Resonator chains of 2-D square dielectric optical microcavities

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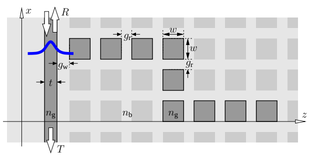
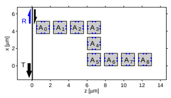
1 *Chains of square 2D microcavities*

Coupled-resonator optical waveguides (CROWs) have been discussed already for some years [1] as a means to realize waveguiding along paths with small-size bends. Concepts based on series of microring resonators or sequences of defects in photonic crystal slabs [2] exist. As an alternative, we consider chains of simple square dielectric cavities, that support a single specific standing wave resonance [3, 4] in the wavelength region of interest. In line with the fourfold symmetry of their resonant field pattern, the individual cavities are arranged sequentially on a discrete rectangular mesh, with guided-wave excitation at one end of the chain. Rigorous semianalytical simulations based on quadridirectional eigenmode propagation (QUEP) [5, 6] enable convenient numerical experiments on these rectangular, piecewise constant configurations.

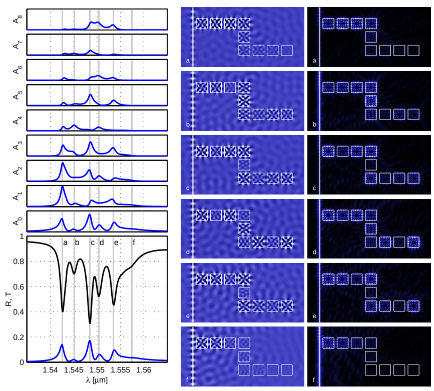


As some step towards an interpretation of the spectral features we look at an intuitive coupled mode theory (CMT) model for the resonator chains. The overall field in the chain is assumed to consist of bidirectional versions of the guided mode of the bus core, with variable local amplitudes, together with the identical, properly positioned resonant field patterns of the individual cavities, each multiplied by a single scalar coefficient. These latter fields can be approximated quite well by a superposition of suitable slab mode profiles, oriented along the two coordinate axes [7]. Then one proceeds along the hybrid CMT approach (HCMT) of Ref. [8]: By variational means [9, 10, 11] one extracts a linear system of equations for the coefficients of the resonator fields, and for the amplitude functions of the bus modes, discretized in terms of finite elements, as unknowns. Note that no free parameters are introduced; the model, however, disregards any radiative losses (so far), and thus cannot be more than an approximation of the resonator chain in a kind of high-Q limit.

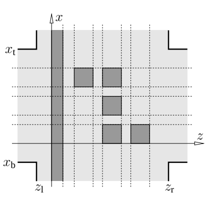
2 *Spectral response, resonant fields*



Parameters, sections 2–6:
$n_b = 1.45$, $n_g = 3.40$, $t = 0.073\,\mu$m, $w = 1.451\,\mu$m,
$g_w = g_r = 0.4\,\mu$m; 2-D, TE, principal electric component E_y.
Spectra:
guided reflection R and transmission T, local intensities A_0–A_8, maximized over 8 observation points per cavity.
Fields:
physical time snapshots E_y and moduli $|E_y|$.

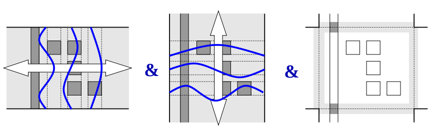


3 *QUEP simulations*



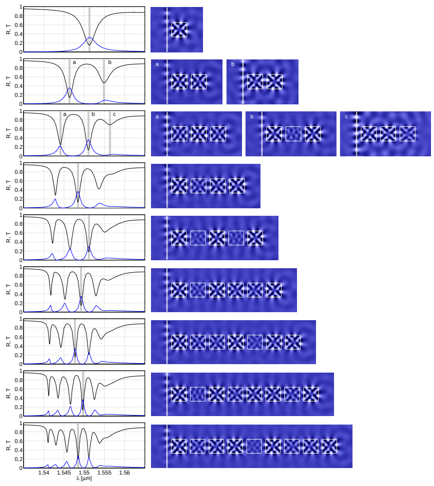
- 2-D Helmholtz problems, TE / TM.
- Piecewise constant, rectangular refractive index; linear, lossless materials.
- Fixed frequency $\omega = k\,c$, vacuum wavelength $\lambda = 2\pi/k$.
- Rectangular interior domain, influx & outflux across all four boundaries, outwards homogeneous external regions.

