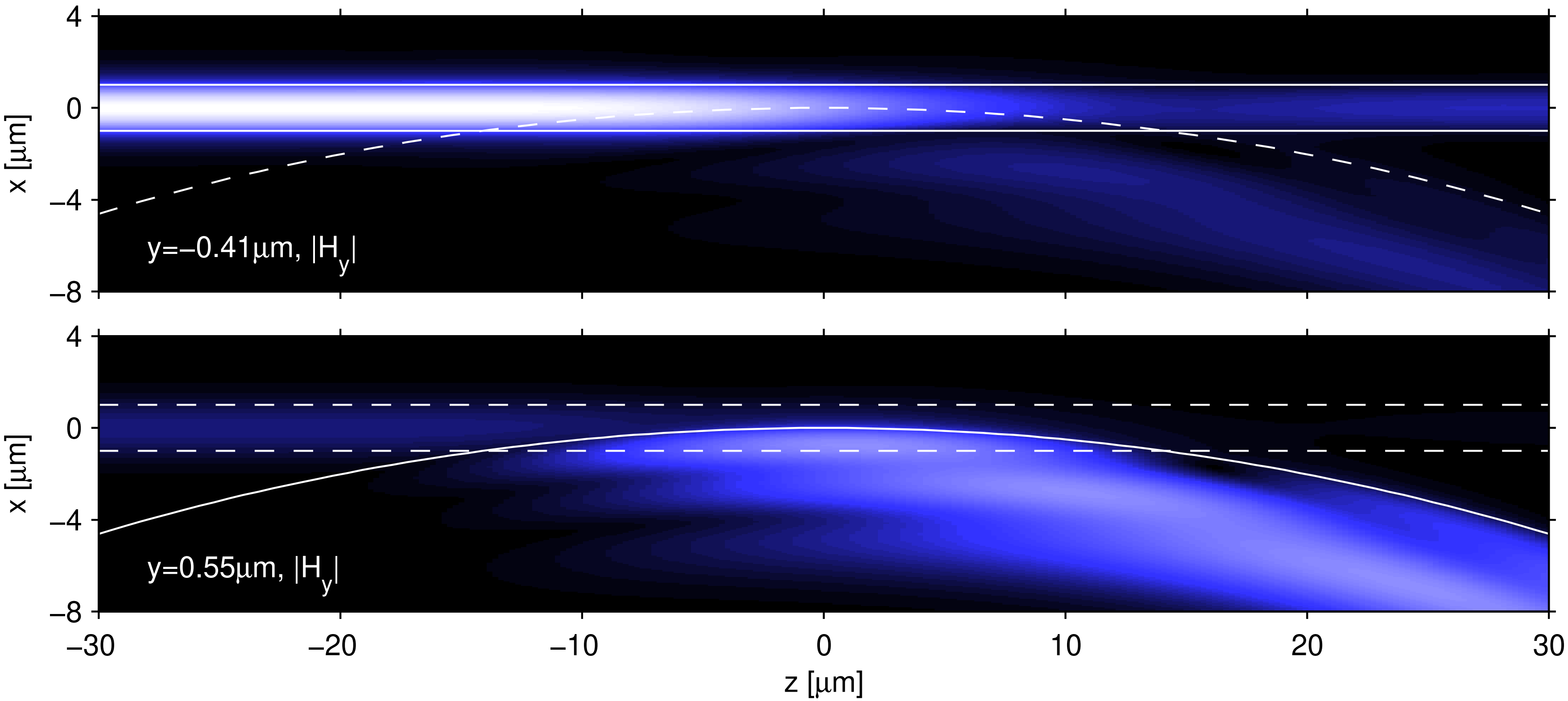
Vertical straight-disk-coupler, scattering matrices

