Perturbational Estimation of Geometry Tolerances for Rectangular Integrated Optics Devices

M. Lohmeyer[*], N. Bahlmann, O. Zhuromskyy, P. Hertel*

UNIVERSITÄT OSNABRÜCK

Department of Physics, Osnabrück, Germany

Contents

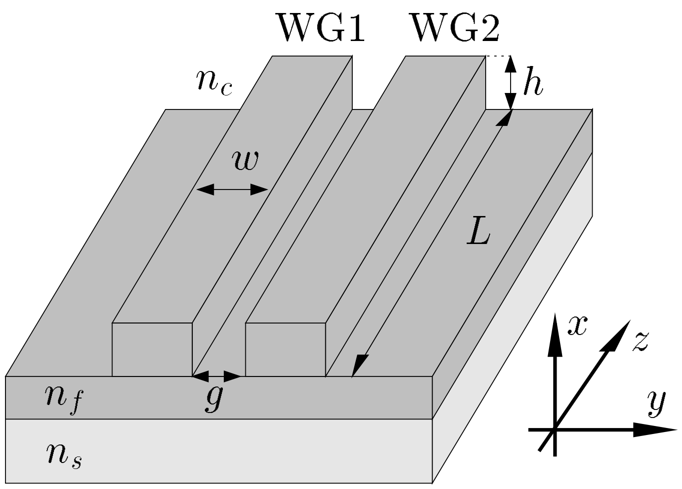
Integrated Optics Devices III — Optoelectronics '99 — Photonics West '99

23 – 29 January 1999 *San Jose, California, USA*

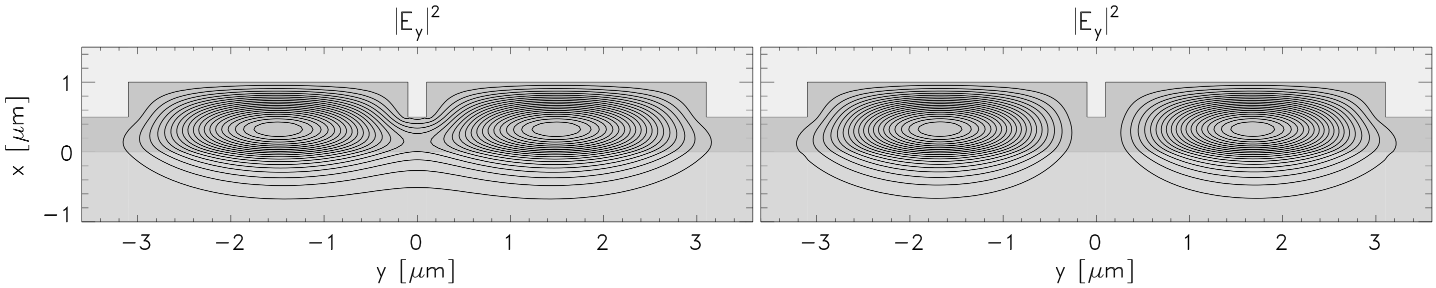
[*] *Department of Physics, University of Osnabrück* *D-49069 Osnabrück, Germany*

Tel.: +49/541/969-2641 Fax: +49/541/969-2670 manfred.lohmeyer@physik.uni-osnabrueck.de

1. *Perturbational Estimation of Geometry Tolerances*



Coupler modes, propagation constants β_s, β_a:

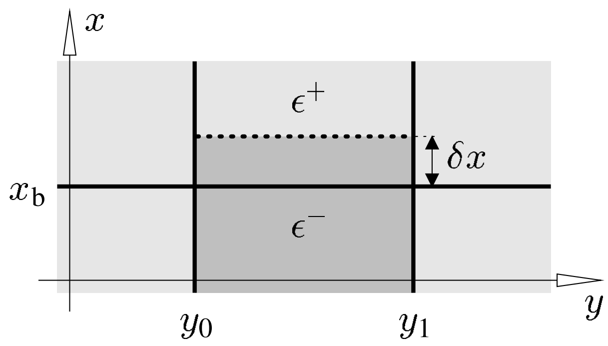


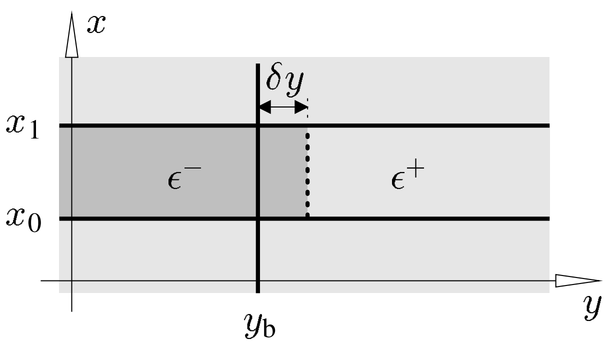
Power transfer WG1 $\longrightarrow$ WG2: $\quad P_{1\to 2} \approx \frac{1}{2}(1 - \cos(\beta_s - \beta_a)L)\,.$