- Assumption $E_y = 0$ ($H_y = 0$) on the external border lines must be reasonable for the problems at hand.
- Division into layers and slices.
- Expansion basis: 1-D modes associated with layers *and* slices; discretization: $E_y = 0$ (TE) at $x = x_b, x_t$, $z = z_l, z_r$.
- Bidirectional projection on all interfaces
  ⇝ solution procedure:

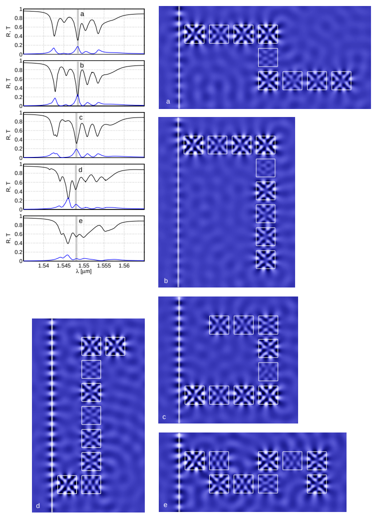


"QUadridirectional Eigenmode Propagation method" (QUEP)

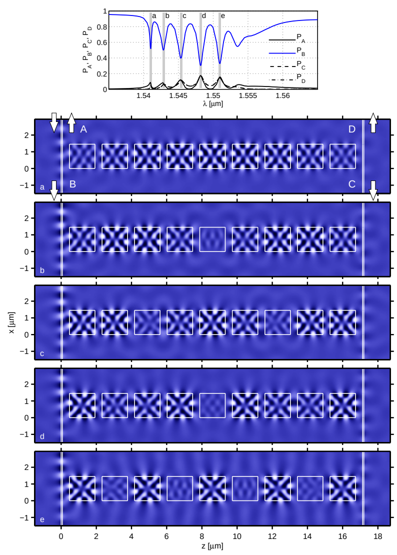
4 *Length*



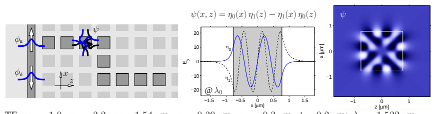
5 *Shape*



6 *Two bus waveguides*



7 *HCMT model*



TE, $n_b = 1.0$, $n_g = 3.2$, $w = 1.54\,\mu$m, $g_r = 0.39\,\mu$m, $g_w = 0.3\,\mu$m, $t = 0.2\,\mu$m; $\lambda_0 = 1.532\,\mu$m.

- Assumption: the optical electromagnetic field is well represented by the template

$$\begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = u(x)\,\boldsymbol{\phi}_u(x,z) + d(x)\,\boldsymbol{\phi}_d(x,z) + \sum_{j=0}^{8} r_j\,\boldsymbol{\psi}_j(x,z)$$

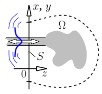
  $\boldsymbol{\phi}_u$, $\boldsymbol{\phi}_d$: guided bus modes, upward and downward traveling,
  $\boldsymbol{\psi}_j$: resonant field pattern of cavity j, here a slab mode superposition,
  $r_0 - r_8$, u, d: unknown amplitudes / functions.

- Discretize $u \to \{u_l\}$ and $d \to \{d_l\}$ by 1-D linear finite elements

$$\rightsquigarrow \quad \begin{pmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{pmatrix}(x,z) = \sum_k a_k\,\boldsymbol{\chi}_k(x,z), \qquad \boldsymbol{\chi}_k\text{: modal elements, } a_k \in \{u_l, d_l, r_j\}.$$

- Variational representation of the Helmholtz problem on Ω with transparent-influx-boundary conditions on S in terms of the functional (here 3-D)

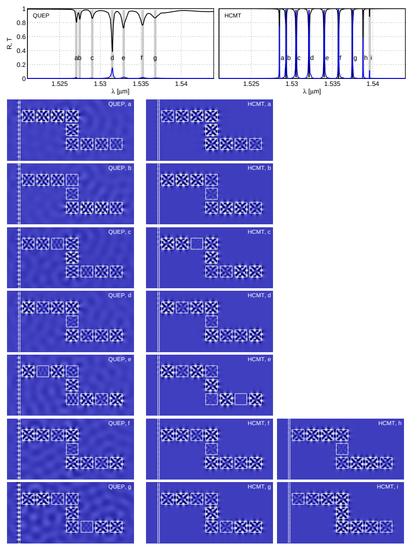
$$\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) = \iiint_\Omega \{\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{H}) + \boldsymbol{H}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) - \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E}^2 + \mathrm{i}\omega\mu_0\boldsymbol{H}^2\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$
$$- \sum_m 2F_m\,\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle\} + \sum_m \frac{1}{2N_m}\{\langle\tilde{\boldsymbol{E}}_m,\boldsymbol{H}\rangle^2 - \langle\boldsymbol{E},\tilde{\boldsymbol{H}}_m\rangle^2\},$$



  - complete set of modes on S, profiles $(\tilde{\boldsymbol{E}}_m, \pm\tilde{\boldsymbol{H}}_m)(x,y)$ ⟷ propagation along $\pm z$, toward the interior / exterior of Ω,
  - $\langle\boldsymbol{A},\boldsymbol{B}\rangle = \iint_S (\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{e}_z\,\mathrm{d}x\,\mathrm{d}y$,
  - F_m: influx, given coefficients of incoming waves.

- Restrict $\mathcal{F}(\boldsymbol{E},\boldsymbol{H}) \to \mathcal{F}_r(a_k)$ to the template, require $\delta_{a_k}\mathcal{F}_r = 0$
  ⇝ system of linear equations in a_k, numerical solution.

8 *HCMT, results*



Acknowledgements

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