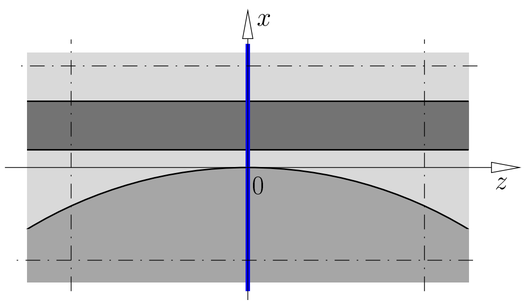


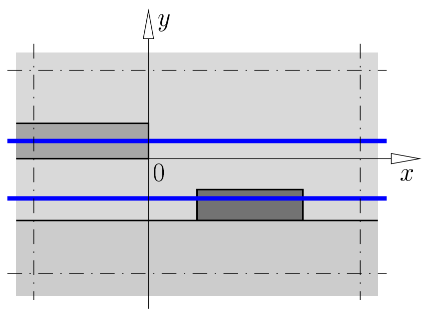


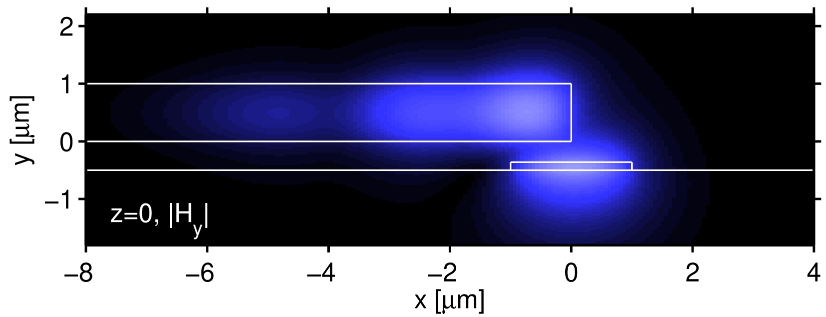
$|\mathrm{S}_{o\,i}|^2$: power transfer mode $i|_{z_\mathrm{i}}$ $\rightarrow$ mode $o|_{z_\mathrm{o}}$.