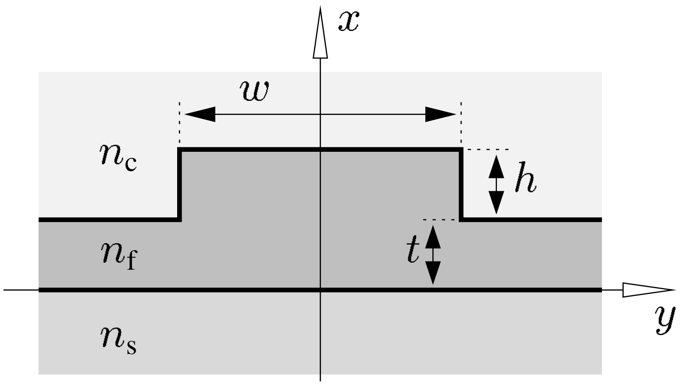
q: any geometry parameter.

$$\frac{\partial P}{\partial q} \quad ? \qquad\qquad \frac{\partial \beta}{\partial q} \quad ?$$

Shift of a boundary in the dielectric profile
$\leftrightarrow$ thin layer perturbations on the strip surface/sidewalls ...







Guided modes: Electromagnetic fields of the form

$$\boldsymbol{E}(x,y)\ \mathrm{e}^{\mathrm{i}(\omega t\,-\,\beta z)}, \quad \boldsymbol{H}(x,y)\ \mathrm{e}^{\mathrm{i}(\omega t\,-\,\beta z)},$$

$\omega\sqrt{\epsilon_0\mu_0} = k = 2\pi/\lambda$ for wavelength λ.
$\boldsymbol{E}(x,y)$, $\boldsymbol{H}(x,y)$: the square integrable mode profile,
β: the corresponding propagation constant.

Maxwell's curl equations:

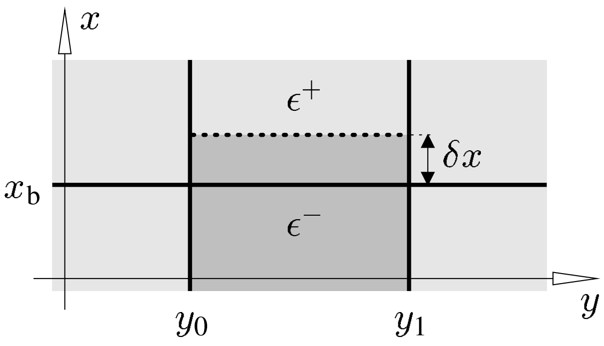
$$(\mathsf{C} + \mathrm{i}\beta\mathsf{R})\boldsymbol{E} = -\mathrm{i}\omega\mu_0\boldsymbol{H}, \quad (\mathsf{C} + \mathrm{i}\beta\mathsf{R})\boldsymbol{H} = \mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E},$$

$$\mathsf{C} = \begin{pmatrix} 0 & 0 & \partial_y \\ 0 & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}, \quad \mathsf{R} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$(\boldsymbol{F}, \boldsymbol{G}) := \iint \boldsymbol{F}^*\boldsymbol{G}\,\mathrm{d}x\,\mathrm{d}y$;
Define B_ϵ as a functional of the independent fields $E_x, \ldots, H_z$:

$$\mathsf{B}_\epsilon(\boldsymbol{E},\boldsymbol{H}) = \frac{\omega\epsilon_0(\boldsymbol{E},\epsilon\boldsymbol{E}) + \omega\mu_0(\boldsymbol{H},\boldsymbol{H}) + \mathrm{i}(\boldsymbol{E},\mathsf{C}\boldsymbol{H}) - \mathrm{i}(\boldsymbol{H},\mathsf{C}\boldsymbol{E})}{(\boldsymbol{E},\mathsf{R}\,\boldsymbol{H}) - (\boldsymbol{H},\mathsf{R}\,\boldsymbol{E})}.$$

B_ϵ is stationary at a valid mode field with $\beta = \mathsf{B}_\epsilon(\boldsymbol{E},\boldsymbol{H})$.

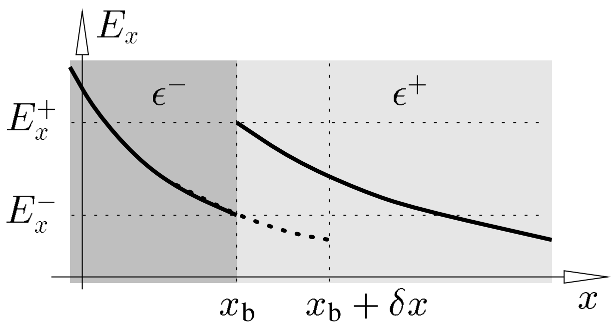


$\square$: the region
$x_b < x < x_b + \delta x$,
$y_0 < y < y_1$.