Vertical straight-disk-coupler, field examples



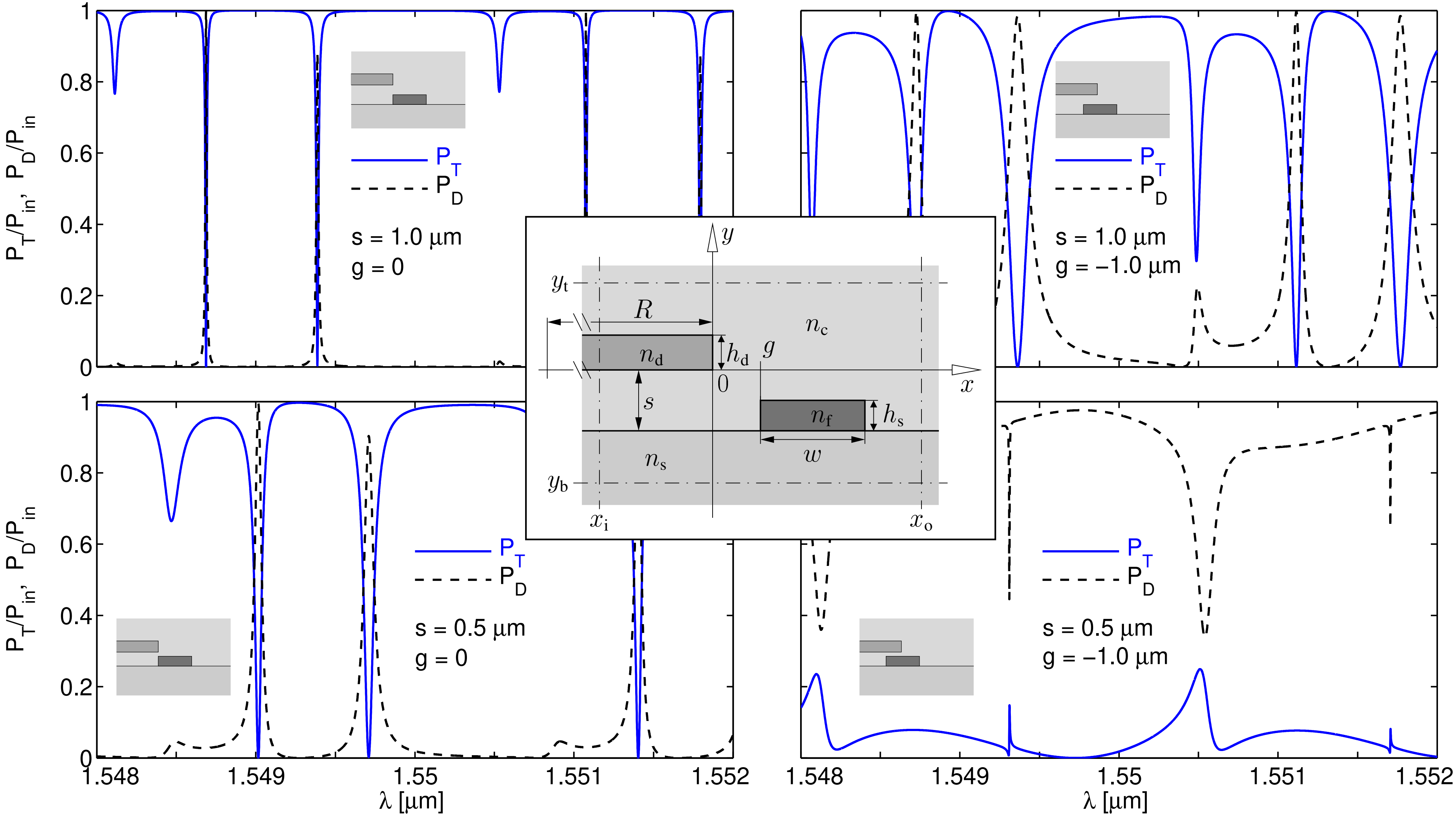




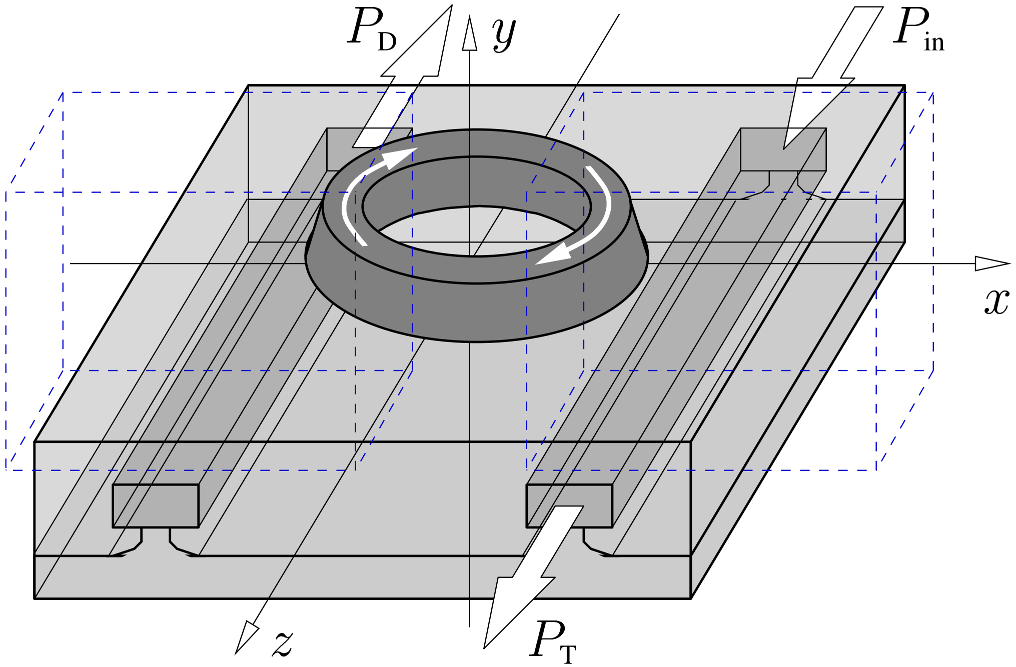


$s = 0.5\,\mu$m, $g = -1.0\,\mu$m; excitation in the bus waveguide.