Shifting the boundary from x_b to $x_b + \delta x$ changes
$\epsilon \longrightarrow \epsilon + \delta\epsilon$,
$\beta \longrightarrow \beta + \delta\beta$,
$(\boldsymbol{E}, \boldsymbol{H}) \longrightarrow (\boldsymbol{E} + \delta\boldsymbol{E}, \boldsymbol{H} + \delta\boldsymbol{H})$,

$$\delta\epsilon\,(x, y) = \begin{cases} (\epsilon^- - \epsilon^+) & \text{for } (x, y) \in \square, \\ 0 & \text{otherwise,} \end{cases}$$

$\beta + \delta\beta = \mathsf{B}_{\epsilon+\delta\epsilon}(\boldsymbol{E} + \delta\boldsymbol{E}, \boldsymbol{H} + \delta\boldsymbol{H})$.

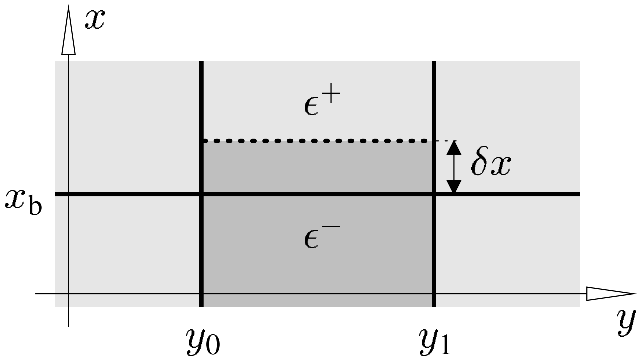


$\delta\boldsymbol{E}$, $\delta\boldsymbol{H}$?

- E_y, E_z, H_x, H_y, H_z are continuous at x_b and at $x_b + \delta x$
  $\rightsquigarrow$ $\delta E_y = \delta E_z = 0$, $\delta H_x = \delta H_y = \delta H_z = 0$.
- E_x jumps at x_b: $\epsilon^- E_x^- = \epsilon^+ E_x^+$, but is continuous at $x_b + \delta x$
  $\rightsquigarrow$
  $$\delta E_x(x, y) = \begin{cases} \dfrac{(\epsilon^+ - \epsilon^-)}{\epsilon^-} E_x(x, y) & \text{for } (x, y) \in \square, \\ 0 & \text{otherwise.} \end{cases}$$
  $E_x + \delta E_x$ is continuous at x_b, but jumps at $x_b + \delta x$.

Result (C. Vassallo: *Optical Waveguide Concepts*, Elsevier 1991):

$$\delta\beta = \frac{\omega\epsilon_0}{2} \frac{(\epsilon^- - \epsilon^+) \iint_\square \left(\dfrac{\epsilon^+}{\epsilon^-} |E_x|^2 + |E_y|^2 + |E_z|^2 \right) \mathrm{d}x\,\mathrm{d}y}{\iint \left(E_x H_y^* - E_y H_x^* \right) \mathrm{d}x\,\mathrm{d}y}.$$



Horizontal boundary shifts:

- hybrid fields

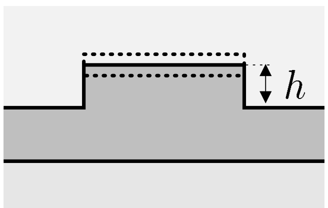
$$\delta\beta = \frac{\omega\epsilon_0}{2} \frac{(\epsilon^- - \epsilon^+) \displaystyle\int_{y_0}^{y_1} \left((E_x^+)^* E_x^- + |E_y|^2 + |E_z|^2\right)(x_b, y)\,\mathrm{d}y}{\displaystyle\iint \left(E_x H_y^* - E_y H_x^*\right)\,\mathrm{d}x\,\mathrm{d}y}\,\delta x\,,$$

- TE-polarized semivectorial fields
  $E_x := 0$, $\beta^{-1}\partial_y E_y \approx 0$ assumed for $x = x_b$, $y_0 < y < y_1$,

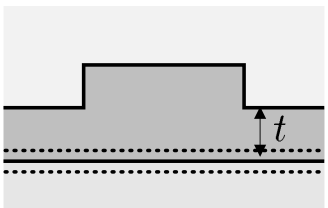
$$\delta\beta = \frac{k_0^2}{2\beta} \frac{(\epsilon^- - \epsilon^+) \displaystyle\int_{y_0}^{y_1} |E_y|^2(x_b, y)\,\mathrm{d}y}{\displaystyle\iint |E_y|^2\mathrm{d}x\,\mathrm{d}y}\,\delta x\,,$$