Multimode microdisk-resonator, spectra



Resonator with hybrid ring cavity



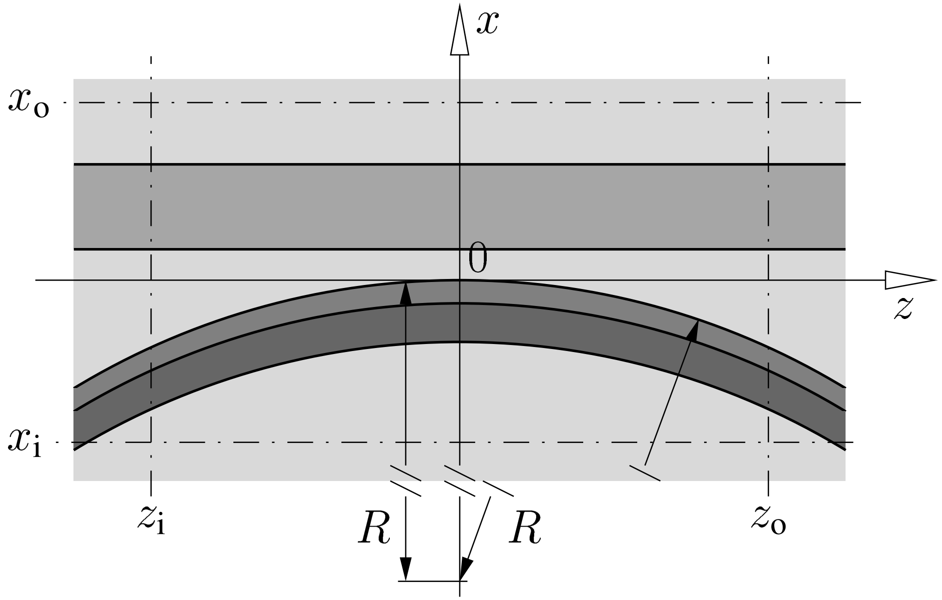
$n_f = 2.009$, $h_s = 0.27\,\mu$m, $w_s = 2.5\,\mu$m,
$n_s = 1.45$, $u = 0.9\,\mu$m

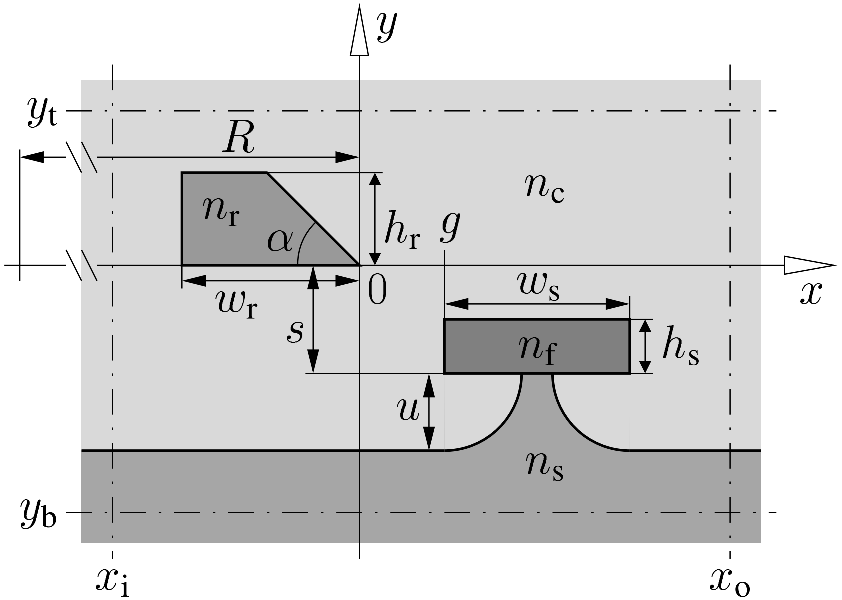
$n_r = 1.6275$, $w_r = 2.0\,\mu$m, $h_r = 1.0\,\mu$m,
$R = 100\,\mu$m, $\alpha = 48°$.

$s = 1.0\,\mu$m, $g = -2.25\,\mu$m, $n_c = 1.412$.

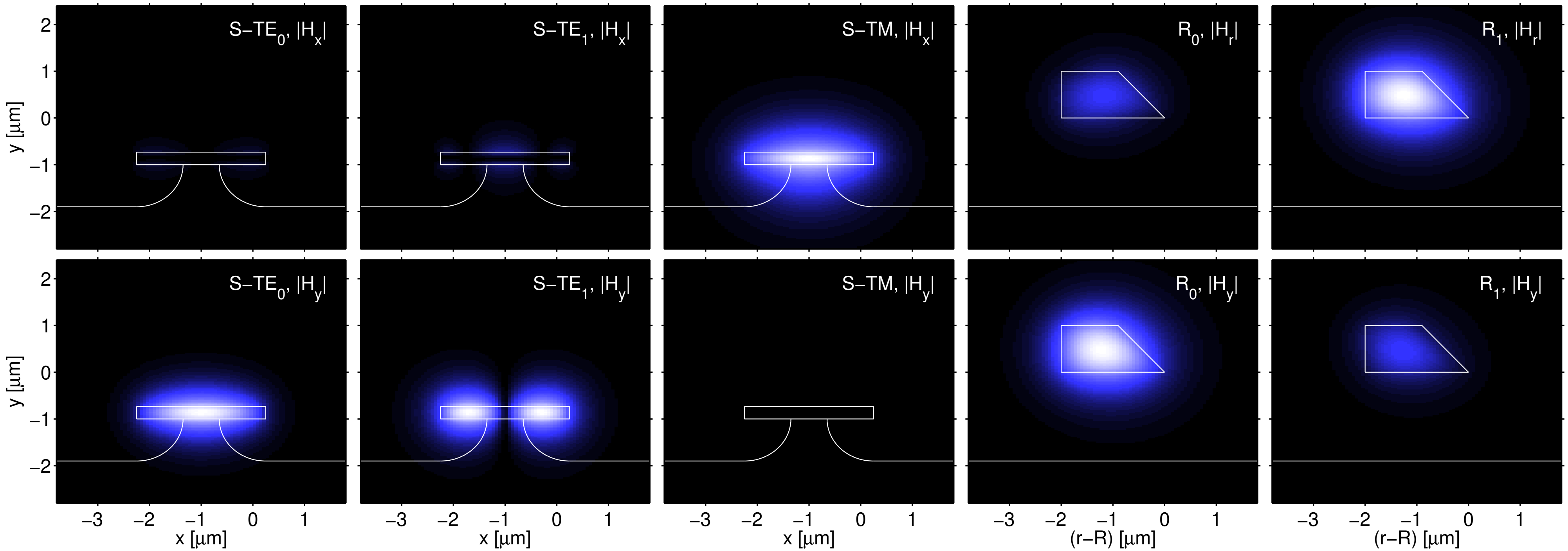
Target wavelength: $\lambda = 1.55\,\mu$m.

CMT computational window:
$[x_i, x_o] = [-12, 4]\,\mu$m,
$[y_b, y_t] = [-3.7, 4.4]\,\mu$m,
$[z_i, z_o] = [-35, 35]\,\mu$m.