- TM-polarized semivectorial fields
  $H_x := 0$, $\beta^{-1}\partial_y H_y \approx 0$ assumed for $x = x_b$, $y_0 < y < y_1$,

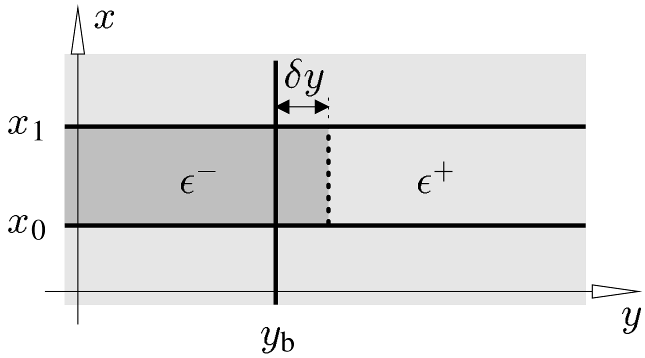
$$\delta\beta = \frac{\beta}{2} \frac{(\epsilon^- - \epsilon^+) \displaystyle\int_{y_0}^{y_1} \left(\frac{1}{\epsilon^-\epsilon^+}|H_y|^2 + \beta^{-2}|\frac{1}{\epsilon}\partial_x H_y|^2\right)(x_b, y)\,\mathrm{d}y}{\displaystyle\iint \frac{1}{\epsilon}|H_y|^2\,\mathrm{d}x\,\mathrm{d}y}\,\delta x\,.$$



$\rightsquigarrow \quad \partial_h\beta$



$\rightsquigarrow \quad \partial_t\beta$



Vertical boundary shifts:

- hybrid fields

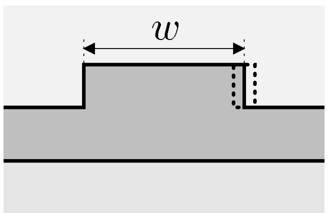
$$\delta\beta = \frac{\omega\epsilon_0}{2}\,\frac{(\epsilon^- - \epsilon^+)\displaystyle\int_{x_0}^{x_1}\Big(|E_x|^2 + (E_y^+)^* E_y^- + |E_z|^2\Big)(x, y_{\mathsf{b}})\,\mathsf{d}x}{\displaystyle\iint \Big(E_x H_y^* - E_y H_x^*\Big)\,\mathsf{d}x\,\mathsf{d}y}\,\delta y\,,$$

- TE-polarized semivectorial fields
  $E_x := 0$

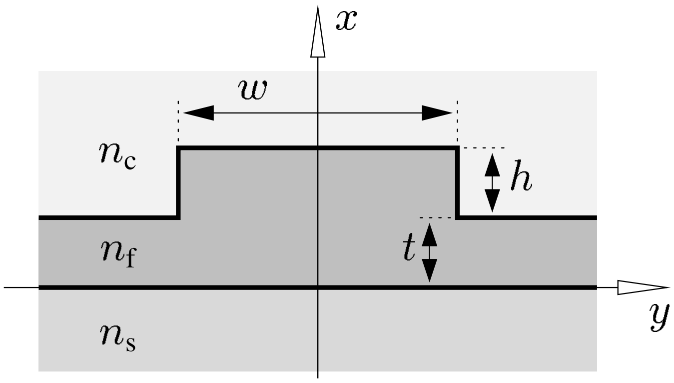
$$\delta\beta = \frac{k_0^2}{2\beta}\,\frac{(\epsilon^- - \epsilon^+)\displaystyle\int_{x_0}^{x_1}\Big((E_y^+)^* E_y^- + \beta^{-2}|\partial_y E_y|^2\Big)(x, y_{\mathsf{b}})\,\mathsf{d}x}{\displaystyle\iint \Big(|E_y|^2 - \beta^{-2} E_y \partial_y^2 E_y^*\Big)\,\mathsf{d}x\,\mathsf{d}y}\,\delta y\,,$$

- TM-polarized semivectorial fields
  $H_x := 0$, $(?)$

$$\delta\beta = \frac{k_0^2}{2\beta}\,\frac{(\epsilon^- - \epsilon^+)\displaystyle\int_{x_0}^{x_1}|H_y|^2(x, y_{\mathsf{b}})\,\mathsf{d}x}{\displaystyle\iint |H_y|^2\mathsf{d}x\,\mathsf{d}y}\,\delta y\,.$$



$\rightsquigarrow \quad \partial_w\beta.$



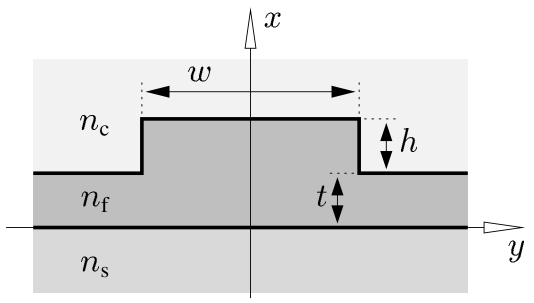
- Refractive indices:
  Use common formulas for small uniform bulk perturbations
  $\rightsquigarrow \quad \partial_{n_s}\beta$, $\partial_{n_f}\beta$, $\partial_{n_c}\beta$.