Hybrid coupler, CMT basis fields & scattering matrix

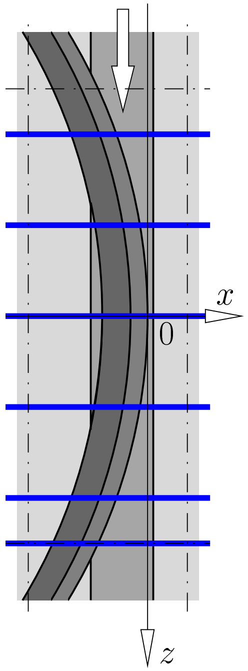


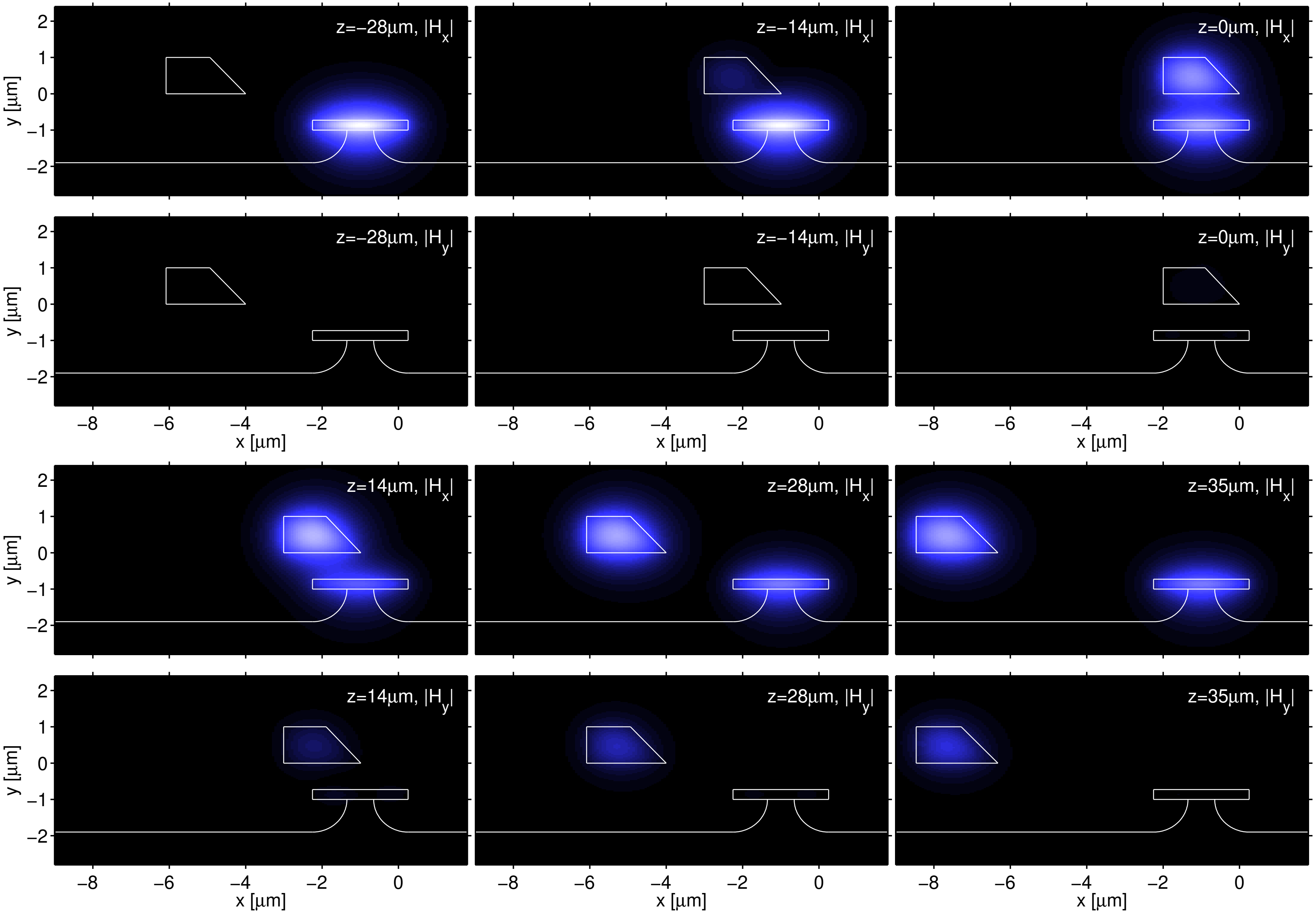
	n_{eff}
S-TE$_0$	1.625326
S-TE$_1$	1.553733
S-TM	1.511020
R$_0$	$1.490975 - \mathrm{i}\, 2.7 \cdot 10^{-7}$
R$_1$	$1.483477 - \mathrm{i}\, 1.2 \cdot 10^{-7}$

CMT $\rightsquigarrow$

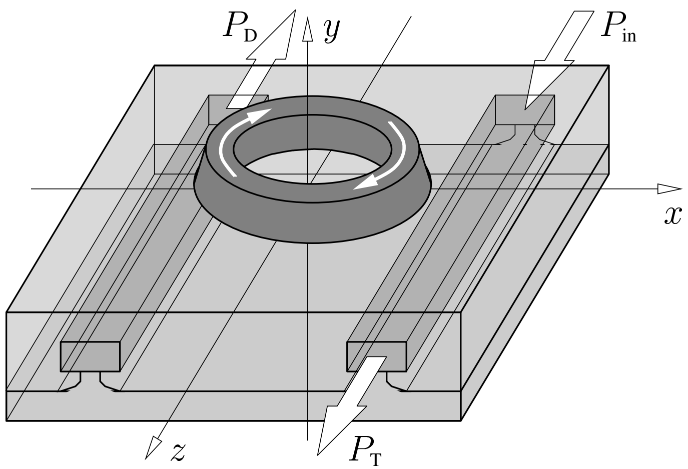
%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	≈ 0	≈ 0	≈ 0	≈ 0
S-TE$_1$	≈ 0	97.8	≈ 0	1.9	0.1
S-TM	≈ 0	≈ 0	26.2	6.3	67.0
R$_0$	≈ 0	2.1	6.7	86.2	5.5
R$_1$	≈ 0	0.1	67.5	5.2	27.3

Hybrid coupler, CMT field example

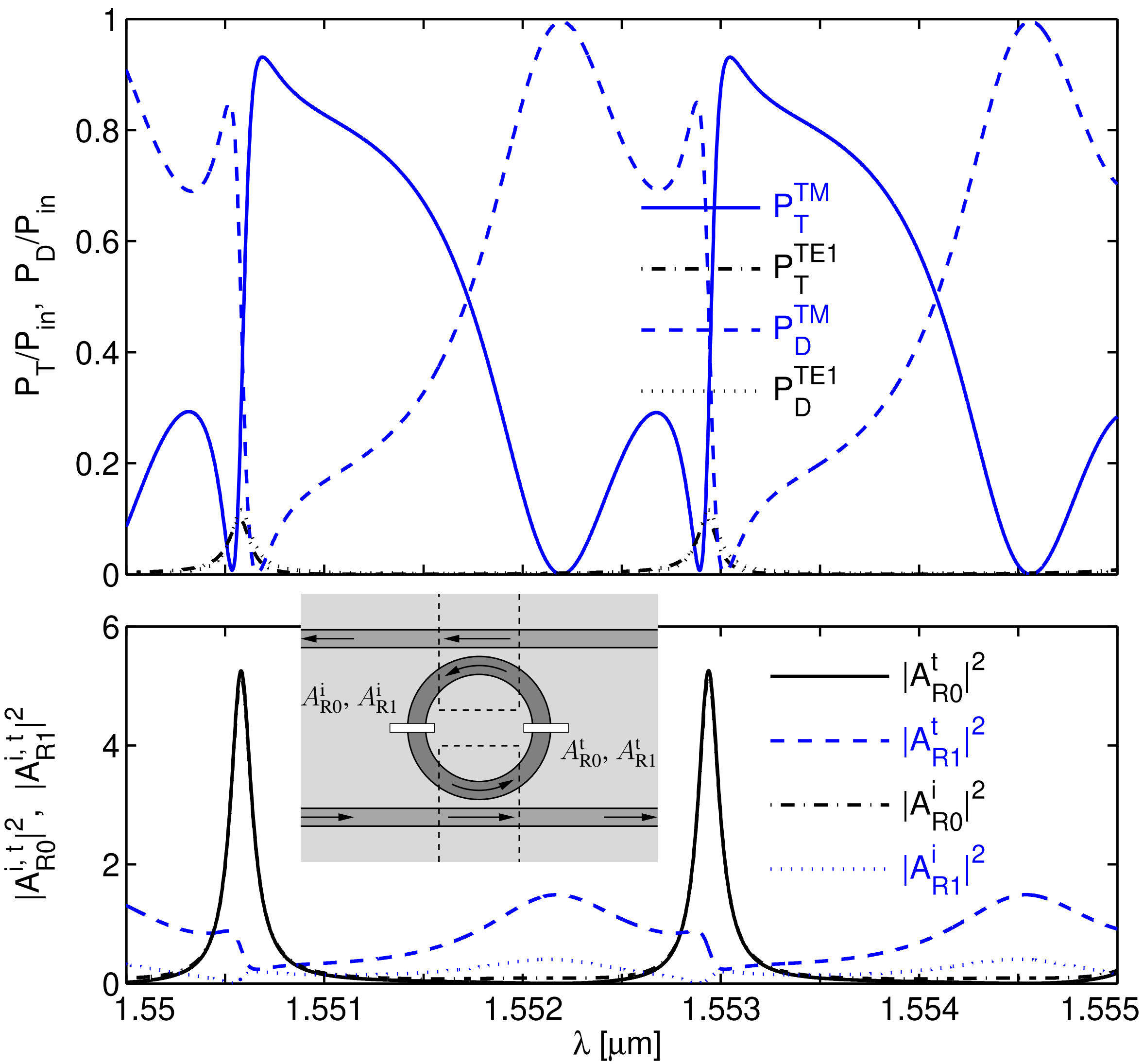




Resonator with hybrid ring cavity, spectrum



%	S-TE$_0$	S-TE$_1$	S-TM	R$_0$	R$_1$
S-TE$_0$	100.0	–	–	–	–
S-TE$_1$	–	97.8	–	1.9	0.1
S-TM	–	–	26.2	6.3	67.0
R$_0$	–	2.1	6.7	86.2	5.5
R$_1$	–	0.1	67.5	5.2	27.3



3-D Microresonator Simulator