- Wavelength:

$$\beta(a\lambda, ah, aw, at) = \frac{1}{a}\beta(\lambda, h, w, t)$$

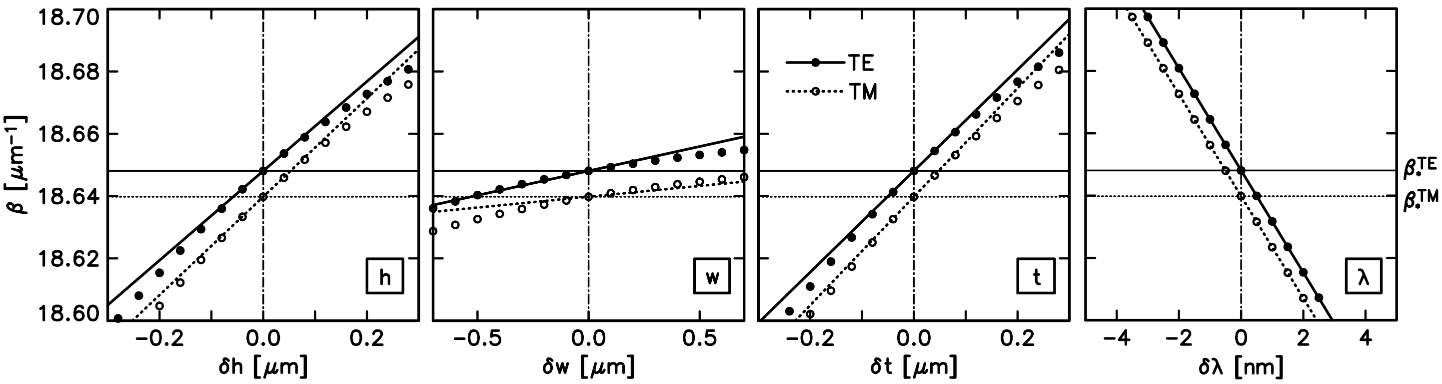
$$\rightsquigarrow \quad \partial_\lambda \beta = -\frac{1}{\lambda}\,(\beta + h\partial_h\beta + w\partial_w\beta + t\partial_t\beta).$$

$\partial_h\beta$, $\partial_w\beta$, $\partial_t\beta$, $\partial_{n_s}\beta$, $\partial_{n_f}\beta$, $\partial_{n_c}\beta$, and $\partial_\lambda\beta$ are at hand.



$n_s = 3.40$, $n_f = 3.44$, $n_c = 1.0$,
$h = 0.5\,\mu$m, $w = 3.0\,\mu$m, $t = 0.5\,\mu$m,
$\lambda = 1.15\,\mu$m

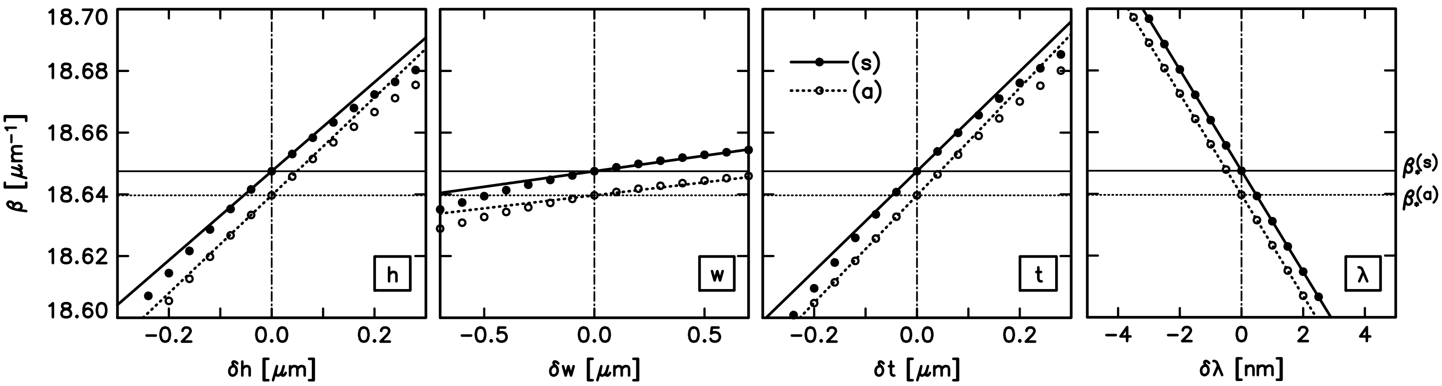
• Semivectorial fields, fundamental modes:



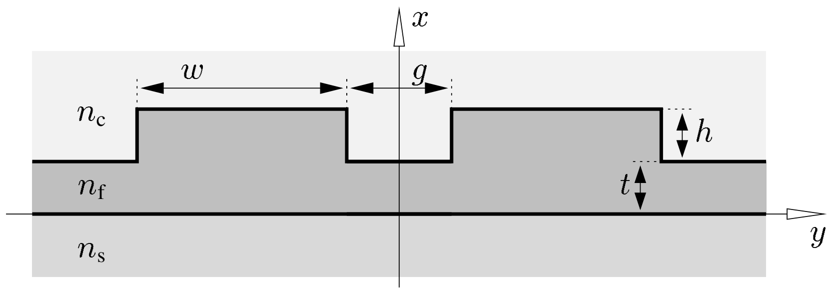
○ ○ ○ ● ● ● : direct computation $\beta(h)$, $\beta(w)$, $\beta(t)$, $\beta(\lambda)$

⋯⋯ ——— : $\beta(q_\circ) + (\delta q)\ \partial_q\beta|_{q_\circ}$, $q \in \{h, w, t, \lambda\}$
with $\partial_q\beta$ evaluated by the perturbation expressions

• Vectorial fields, fundamental modes of definite symmetry:

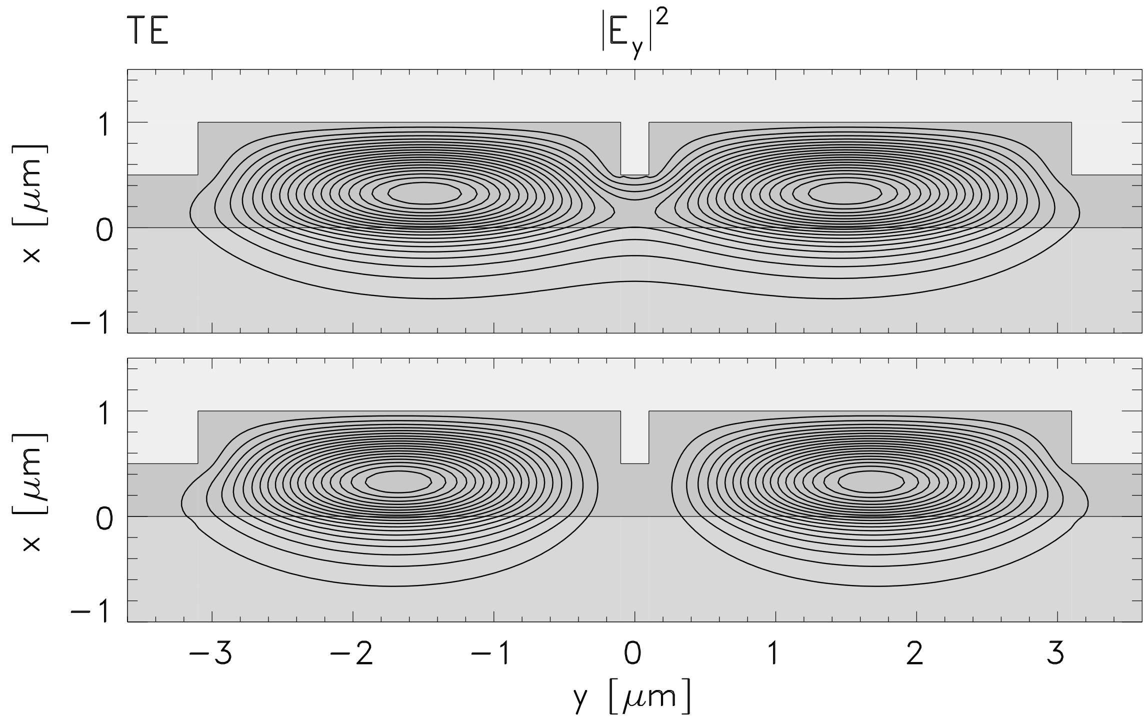


	Symmetry $y \to -y$		Polarization-character
	H_x, E_y, H_z	E_x, H_y, E_z	
(s)	even	odd	TE
(a)	odd	even	TM



$n_s = 3.40$, $n_f = 3.44$,
$n_c = 1.0$, $\lambda = 1.15\,\mu$m,
$h = 0.5\,\mu$m, $w = 3.0\,\mu$m,
$t = 0.5\,\mu$m, $g = 0.2\,\mu$m

Mode profiles:



Power transmission:

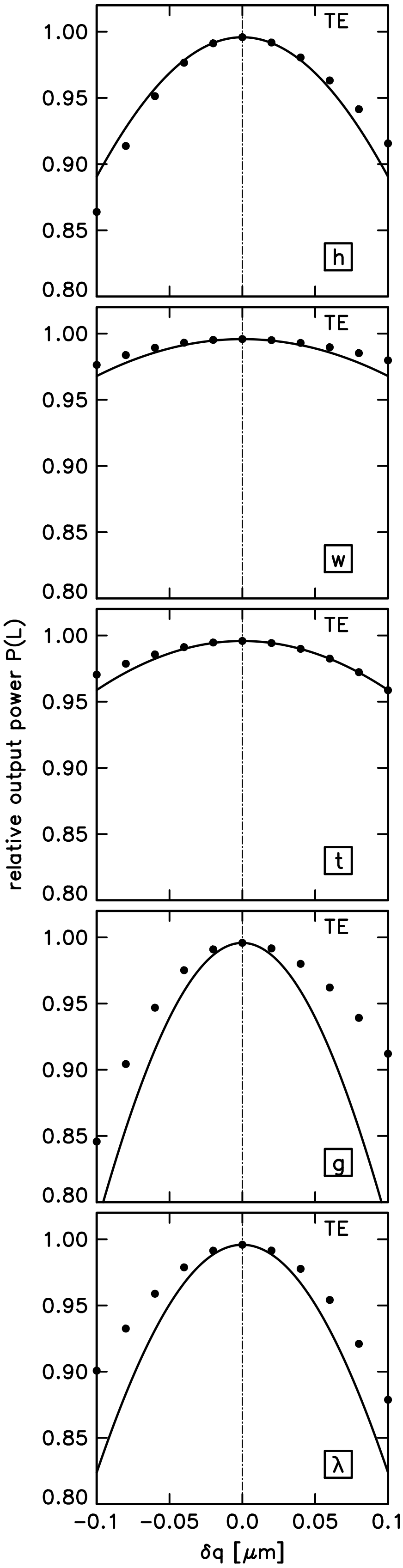
ϕ_l, ϕ_r: the modes of the isolated left and right channels
ψ_s, ψ_a: the symmetric/antisymmetric supermode
β_s, β_a: their propagation constants
$\langle\, .\, ,\, .\, \rangle$: a suitable bilinear product

$\rightsquigarrow$ power output to ϕ_r at $z = L$, excitation by ϕ_l at $z = 0$:

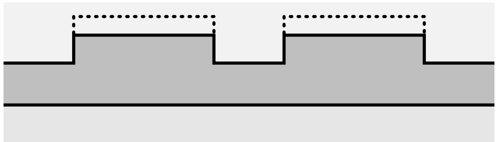
$$P(L) = w_s^2 + w_a^2 + 2w_s w_a \cos(\beta_s - \beta_a)L$$
$$w_j = \langle \phi_r, \psi_j \rangle\, \langle \psi_j, \phi_l \rangle,\ \ j = s, a, \quad w_s w_a < 0.$$

Coupling length: $L_c = \pi/(\beta_s - \beta_a) = 650\,\mu$m

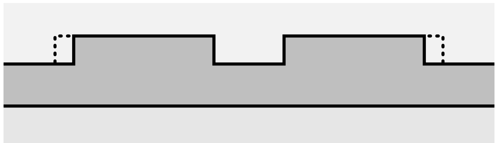
Influence of geometry alterations δq on the power transmission:



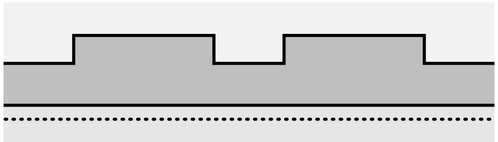
$q = h$:



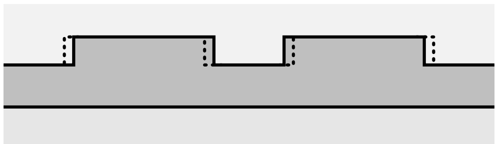
$q = w$:



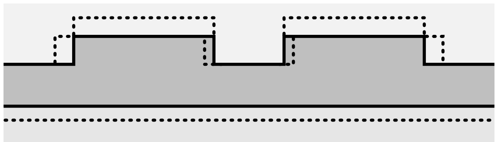
$q = t$:



$q = g$:



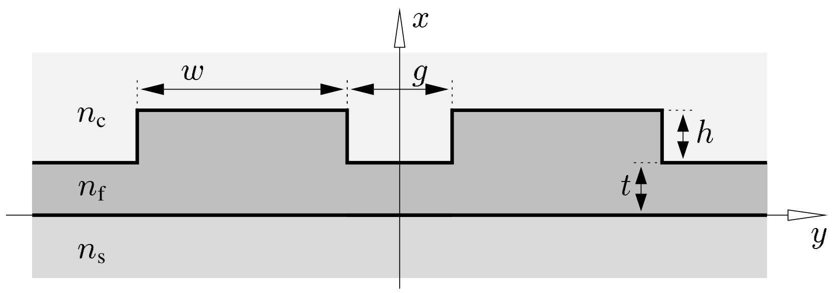
$q = \lambda$:



Markers: $P(L)$ evaluated with directly calculated $\beta(q + \delta q)$

Lines: $P(L)$ evaluated with perturbational estimates $\beta(q) + \delta q \, \partial_q \beta$

$$P(L) = w_\mathsf{s}^2 + w_\mathsf{a}^2 + 2 w_\mathsf{s} w_\mathsf{a} \cos(\beta_\mathsf{s} - \beta_\mathsf{a}) L$$



$n_s = 3.40$, $n_f = 3.44$,
$n_c = 1.0$, $\lambda = 1.15\,\mu$m,
$h = 0.5\,\mu$m, $w = 3.0\,\mu$m,
$t = 0.5\,\mu$m, $g = 0.2\,\mu$m

Tolerances:

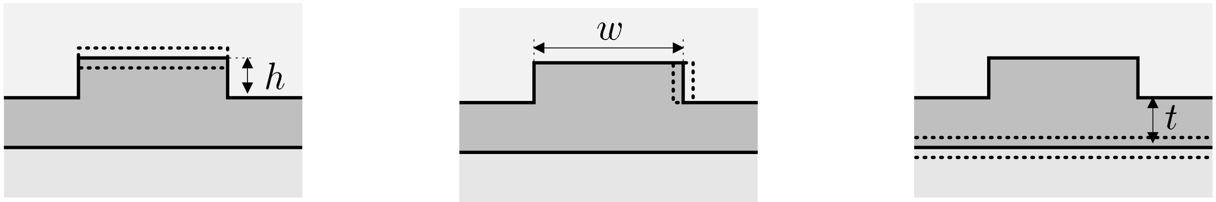
Assume $|w_s| \approx |w_a| \approx 1/2$; $P_q(L)$ depends on a parameter q mainly via the propagation constants.
$\rightsquigarrow$ Expanding $P_{q+\delta q}(L)$ yields tolerance formulas for
$q \in \{h, w, t, g, \lambda, n_s, n_f, n_c\}$:

At $L = L_c$:

$$\Delta q = \frac{2\sqrt{\Delta P}}{L|\partial_q\beta_s - \partial_q\beta_a|},$$

$$P_{q+\delta q} > P_q - \Delta P \quad \text{for} \quad -\Delta q < \delta q < \Delta q.$$

TE		$\frac{\partial_h\beta}{\mu\text{m}^{-2}}$	$\frac{\partial_w\beta}{\mu\text{m}^{-2}}$	$\frac{\partial_t\beta}{\mu\text{m}^{-2}}$	$\frac{\partial_g\beta}{\mu\text{m}^{-2}}$	$\frac{\partial_\lambda\beta}{\mu\text{m}^{-2}}$
(s)	FD	0.137	0.010	0.168	−0.006	−16.382
	PT	0.138	0.013	0.167	−0.009	−16.398
(a)	FD	0.147	0.015	0.162	0.004	−16.392
	PT	0.148	0.018	0.161	0.005	−16.411
$\Delta P = 1\%$		Δh/nm	Δw/nm	Δt/nm	Δg/nm	$\Delta\lambda$/nm
$L = L_c$	NUM	29	74	52	28	30
	PT	30	59	51	21	23



- Propagation constant changes with respect to geometry alterations can be estimated by line integrals of the mode fields along relevant dielectric interfaces.

- Comparison of gradients from equivalent boundary shifts and comparison with conventionally computed derivatives provide a self consistency test for the underlying mode solver.

- Accuracy is sufficient to yield reasonable tolerance estimates for our simple coupler, from one mode analysis only.

- With the geometry perturbation formulas and analogous expressions for refractive indices and wavelength, derivatives of the propagation constants with respect to all relevant parameters are at hand. Regarding the automated optimization of a device geometry, this should reduce the computational effort significantly.

- Guidelines for geometry tolerant devices:

$$\Delta q = \frac{2\sqrt{\Delta P}}{L|\partial_q \beta_{\mathsf{s}} - \partial_q \beta_{\mathsf{a}}|}\,, \quad \delta\beta = \frac{k_0^2}{2\beta}\, \frac{(\epsilon^- - \epsilon^+) \displaystyle\int_{y_0}^{y_1} |E_y|^2(x_{\mathsf{b}}, y)\, \mathrm{d}y}{\displaystyle\iint |E_y|^2 \mathrm{d}x\, \mathrm{d}y}\, \delta x\,.$$

  The total length should be kept short, intensities on dielectric surfaces and refractive index contrasts must be low, and only modes with similar profiles should interfere.